The distortive costs of income taxation

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We determine the marginal distortive costs of raising marginal income taxes, allowing for both intensive and extensive responses in taxable income. Contrary to earlier literature, we provide these costs for marginal tax rates at every point in the income distribution, while duly taking account of nonlinearities in the tax schedule. We present results for the United States in a way we believe is more informative for actual tax reform than traditional optimal tax simulations.

1 Introduction

Modern tax systems are characterized by many nonlinearities in their income tax schedules. Effective marginal tax rates vary from one income level to the other due to a large number of different tax policies that each have a distinct impact on the shape of the income tax schedule – think tax brackets, tax credits, income-contingent transfers, and more. Consequently, policy makers have many instruments at their disposal that all affect marginal tax rates in different ways. This yields an abundance of issues to consider when contemplating tax reform: which tax instruments should be adjusted? how should they be adjusted? given a limited capacity for reform, which reforms should be prioritized over others? The

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distortive costs of marginal taxes are (or at least ought to be) a crucial input to answering any of these questions. All else equal, one would like to adjust tax instruments in a way that minimizes the economic costs of tax distortions, and prioritize those tax reforms that are expected to yield the largest reduction (or smallest increase) in the total dead-weight loss.

It is therefore striking that there are – to the best of our knowledge – no estimates of tax distortions that properly take into account the dual facts that (i) tax schedules are typically nonlinear and (ii) most tax reforms have the potential to affect marginal tax rates at various levels of taxable income. While there are studies that take into account the fact that tax schedules are nonlinear, these typically only determine the marginal dead-weight loss of simultaneously raising marginal tax rates at every point of the income distribution (e.g., Blomquist and Simula, 2016). While there are studies that provide multiple marginal dead-weight loss estimates for more limited income intervals, these typically do not take into account the fact that tax schedules are nonlinear (e.g., Kleven and Kreiner, 2006; Eissa, Kleven, and Kreiner, 2008).

In this paper, we derive the marginal dead-weight loss associated with raising marginal income tax rates. Specifically, we provide an answer to the following question. How large are the tax-revenue losses associated with the behavioral responses to an increase in a marginal tax rate that redistributes one unit of resources from the private sector to the public sector? We allow for behavioral responses of taxable income at the intensive margin (both income and substitution effects) and the extensive margin (labor participation). We moreover take into account the fact that different groups in the population face different tax schedules based on their marital status and number of dependents. We determine the distortive costs of taxation at every point in the income distribution. By plotting distortive costs against taxable income, we obtain a straightforward guide to tax reform. For an efficient tax system, the distortive costs must be below one for all income levels. At any income level for which the distortive costs exceed one, a reduction in marginal taxes yields a Pareto improvement. Moreover, as long as policy makers care weakly more about the income of poor individuals than about the income of rich individuals, the distortive costs must be positive and weakly increasing with income. Thus, marginal

taxes ought to be raised at interior local minimums and lowered at local maximums.

Related literature... Longstanding literature on the measurement of dead-weight loss (e.g., Harberger, 1964a,b; Auerbach, 1985; Auerbach and Hines, 2002; Kleven and Kreiner, 2006; Eissa, Kleven, and Kreiner, 2008; Saez, Slemrod, and Giertz, 2012; Blomquist and Simula, 2016). Traditional literature on optimal taxation that bases policy implications on simulations for given social preferences (e.g., Mirrlees, 1971; Tuomala, 1990; Saez, 2001, 2002; Blundell and Shephard, 2012; Jacquet, Lehmann, and Van der Linden, 2013; Jacquet and Lehmann, 2015). Recent studies deriving implicit welfare weights (e.g., Bourguignon and Spadaro, 2012; Bargain and Keane, 2010; Bargain et al., 2010; Hendren, 2014; Zoutman, Jacobs, and Jongen, 2016).

Road map...

2 Theory

Imagine that the government wants to raise the marginal tax rate at a certain level of income, such that it raises a total of one dollar of revenue from the people who earn more than that level of income. How much revenue is lost due to the behavioral responses associated with this change in the tax rate? In this section, we derive the answer to this question in a relatively informal way. The Appendix provides the formal underpinning of the results.

2.1 Substitution effects

We assume there is a continuum of individuals with mass one. This continuum can be partitioned in I groups that each face a different tax schedule – think of different combinations of marital status and numbers of dependants. Individuals in group $i \in \{1, ..., I\}$ face the tax schedule $T^i(z)$, which is a function of taxable income z. It is assumed that $T^i(z)$ is twice differentiable for all i and z. The income levels of individuals in group i are distributed over support $[0, \infty]$ according to the cumulative distribution function $H^i(z)$ with corresponding density $h^i(z) \equiv \partial H^i(z)/\partial z^i$. Consider

raising the marginal tax rate for group i by $\mathrm{d}\kappa$ over an infinitesimally small interval δz in the neighborhood of income level z^*. This raises $\delta z \mathrm{d}\kappa$ additional tax revenue from every group-i individual earning more than z^*. The mechanical increase in tax revenue is given by:

$$(1) \qquad\qquad (1 - H^i(z^*)) \cdot \delta z \mathrm{d}\kappa,$$

Thus, from all $1 - H^i(z^*)$ group-i individuals who earn more than z^*, the reform raises $\delta z \mathrm{d}\kappa$ units of tax revenue. Notice that eq. (1) merely reflects a transfer of resources from the private to the public sector, and therefore does not reflect any distortive costs.

At the same time, the reform generates behavioral responses as individuals adjust their taxable income to the changes in the tax schedule. We first focus on the substitution effects of the tax reform, i.e., on the behavioral responses of the individuals with income z^* who see their marginal tax rate increase. The distortive effect of the tax reform is given by the loss in tax revenue due to such behavioral responses. It equals:

$$(2) \qquad\qquad T_z^i(z^*) \cdot \frac{z^* \bar{e}_c^i(z^*)}{1 - T_z^i(z^*)} \cdot h^i(z^*)\delta z \cdot \mathrm{d}\kappa,$$

with $T_z^i(z) \equiv \partial T^i(z)/\partial z$ the marginal tax rate and $\bar{e}_c^i(z)$ the group-i average compensated net-of-tax rate elasticity of taxable income at income level z.[1] In words: the reform raises marginal tax rates by $\mathrm{d}\kappa$ for $h^i(z^*)\delta z$ individuals. For every unit increase in the marginal tax rate, these individuals reduce their taxable income by $z^* \bar{e}_c^i(z)/(1 - T_z^i(z^*))$. And for every unit reduction in taxable income, tax revenue declines by $T_z^i(z^*)$. Multiplying these terms yields the expression in eq. (2).

Now consider the case that $\mathrm{d}\kappa$ is such that the reform raises exactly one unit of income from the totality of group-i individuals earning more than z^*. Eq. (1) then implies that $\delta z \mathrm{d}\kappa = 1/(1 - H^i(z^*))$. Substituting into eq. (2) and rearranging yields the following expression for the distortive costs

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[1]These elasticities represent behavioral responses along the nonlinear budget curve (cf. Jacquet, Lehmann, and Van der Linden, 2013) rather than elasticities along a linearized virtual budget line (Saez, 2001). With knowledge on the curvature of the tax schedule (i.e., the first and second derivatives of the tax function) it is straightforward to express the former in terms of the latter; also see the discussion in Gerritsen (2016); Blomquist and Simula (2016).

of the reform:

$$(3) \qquad \Delta_S^i(z^*) \equiv \frac{z^* h^i(z^*)}{1 - H^i(z^*)} \cdot \frac{T_z^i(z^*)}{1 - T_z^i(z^*)} \cdot \bar{e}_c^i(z^*).$$

The subscript S in $\Delta_S^i(\cdot)$ stresses the fact that it refers to the tax hike's substitution effects. Eq. (3) gives the substitution effects' distortive costs of raising one unit of income from group-i individuals with income above z^*. Notice that, to determine these costs empirically, all we need is information on the income distribution, the tax schedule, and the compensated elasticity.

2.2 Income effects

An increase in marginal tax rates might affect taxable income through income effects as well as substitution effects. Indeed, the $\delta z \mathrm{d}\kappa$-reform might generate income effects for all group-i individuals who earn more than z^*. These income effects affect tax revenue. The tax revenue *costs* due to this effect are given by:

$$(4) \qquad \int_{z^*}^{\infty} \Big(T_z^i(z) \cdot \bar{\eta}^i(z) \Big) \mathrm{d}H^i(z) \cdot \delta z \mathrm{d}\kappa,$$

where $\bar{\eta}^i(z)$ gives the group-i average effect of net disposable income on taxable income at income level z.[2] In words: the reform raises $\delta z \mathrm{d}\kappa$ units of income from all individuals with income above z^*. A unit reduction in their disposable income leads to an $\bar{\eta}^i(z)$ reduction in taxable income. Every unit reduction in taxable income leads to a $T_z^i(z)$ reduction in tax revenue. Multiplying terms and integrating over all individuals who earn more than z^* yields the expression in eq. (4). We typically observe that higher disposable income leads to a drop in taxable income. In that case, $\bar{\eta}^i(z) < 0$ and the income effects of a tax increase yield revenue gains as long as marginal taxes are positive.

Again, we consider $\delta z \mathrm{d}\kappa = 1/(1 - H^i(z^*))$. This implies that the revenue

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[2]As with the compensated elasticity, this income effect denotes behavioral responses along the nonlinear budget curve.

loss from the income effects is given by:

$$(5) \qquad \Delta_I^i(z^*) \equiv \frac{\int_{z^*}^{\infty} T_z^i(z) \bar{\eta}^i(z) \mathrm{d}H^i(z)}{1 - H^i(z^*)},$$

where the subscript I in $\Delta_I^i(z)$ stresses the fact that it refers to the tax hike's income effects. Thus, eq. (5) measures the distortive costs associated with the income effects of raising one unit of tax revenue from group-i individuals with income above z^*. As long as $\bar{\eta}^i(z) \leq 0$ and $T(z) \geq 0$ for all $z > z^*$, then $\Delta_I^i(z^*) \leq 0$ and the income effects result in a tax-revenue gain. With knowledge of the income distribution, the tax schedule, and income effects, we can determine these distortions empirically.

2.3 Participation effects

Now consider the existence of participation effects. This implies that the $\delta z \mathrm{d}\kappa$-reform reduces labor market participation among group-i individuals who earn more than z^*. Taxes paid by non-participants are denoted by $T(0)$. The tax revenue losses of the participation response are given by:

$$(6) \qquad \int_{z^*}^{\infty} \Big((T(z) - T(0)) \cdot \zeta^i(z) \Big) \mathrm{d}H^i(z) \cdot \delta z \mathrm{d}\kappa,$$

with $\zeta^i(z)$ the average group-i change in the participation probability of (or the percentage change in) individuals who earn taxable income z due to an increase in income when working. In words: the reform raises $\delta z \mathrm{d}\kappa$ units of income from all individuals with income above z^*. A unit reduction in their disposable income leads a share $\zeta^i(z)$ of individuals with income z to stop working. Every individual who stops working generates a tax revenue loss equal to $T^i(z) - T^i(0)$. Multiplying terms and integrating over all individuals who earn more than z^* yields the expression in eq. (6).

Again, we consider $\delta z \mathrm{d}\kappa = 1/(1 - H^i(z^*))$. This implies that the tax-revenue losses from the participation effect are given by:

$$(7) \qquad \Delta_P^i(z^*) \equiv \frac{\int_{z^*}^{\infty} (T(z) - T(0)) \zeta^i(z) \mathrm{d}H^i(z)}{1 - H^i(z^*)},$$

where the subscript P in $\Delta_P^i(z)$ stresses the fact that it refers to the partic-

ipation effect. Thus, eq. (7) measures the distortive costs, associated with the participation effect, of raising one unit of tax revenue from group-i individuals with income above z^*.

2.4 Total distortive costs

Total distortive effects of the reform are given by:

$$(8) \qquad \Delta^i(z^*) \equiv \Delta^i_S(z^*) + \Delta^i_I(z^*) + \Delta^i_P(z^*),$$

which in the optimum equals $1 - \bar{a}^i_{z>z^*}$.

2.5 Aggregating groups

Denote the population share of group-i individuals as n^i. In that case, we can write the distortive costs of raising the marginal tax revenue for all individuals at z^* as:

$$(9) \qquad \Delta(z^*) \equiv \frac{\sum_{i=1}^I n^i(1 - H^i(z^*))\Delta^i(z^*)}{\sum_{i=1}^I n^i(1 - H^i(z^*))}.$$

2.6 Discussion

A discussion on why $\Delta^i(z)$ and $\Delta(z)$ provide useful measures of the distortiveness of marginal taxes.

3 Simulations

Calibrating $\Delta^i(z^*)$ for various groups on the basis of U.S. data.

3.1 Elasticities

The average compensated elasticity $\bar{e}^i_c(z)$ represents changes in taxable income along the nonlinear budget curve. Empirical studies typically instrument for the marginal tax rate when estimating elasticities. As a result,

these estimates represent changes along a linear 'virtual' budget line. Denoting this virtual elasticity as $\hat{e}_c^i(z)$, we can write:

$$(10) \qquad \left(1 + \frac{T_{zz}^i(z)z}{1 - T_z^i(z)}\hat{e}_c^i(z)\right)\bar{e}_c^i(z) = \hat{e}_c^i(z).$$

Similarly, for the income effect we can write:

$$(11) \qquad \left(1 + \frac{T_{zz}^i(z)z}{1 - T_z^i(z)}\hat{e}_c^i(z)\right)\bar{\eta}^i(z) = \hat{\eta}^i(z),$$

where $\hat{\eta}^i(z) = (1 - T_z^i(z))(\hat{e}_u^i(z) - \hat{e}_c^i(z))$, with $\hat{e}_u^i(z)$ the uncompensated net-of-tax rate elasticity of taxable income, defined as a change along the virtual budget line.

Substituting for $\bar{e}_c^i(z)$ and $\bar{\eta}^i(z)$ from eqs. (10) and (11) into eqs. (3) and (5), we thus get the distortive costs in terms of first and second derivatives of the tax schedule, the c.d.f. and p.d.f. of the income distribution, and estimates of compensated and uncompensated elasticities along a virtual budget line. [AG: what are good estimates for the participation response?]

3.2 Smoothing the tax schedule and the income distribution

We obtain data on the income distribution from the CPS labor extracts. This is complemented by data on tax schedules from the NBER Tax Calculator TAXSIM.

3.3 Results

4 Conclusion

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Appendix

There is a mass-one continuum of individuals. Every individual is part of a specific demographic group that is denoted by $j \in \{1, ..., J\}$. Any demographic group j consists of a continuum of individuals denoted by $i^j \in I^j$. The group of any individual is exogenously given. Every individual decides to participate in the labor market or not. The participation of individual i^j is represented by the indicator variable $p^{ij} \in \{0, 1\}$, which takes on the value 1 in case of participation and the value 0 in case of non-participation. Non-participants earn zero income. Conditional on participation, individual i^j decides to earn z^{ij}. Thus, income of individual i^j equals $p^{ij} z^{ij}$.

Every demographic group j faces its own tax schedule. The participation-conditional tax burden of individual i^j is affected by his income z^{ij} as well as by potential reforms to the tax schedule. To capture this, we write his tax burden as:

$$(12) \qquad T^j(z^{ij}, \kappa) = \mathcal{T}^j(z^{ij}) + \kappa \tau^j(z^{ij}),$$

where we call $\tau^j(z^{ij})$ the reform function and κ the reform parameter. Both $\tau^j(z^{ij})$ and κ may take on any arbitrary value, whereas $\mathcal{T}^j(z^{ij})$ ensures that $T^j(z^{ij})$ equals any pre-reform tax schedule. We can study the marginal effects of a policy reform that raises the tax schedule by $\tau^j(z^{ij})$ by considering a small change $d\kappa$.

We assume that participation-conditional income z^{ij} is differentiable in κ for all $i^j \in I^j$. In other words: marginal changes in the tax schedule lead to marginal changes in individuals' income conditional on participation. This implies that we can simply take the derivative of eq. (12) to obtain the effects of a tax reform on on an individual's participation-conditional tax burden:

$$(13) \qquad dT^j(z^{ij}, \kappa) = \left(\tau^j(z^{ij}) + T^j_z(z^{ij}, \kappa) \frac{dz^{ij}}{d\kappa} \right) d\kappa.$$

We furthermore assume that individuals only adjust their labor earnings in response to changes in their *own* marginal and average tax rates. This allows us to write $dz^{ij}/d\kappa$ in terms of income and substitution effects. To

this end, we first introduce the following elasticity concepts:

$$(14) \qquad e_c^{ij} \;\equiv\; \frac{1 - T_z^j(z^{ij}, \kappa)}{z^{ij}} \left. \frac{\mathrm{d}z^{ij}}{-\tau_z^j(z^{ij})\mathrm{d}\kappa} \right|_{\tau^j(z^{ij})=0},$$

$$(15) \qquad \eta^{ij} \;\equiv\; \left(1 - T_z^j(z^{ij}, \kappa)\right) \left. \frac{\mathrm{d}z^{ij}}{-\tau^j(z^{ij})\mathrm{d}\kappa} \right|_{\tau_z^j(z^{ij})=0}.$$

This notation, along with the assumption that income of individual i^j is only affected by the 'own' tax rates, allows us to write:

$$(16) \qquad \frac{\mathrm{d}z^{ij}}{\mathrm{d}\kappa} = -\frac{z^{ij}}{1 - T_z^j(z^{ij}, \kappa)} \left(e_c^{ij}\tau_z^j(z^{ij}) + \eta^{ij}\frac{\tau^j(z^{ij})}{z^{ij}} \right),$$

and thus:

$$(17)$$
$$\mathrm{d}T^j(z^{ij}, \kappa) = \left(\tau^j(z^{ij}) - \frac{T_z^j(z^{ij}, \kappa)}{1 - T_z^j(z^{ij}, \kappa)} \left(z^{ij}e_c^{ij}\tau_z^j(z^{ij}) + \eta^{ij}\tau^j(z^{ij}) \right) \right) \mathrm{d}\kappa.$$

Total tax revenue from demographic group j is given by:

$$(18) \qquad R^j \equiv \int_{I^j} \left(p^{ij}T^j(z^{ij}, \kappa) + (1 - p^{ij})T^j(0, \kappa) \right) \mathrm{d}i^j.$$

We write the share of group-j individuals with participation-conditional income z^{ij} that decide to participate as π^{ij}. Thus, we can write:

$$(19) \qquad R^j = \int_I \left(\pi^{ij}T^j(z^{ij}, \kappa) + (1 - \pi^{ij})T^j(0, \kappa) \right) \mathrm{d}i.$$

Denoting population shares for demographic group j by n^j such that $n^1 + \ldots + n^J = 1$, we can write total tax revenue as:

$$(20) \qquad R \equiv \sum_{j=1}^{J} n^j \int_{I^j} \left(\pi^{ij}T^j(z^{ij}, \kappa) + (1 - \pi^{ij})T^j(0, \kappa) \right) \mathrm{d}i^j.$$

Taking the derivative with respect to κ yields the tax-revenue effects of a reform:

$$(21)$$
$$\mathrm{d}R \equiv \sum_{j=1}^{J} n^j \int_{I^j} \left(\pi^{ij} \left(\tau^j(z^{ij}) - \frac{T_z^j(z^{ij}, \kappa)}{1 - T_z^j(z^{ij}, \kappa)} \left(z^{ij}e_c^{ij}\tau_z^j(z^{ij}) + \eta^{ij}\tau^j(z^{ij}) \right) \right) + (1 - \pi^j) \left(\tau^j(0) \right) + \left($$

We focus on marginal tax reforms in the interior of the income distribution and therefore set $\tau^j(0) = 0$. We furthermore denote the semi-elasticity of participation as the percentage increase in participation due to a unit increase of in-work benefits:

$$(22) \qquad \zeta^{ij} \equiv \frac{1}{\pi^{ij}} \frac{\mathrm{d}\pi^{ij}}{-\tau^j(z^{ij})\mathrm{d}\kappa}.$$

Assuming that the participation decision is only determined by the amount of in-work benefits, we can write:

$$(23) \qquad \frac{\mathrm{d}\pi^{ij}}{\mathrm{d}\kappa} = -\pi^{ij}\zeta^{ij}\tau^j(z^{ij}).$$

Substituting this back into eq. (21) and rearranging yields:

(24)
$$\mathrm{d}R \equiv \sum_{j=1}^{J} n^j \int_{I^j} \pi^{ij} \left(\tau^j(z^{ij}) - \frac{T_z^j(z^{ij})}{1 - T_z^j(z^{ij})} z^{ij} e_c^{ij} \tau_z^j(z^{ij}) - \left(\frac{T_z^j(z^{ij})}{1 - T_z^j(z^{ij})} \eta^{ij} + \left(T^j(z^{ij}) - T^j(0)\right) \zeta^{ij} \right) \right)$$

where we suppressed the function argument κ for brevity.

Now imagine we raise the marginal tax at z^* by $\tau_z^j(z^*)\mathrm{d}\kappa$ for all groups j such that all individuals with income above z^* pay additional taxes $\tau^j(z^{ij})\mathrm{d}\kappa = \mathrm{d}\kappa$. This implies that $\tau^j(z^{ij}) = 1$ for all $z^{ij} > z^*$ and for all j, and $\tau_z^j(z^*)\mathrm{d}\kappa = \mathrm{d}\kappa/\mathrm{d}z$ for all j. Thus, we can write:

(25)
$$\mathrm{d}R \equiv \sum_{j=1}^{J} n^j \left[\int_{I^j:z^{ij}>z^*} \pi^{ij} \left(1 - \left(\frac{T_z^j(z^{ij})}{1 - T_z^j(z^{ij})} \eta^{ij} + \left(T^j(z^{ij}) - T^j(0)\right) \zeta^{ij} \right) \right) \mathrm{d}i^j - \int_{I^j:z^{ij}=z^*} \pi^{ij} \frac{T}{1-} \right.$$

Or, written in terms of group-specific cdfs $H^j(z) \equiv \int_{I^j:z^{ij}<z} \mathrm{d}i^j$ and pdfs $h^j(z) \equiv \mathrm{d}H_z^j(z)/\mathrm{d}z = \int_{I^j:z^{ij}=z} \mathrm{d}i^j/\mathrm{d}z$:

(26)
$$\mathrm{d}R \equiv \sum_{j=1}^{J} n^j \left[\int_{z^*}^{\infty} \pi^{ij} \left(1 - \left(\frac{T_z^j(z^{ij})}{1 - T_z^j(z^{ij})} \eta^{ij} + \left(T^j(z^{ij}) - T^j(0)\right) \zeta^{ij} \right) \right) \mathrm{d}H^j(z^{ij}) - \pi^{*j} \frac{T_z^j(z^*)}{1 - T_z^j(z^*)} \right.$$

Thus, the mechanical revenue gain equals:

$$(27) \qquad \sum_{j=1}^{J} n^j \left[\frac{\int_{z^*}^{\infty} \pi^{ij} \mathrm{d}H^j(z^{ij})}{1 - H^j(z^{ij})} \right] (1 - H^j(z^{ij})) \equiv \sum_{j=1}^{J} n^j \bar{\pi}^{*j}(1 - H^j(z^{ij})).$$

13

The distortive costs due to the substitution effect are given by:

$$(28) \qquad \sum_{j=1}^{J} n^j \pi^{*j} \frac{T_z^j(z^*)}{1 - T_z^j(z^*)} z^* \bar{e}_c^{*j} h^j(z^*)$$

Thus, under the condition of full participation (gotta fix this), we get that the distortive costs per euro of mechanical tax revenue due to the substitution effect equals:

$$(29) \qquad \frac{1}{\sum_{j=1}^{J} n^j (1 - H^j(z^*))} \sum_{j=1}^{J} n^j \frac{T_z^j(z^*)}{1 - T_z^j(z^*)} h^j(z^*) z^* \bar{e}_c^{*j}$$

Write the 'total' cdf as:

$$(30) \qquad H(z) = \sum_{j=1}^{J} n^j H(z) / \sum_{j=1}^{J} n^j \qquad \Leftrightarrow \qquad h(z) = \sum_{j=1}^{J} n^j h(z) / \sum_{j=1}^{J} n^j$$

and the average marginal tax wedge at a certain income level as:

$$(31) \qquad \overline{\frac{T_z^j(z^*)}{1 - T_z^j(z^*)}} = \frac{1}{\sum_{j=1}^{J} n^j h^j(z^*)} \sum_{j=1}^{J} n^j h^j(z^*) \frac{T_z^j(z^*)}{1 - T_z^j(z^*)}$$

Then we get:

$$(32) \qquad \frac{h(z^*) z^*}{1 - H(z^*)} \overline{\frac{T_z^j(z^*)}{1 - T_z^j(z^*)}} \bar{e}_c^{*j}$$