Sizing Up the Aggregate Frisch Elasticity from the U.S. Budget Sequestration "Experiment"*

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Abstract

The aggregate Frisch elasticity is critical for the evaluation of the outcomes of alternative fiscal and monetary policies as well as of competing interpretations of important macroeconomic phenomena, but consensus about its magnitude remains elusive. The temporary government spending cuts initiated by a 2013 U.S. budget sequestration procedure had experiment-like features almost ideal for measuring that elasticity from the effects that that policy development induced on macroeconomic variables. The paper inspects those effects with a novel methodology that borrows its main elements from the “event study” and Business Cycle Accounting traditions. Its main finding is that studies that calibrate the aggregate Frisch elasticity to the budget sequestration evidence should favor values for that elasticity in the low end of the range of those that have been proposed as empirically plausible in the literature.


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1 Introduction

The aggregate intertemporal labor-supply elasticity plays a prominent role in controlling the transitional dynamics of macroeconomic variables in response to a variety of shocks. Its exact value is, therefore, critical for the empirical relevance of alternative interpretations of many macroeconomic phenomena. For example, the view that a significant fraction of economic fluctuations at business cycle frequencies is accounted for by total factor productivity shocks is consistent with the evidence only if the intertemporal labor substitution elasticity is large. Other critical policy-related questions have a stake on the issue as well, because the value of that elasticity is key not only for correctly estimating the fraction of tax cuts eventually self-financed through endogenous labor supply feedback effects, but also for establishing the empirical relevance of the expectationally driven liquidity traps studied by Mertens and Ravn (2014).\(^1\)

Noticing the absence of empirical estimates of the aggregate intertemporal labor substitution elasticity, despite its importance for answering many macroeconomic questions, Lucas and Rapping (1969) provided the first one that initiated a voluminous research agenda in subsequent years. The disparity of estimates of that elasticity obtained since then has been disconcerting to the profession, as they are too far apart to settle controversies about alternative interpretations of the macroeconomic phenomena or about the effects of economic policies mentioned above. Progress in narrowing down the range of empirically plausible values of the intertemporal labor supply elasticity has been hampered by a number of conceptual misunderstandings and empirical obstacles.

One of the difficulties at the conceptual level has been some confusion about the correspondence between different empirical estimates of the real wage labor supply elasticity and their theoretical counterparts. As made clear by Lucas and Rapping, the elasticity relevant

\(^1\)The connection between the Frisch elasticity and the liquidity traps analyzed by Mertens and Ravn stands out more clearly in the 2010 working paper version of their published one just mentioned.
for the study of the dynamics of macroeconomic variables is that which associates the change in the quantity of man-hours supplied with temporary changes in the real wage, all else equal. More formally, the concept of interest to macroeconomists is the *marginal-utility-of-wealth-held-constant* aggregate labor supply real wage elasticity, or aggregate Frisch elasticity for the sake of brevity.

Accordingly, that is the elasticity that Lucas and Rapping proceeded to infer from aggregate variables in their seminal paper, regardless of whether the observed changes in hours worked were coming from the intensive margin of the households’ labor supply decision (variations in the number of hours worked by already employed individuals) or from the extensive margin of that decision (movements in and out of employment for an indivisible number of hours). But this is not the Frisch elasticity concept reported by many microeconomic studies, often times focused just on the response of hours worked to a temporary real wage change of those already employed. The lack of distinction between the Frisch elasticity of just the intensive margin of the labor supply and that capturing both margins has often times derailed the debates about the empirically relevant value of the aggregate Frisch elasticity, as defined in the previous paragraph.

Those misunderstanding didn’t prevent Prescott (1986) from endorsing the Lucas-Rapping assumption that the aggregate Frisch elasticity could be summarized by a single parameter. Aware, however, of the issues of interpretation potentially raised by not treating the two different margins of the labor supply just described separately, Prescott proposed in that same paper that the value of the parameter capturing the aggregate Frisch elasticity should be set respecting the "calibration principle" that parameter values of models addressing quantitative macroeconomic questions "*cannot be specific to the phenomena being studied.*" The assumption gained further popularity and became widely adopted in the profession after Rogerson (1988) rigorously established its theoretical foundations previously hinted at by Prescott himself.
To facilitate comparison with previous studies, the present paper goes along with the common practice of summarizing both the extensive and intensive margin of the labor supply in a single aggregate Frisch elasticity parameter. Furthermore, in the spirit of Prescott (2004) and Chetty, Guren, Manoli, and Weber (2013), the paper also exploits a fiscal policy development to size up the empirical relevance of alternative values of that elasticity, in the wide range of them that have been argued to possess that attribute in the literature. A brief discussion of the studies by Prescott and by Chetty et al. just mentioned will be helpful to put in perspective the contribution and findings of this paper.

Faithful to the quantitative discipline he had proposed thirty years earlier, Prescott (2004) calibrated the value of the aggregate Frisch elasticity relevant for the study of business cycles to that which accounted for differences in the households’ allocation of time to work across countries with different labor income tax rates. The same calibration approach led Chetty et al. to infer, from one-time fiscal policy regime changes with "natural experiment" features implemented in Iceland in 1987 and in Canada in the 1990s, the value that should be assigned to the aggregate Frisch elasticity in representative agent macroeconomic models.

The findings of these two studies did little to close the gap between existing estimates of that elasticity. Although both exploited fiscal policy to pin down the value of the aggregate Frisch elasticity, Prescott concluded that the cross-country evidence he examined was consistent with a large value of 3.0 for the parameter summarizing that elasticity, while Chetty et al. argued that the evidence for Canada and Iceland they studied suggested "to calibrate representative agent macro models to match a Frisch elasticity of aggregate hours of 0.75." These differences are not trivial, because Prescott (2006) has argued that a Frisch elasticity at least as large as 3.0 is needed for total factor productivity and other real—as opposed to nominal—shocks to contribute significantly to business cycle fluctuations.\(^2\)

\(^2\) Prescott (2006) obtains the high value of 3.0 for the aggregate Frisch elasticity by assuming an inter-temporal elasticity of leisure substitution equal to one or greater and restricting the utility function to be of the Cobb-Douglas type.
In the light of those contradictory findings and their very different implications, this paper couldn’t resist the temptation of checking the magnitude of the aggregate Frisch elasticity parameter favored by the evidence associated with another fiscal policy event with natural experiment features, the largely unanticipated decade-long non-negligible cuts in government discretionary expenditures in the U.S. initiated with a "budget sequestration" procedure in 2013. The nature and timing of those cuts suggested that such a task could be undertaken with a methodological approach that borrowed its main elements from the "event study" and "business cycle accounting" traditions. The novel methodology proposed by the paper, a contribution in its own right, made it possible to circumvent two limitations of the studies by Prescott and Chetty et al.

As pointed out by Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulàlia-Llopis (2012), the parameter value estimated by Prescott may have little to say about the willingness of households to substitute leisure across periods, because the evidence he examined referred only to permanent after-tax real wage differentials, whereas the aggregate Frisch elasticity measures the response of aggregate labor input to temporary real wage changes. In fact, Chetty et al. argue that what Prescott estimated is really the "aggregate Hicks elasticity " of labor supply, not the aggregate Frisch elasticity. The event study approach adopted in this paper avoids this pitfall by inspecting the response, on impact, of aggregate labor hours to the temporary sequence of spending cuts–and corresponding temporary changes in real wages induced by general equilibrium effects–mandated by the U.S. legislation under the unusual circumstances that will be discussed later.

As to the study by Chetty et al., the discrepancy of their low estimate of the aggregate Frisch elasticity with the much higher one reported by Prescott could be dismissed as the result of an invalid extrapolation to the whole economy of the labor market outcomes observed for the particular households affected by the fiscal policies implemented in Canada and Iceland. The "business cycle accounting" (BCA hereafter) element of the methodology
proposed in this paper addresses this objection, because it makes it possible to gauge alternative values of the aggregate Frisch elasticity with a widely accepted statistical metric by considering only the evidence for aggregate variables. An additional important advantage of the BCA methodology is that it replicates the data exactly. As shown in the paper, this property will be particularly useful for establishing the consistency between, on one hand, the Frisch elasticity value that in the model economy attributes the largest fraction of the economic fluctuations observed in the actual one to total factor productivity shocks and, on the other hand, the value of that elasticity suggested by the evidence associated with U.S. budget sequestration spending cuts that is the focus of this paper.

It is only fair to recognize, however, that overcoming some of the objections to previous attempts to gauge the aggregate Frisch elasticity from the evidence associated with fiscal policy developments came at a cost. In particular, it would have been too ambitious to fully calibrate the aggregate Frisch elasticity with the limited evidence that can be scrutinized with the "event study" approach adopted for this paper. Instead, the paper considers several values of that elasticity that have been proposed to be empirically relevant in the literature, in the 0.5-3.0 range encompassing the values reported by Prescott and Chetty et al., with the modest goal of establishing which of them is favored, according to the well established likelihood metric, by the dynamics of U.S. macroeconomic variables in the year when the budget sequestration cuts became effective.

The main finding of the paper is that the evidence associated with that temporary fiscal policy regime change is more consistent with the low values than with the high values of the Frisch elasticity in the range mentioned above. The opposite is true for the value of the Frisch elasticity that maximizes the importance of total factor productivity shocks in business cycle fluctuations in the same model economy. It follows that the model economy would fail to attribute a large fraction of those fluctuations to total factor productivity shocks if the value of the parameter capturing the aggregate Frisch elasticity were set, respecting the calibration
principle, to that that accounts best, among those considered by the paper, for the dynamic of macroeconomic variables induced by the budget sequestration spending cuts.

Thus, the paper contributes, from the perspective of a fiscal policy development with a natural experiment flavor, to a body of evidence suggesting that the magnitude of the aggregate Frisch elasticity, as defined for the purpose of this paper, is rather moderate, if not low. There is no shortage of microeconomic studies reaching a similar conclusion from data on the extensive and intensive margins of the labor supply at the level of households. Those by Fiorito and Zanella (2012), and Heathcote, Storesletten, and Violante (2010, 2014) have provided convincing evidence that the aggregate Frisch elasticity is not much higher than 1.7, nearly half the value needed, according to Prescott (2006), to assign to total factor productivity shocks the prominent role in business cycle fluctuations. Interesting enough, that upper bound is not too far from the value of 1.4 reported by Lucas and Rapping in the paper mentioned earlier that kicked off the quantitative research agenda on the empirical value of the aggregate Frisch elasticity. Subsequent studies looking also at just aggregate variables, as the one by Ríos-Rull et al. mentioned before, largely inspired by the earlier estimate reported by Smets and Wouters (2007), also suggest an upper bound for the Frisch elasticity that is lower than that that would support the hypothesis that business cycles are driven mostly by shocks to the "efficiency wedge," to use the terminology of the BCA approach adopted by the paper.

The rest of the paper is organized as follows. Section 2 goes over some background material and chronology of events. Section 3 presents an overview of the methodology blending an event study approach with a business cycle accounting analytical framework adopted by the paper, and goes over measurement issues that motivated many of the assumptions and details of specification of the model presented in Section 4. Section 5 discusses first intuitively, and then in more detail, the adaptation of the BCA approach and the statistical tools that the paper exploits to infer the aggregate Frisch elasticity values most likely to account
for the budget sequestration evidence. Section 6 reports the findings. Section 7 concludes.

2 Background Material

2.1 The Budget Sequestration: Relevant Chronology and Details

In the U.S., the government can borrow to finance any shortfall of revenues relative to expenditures up to a certain "debt ceiling" set by Congress. The authorization step is usually a formality, as it simply provides the U.S. Treasury the means to pay for government spending previously agreed upon. That was not the case, however, in January 2011, when the U.S. Treasury request for a debt ceiling increase was opposed by lawmakers concerned with the explosive debt scenario that the Congressional Budget Office (CBO hereafter) had projected in a June 2010 report. These legislators demanded that any increase in the debt ceiling should be accompanied with fiscal deficit reduction measures that prevented the materialization of that debt scenario. There was, however, much disagreement over the specific measures and the prolonged negotiations brought the U.S. to the brink of default. A last minute deal, the Budget Control Act signed into law on August 2, 2011, avoided that outcome but included two unusual provisions intended to prevent the government debt from exploding.

One created a bipartisan Joint Select Committee on Deficit Reduction in charge of prescribing measures to reduce fiscal deficits in a cumulative amount of $1.5 trillion (about 10% of GDP at the time) over fiscal years 2012-2021. The other unconventional provision was a contingent clause stipulating that if the Joint Committee failed to propose, or Congress failed subsequently to enact, legislation to cut the deficit by at least $1.2 trillion by January 15, 2012, existing caps on budget authority to spend would be reduced in that cumulative amount, including savings in servicing the government debt, starting in January 2013 and continuing through fiscal year 2021. In practice, this contingent clause meant that the fiscal
stabilization that the Budget Control Act sought to ensure would be delivered either by the deliberate measures eventually proposed by the Joint Select Committee or by automatic spending cuts evenly split between discretionary defense and non-defense programs.

From an operational point of view, the "budget sequestration" procedure prescribed in the legislation was necessary to revoke or "sequester" previously authorized expenditures above the new spending caps. This is the reason why all the spending cuts eventually triggered by the contingent clause are generically referred to as "budget sequestration" spending cuts, even if not all of them applied to already authorized expenditures.

An important detail for building a model economy that adequately captures critical features of the actual one under study is that it was understood that the sizable federal government outlays on civil and military payrolls would be largely excluded from budget sequestration. The measure wasn’t projected, therefore, to notably affect public sector employment. In addition, the Budget Control Act also shielded most mandatory programs from the sequester. These two exclusions make it possible to circumvent some of the measurement difficulties mentioned in the introduction, with the assumption that government doesn’t contribute to value added and that the spending cuts simply reduce the government absorption of goods and services exclusively produced by the private sector.

According to the Congressional Budget Office (2012), if implemented, the budget sequestration spending cuts would lower that absorption as a share of GDP to the lowest levels on record.³ Despite the incentives to avert this, in principle, unpalatable extreme spending austerity, the Joint Committee announced on November 2011 that, "after months of hard work and intense deliberations", it had concluded that it wouldn’t be possible to reach an agreement on an alternative fiscal deficit reduction package before the January 15, 2012 deadline specified in the Budget Control Act.

Still, the fact that the cuts would reduce discretionary spending as a share of GDP to

³ More specifically, in table 1-1 of the cited CBO report, discretionary spending at the end of the sequestration period, in 2021, was projected to represent 5.7 of GDP, the lowest level since at least 1972.
levels not seen before, eventually impairing the ability of government agencies to adequately perform core functions, kept alive throughout all of 2012 the hopes that Congress would eventually act to avoid them. Such hopes weren’t misplaced, given that lawmakers were considering whether or not to extend tax cuts enacted in 2001 and 2003 due to expire precisely that year. It was plausible to speculate that the negotiations inevitably required to change the tax code would offer legislators a golden opportunity to come up with alternative deficit reduction measures that met the conditions to cancel, or at least suspend, the budget sequestration. Such speculation may have been reinforced by repeated public statements from Congress and even the President insisting on their determination to find a compromise.4

It is fair to conjecture, then, that at the end of 2012 the credibility of a budget sequestration, so widely regarded as unreasonable in depth and scope, was very low. That was indeed the case, as anticipated in the introduction, by the standards of the statistical metric proposed by Hu and Zarazaga (2016). The same study uncovers evidence, however, that that perception seems to have changed in 2013, conceivably prompted by the enactment of the Taxpayer Relief Act at the dawn of that year. The passage of this law may have convinced households and businesses that the budget sequestration was no longer a distant, unlikely prospect when lawmakers did modify the tax code, as speculated, but failed to take any substantial action with respect to sequestration, other than postponing its implementation by two months, from its originally slated date, January 2, 2013, to March 1, 2013.

That assessment, along with the brief chronicle of events described above, suggest that the U.S. economy registered the effects of the sequester with particular intensity in 2013 and that it is the evidence for that year, therefore, that offers the best chance to obtain reliable readings of the elusive value of the aggregate Frisch elasticity most likely to have accounted for it.

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4 According to press reports, the Department of Defense, one of the federal agencies that would be hit particularly hard by the spending cuts, wasn’t making any contingent plans to deal with them as late as September 2012.
An additional argument supporting the view that the evidence examined in this paper is particularly suitable for its intended purpose is the observation that the inclusion of the budget sequestration clause in the legislation just described didn’t have any obvious countercyclical motivation. A careful reading of the arguments offered in support of the law suggests that the primary factor behind the sequestration threat was the long-run concern created by inherited past budget deficits, as embodied in a historically high government debt to output ratio. This is one of the criteria by which tax changes (or spending changes in the case of this paper) qualify as "exogenous" in the taxonomy and terminology proposed by Romer and Romer (2010). In those authors’ view, it is precisely the responses of economic variables to those exogenous changes that generates evidence with the experimental attributes ideal for gauging economic relationships or, by extension, parameters values governing them, such as the impact of tax changes on economic activity or the Frisch elasticity parameter that is the focus of this paper.

2.2 Estimated Size of the Budget Sequestration Spending Cuts

As it could be expected, the inferences about the value of the Frisch elasticity obtained by examining the performance of macroeconomic variables presumably under the influence of the budget sequestration spending cuts will depend crucially on the exact magnitude and distribution over time of the those cuts in real terms. That information is not readily available, because government budgets are typically approved in nominal terms. Even then, the transformation of nominal spending cuts into real ones requires to make assumptions about the evolution of the inflation rate over the budget sequestration period. In addition, it is necessary to make assumptions about the growth rate of private sector output, because in the model economy the spending cuts will be introduced as a policy regime change that shifts downwards for its duration the stochastic process otherwise governing over time the evolution of the ratio of the government absorption of goods and services to private sector
In any case, the calculation of the statutory spending cuts implied by the budget sequestration in real terms must start from a reliable sequence of nominal ones, fortunately made available by the Congressional Budget Office (2013), as summarized in the second column of Table 1. The nominal values were converted to real ones by assuming an annual expected inflation rate equal to the Federal Reserve target for that variable, 2%, and an annual growth rate of real output of 2% also, consistent with the long-run growth rate of this variable suggested by the calibration of the model discussed later. More specifically, these rates were used to construct series for the price index for non-durable goods and services and for real GDP for the intended duration of the sequester, the period 2013-2021, taking as reference the values of those indices observed in 2012.

The third column in Table 1 documents the spending cuts in real terms implied by the budget sequestration adopted by the paper, calculated by dividing the corresponding nominal sequence by the indices just described and grossing up the result by 15 percent. The last step was dictated by internal consistency with the empirical methodology, which when appropriate represents variables as shares of output produced by private sector businesses, about that percentage lower than total output when the value added by government agencies and enterprises is excluded from it.

The Budget Control Act did not stipulate spending caps past the year 2021, so it didn’t impose any legal restrictions on the level of government absorption of goods and services as a share of GDP in the long run. This long run value is needed, however, because the steady state equilibrium of the model economy will be an important reference point for the empirical implementation of the adapted BCA approach. The above developments suggest that the budget sequestration was a fiscal stabilization measure of last resort and, as such, not intended to persist beyond the period explicitly established in the legislation. Accordingly, the paper assumes that at the end of the budget sequestration period, the government
absorption of goods and services as a share of GDP returns to its historical average, as measured in section 4.4 of the paper.

[insert Table 1 about here]

3 Overview of the methodology

Given its potential repercussions, the reliability of the measuring device was essential for the delicate task of teasing out the aggregate Frisch elasticity from the evidence provided by the budget sequestration event. It seemed reasonable to expect that confidence on the resulting measurements would be enhanced if obtained with an analytical framework with two qualities usually perceived as highly desirable: 1) the capability of taking into account general equilibrium considerations when evaluating the endogenous response of macroeconomic variables to shocks and policy regime changes, and 2) the flexibility to accommodate different views about the features of the economic environment ultimately responsible for those responses.

As suggested in the introduction, those considerations led almost naturally to the business cycle accounting (BCA hereafter) approach of Chari, Kehoe, and McGrattan (2007)—"CKM"—which introduces in a widely used frictionless neoclassical growth model auxiliary variables ("wedges"), that stand in for a variety of distortions (financial and/or nominal in nature) often proposed as essential for the correct interpretation of business cycles and other phenomena.

The BCA approach has several advantages. First, it builds the model economy around the structural framework of the well-established neoclassical growth model, thereby addressing Attanasio’s (2013) concern that the evidence examined by microeconomic studies might be misinterpreted without accounting for the general equilibrium effects of economic shocks and policy regime changes.
Second, related to the previous advantage, it explicitly incorporates the capital stock into
the analysis and eliminates, therefore, a potential bias in the Frisch elasticity estimates from
Chetty et al., whose measuring device is the model without capital accumulation proposed
by Rogerson and Wallenius (2009). As is well known, intertemporal variations in the labor
supply arise in part from deviations of the capital stock from its trend. Thus, one possible
reason for the weak response of aggregate hours in Canada and Iceland in the policy episodes
analyzed by Chetty et al. (2013) might be that those countries’ capital stocks were above
trend.

Third, although typically anchored in a frictionless benchmark structural model, the
BCA approach is flexible enough, as already anticipated, to accommodate several frictions
considered important in the literature, introduced via the parsimonious shortcut of wedges
that close the gaps that appear in optimality and equilibrium conditions when theoretical
variables are replaced by their empirical counterparts. In that sense, the paper pays heed to
the advice of Chetty et al. that incorporating wedges to the analysis may yield additional
insights in the quest of obtaining reliable estimates of the Frisch elasticity.

A fourth advantage, particularly important for this paper, is that the BCA approach
renders itself easily to a state-space representation of the wedges that replicates the data
exactly, a feature that will be exploited to rank alternative Frisch elasticity values by their
relative ability to account for the observed 2013 performance of macroeconomic variables.

The reason to scrutinize particularly closely the available evidence for 2013 is that the
analysis in Hu and Zarazaga (2016) suggests that economic agents didn’t factor the seque-
stration in their decisions until that year. This seems a plausible assessment, given that it
could have been reached as well with the narrative approach of Romer and Romer (2010).
In keeping the focus on a relatively short period of time after the budget sequestration was
triggered, the paper follows the standard practice in the "event study" literature, which has
shown that new and/or additional insights can be gained on unresolved issues or questions
by keeping track of the evolution of economic variables of interest over a relative narrow window of time after the announcement and/or implementation of a policy event.

In the process of blending the event study and BCA approaches with the methodology proposed in this paper, it was necessary to deal with some measurement and model parameterization issues. On the measurement front, the accuracy of the inferences could have suffered from the lack of correspondence between the variables in the model economy and their empirical counterparts for the reasons noted by Gomme and Rupert (2007). This problem was minimized by adopting those authors’ "private sector output" methodology for measuring variables in the actual economy and by introducing in the model economy an external-like sector with the "minimalist" approach of Trabandt and Uhlig (2011).

On the parameterization front, the challenge was to allay concerns about the accuracy of the results obtained with the computational techniques employed in the paper. This potential source of bias was addressed by specifying a preference ordering of consumption-leisure choices consistent with balanced growth and a constant aggregate Frisch elasticity, that is, invariant to the fraction of time that households are at work and, therefore, independent of the deviations of the economy from its steady state. In addition, to facilitate comparison of the results with those obtained by Chetty et al. (2013) and Prescott (2004), it was further assumed that preferences are separable in consumption and leisure, with the obvious implication that the conclusions of the paper will be conditional on the leisure-held constant intertemporal elasticity of substitution in consumption being equal to one.

Before getting any deeper into the details, it will informative to provide a brief intuitive account of the economic mechanisms activated by the budget sequestration that will, in principle, allow the methodology just described to tease out the aggregate Frisch elasticity value most consistent with the evidence associated with that temporary policy regime change. As discussed later, those cuts fell mostly on government consumption. In the model, the reduction of government absorption of private sector output leaves more of it available for private
consumption. But given the temporary nature of the government consumption decline, the consumption-smoothing motive will induce households to save part of the output freed up by the government. In any case, the resulting change in consumption implies a corresponding change in the opposite direction of the marginal utility of this variable which, through the standard intratemporal optimality considerations, must be accompanied by a commensurate variation of the marginal utility of leisure and, therefore, of the fraction of time households allocate to work. The magnitude of the labor supply response depends, of course, on the aggregate Frisch elasticity. The model takes into account all the general equilibrium effects of the forces just described by filling in, so to speak, with the appropriate wedges any gaps in the model equilibrium conditions otherwise implied by the stand-in household’s decision rules. Alternative candidate values of the Frisch elasticity will induce different configurations of the wedges in 2013, the likelihood of each of which can be calculated to identify the particular configuration and, therefore, corresponding Frisch elasticity value that accounts best for the evidence under study.

3.1 Measurement Issues

For the reasons given earlier, inferences about the value of the Frisch elasticity will be obtained from a limited number of observations. It was thus important to eliminate from them the potentially severe measurement errors introduced by a lack of correspondence between variables in the model economy and their empirical counterparts.

The discrepancy should be of particular concern when government activities—in the form of reduction of government spending of goods and services in the present paper—play a critical role in the analysis, and the output originated in those activities represents a significant fraction of overall output. For the case of the U.S., that fraction is about 19% of GDP, large enough to give rise to imprecise inferences if the model economy is built on the assumption that the quantities of all types of goods and services produced and used up in
the actual one reflect the interaction of optimizing private agents that value them at market prices, when in reality that is not the case of the mostly non-market activities typically conducted by government agencies, not driven by profit motives.

Gomme and Rupert have proposed to mitigate this conceptual mismatch by adjusting the data in a manner consistent with the behavior of economic agents assumed in the model economy. Several steps are involved in this adjustment, but the one that is important to highlight for the purposes of this paper is that since the model economy assumes that all output is produced by profit-maximizing firms, the appropriate counterpart in the actual economy is constructed by subtracting from real GDP the value added by the general government in the process of producing non-market goods and services. This "private sector economy" approach is not an obstacle to make inferences about the Frisch elasticity values from the responses of macroeconomic variables to the budget sequestration because, as mentioned earlier, the spending cuts implied by that measure fell mostly on the government absorption of the goods and services produced by the private sector, rather than on the value added by the government, represented mostly by the compensation of the labor services provided by government employees. The data necessary to obtain as just indicated the historical series of private sector output are available at an annual frequency only since 1977. The analysis in this paper uses therefore data from that year until 2013, the year that the budget sequestration began.

As hinted at above, further adjustments are necessary to make the data consistent with the conceptual entities in the model economy, but a thorough discussion of them is tedious and would detract from the main focus of the paper. Interested readers will be able to find the relevant details in Kydland and Zarazaga (2016).
4 The Model Economy

Given that the model economy is built around the framework provided by the neoclassical growth model, it was appropriate to confine the specification of preferences, technology, and government policies to those in the class consistent with balanced growth, as characterized by King, Plosser, and Rebelo (1988a, b). As established by those authors, the balanced growth condition imposes the additional restriction that, in the presence of investment-specific technological progress, the production function must be such that it permits to represent that progress as if labor-augmenting. Following Greenwood, Hercowitz, and Krusell (1997), this requirement is met by adopting a Cobb-Douglas production function and by representing macroeconomic variables in real terms, when appropriate, in units of the consumption good. The latter are obtained by dividing the relevant nominal variables by a price index of non-durable goods and services. Together, these assumptions and transformations imply that technological progress from all sources is summarized by total factor productivity growth.

The model economy is assumed to display total factor productivity and population secular deterministic growth, but for computational reasons, it was more convenient to represent it as an economy without growth. To that end, all variables that would otherwise display secular growth were detrended as dictated by theory and their counterparts in the actual economy by the growth rate of total factor productivity and of population 16 years of age and older.

Typically no adjustment is needed for labor input, because according to theory it shouldn’t display secular growth along a balanced-growth path. In the case of the U.S., however, this variable, empirically captured by the fraction of available time that households are on average actually at work, grew steadily since the end of the Second World War until approximately the beginning of the 21st century. This theory-contradicting performance was largely the result of an irreversible increase in women’s labor force participation rate that seemed to have reached its limit by the end of the period just mentioned. If ignored, this seemingly
transitory upward drift in labor input could introduce measurement errors that could distort to an unknown extent the inferences about the Frisch elasticity values that will be obtained later on. In the absence of a clear guidance from theory, the drift just mentioned was removed from labor input with a linear interpolation version of the procedure proposed in Kydland and Zarazaga.

4.1 The Stand-in Household’s Choice Problem

The model economy is assumed to be inhabited by an infinitely-lived household, which stands for the large number of them present in the actual economy, and whose preferences can be ordered by a time-separable Constant Frisch Elasticity (CFE hereafter) utility function defined over infinite streams of consumption, \( \{c_t\}_t^\infty \), and the fraction of available time devoted to work, \( \{h_t\}_t^\infty \).

In addition to being consistent with balanced growth, this utility function is the only one that allows consumption and leisure to be eventually non-separable within periods without tying the Frisch elasticity value to the fraction of time that the representative household is at work. As mentioned in the introduction, this feature was an important consideration for adopting the CFE utility function specification, given that the equilibrium allocations are computed with perturbation techniques that approximate the private sector’s decision rules in the neighborhood of the non-stochastic steady state of the model economy. The unavoidable approximation errors introduced by this computational technique are likely to be compounded by alternative utility function specifications typically proposed in the literature, such as in Cooley and Prescott (1995), which implies that the Frisch elasticity changes with the fraction of available time devoted to market activities and is different, therefore, at and away from the steady-state.\(^5\)

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\(^5\)As mentioned in the introduction, Ríos-Rull et al. (2012) question the overall ability of this utility function specification to capture the willingness of households to substitute hours worked across time.
For continuity with previous studies that have exploited fiscal policy developments for the same purpose as this paper, especially those of Chetty et al. (2013) and Prescott (2004), consumption and leisure are assumed to be intratemporally separable. Combined with the requirement that the model predictions must be consistent with balanced growth, this assumption implies a logarithmic utility function for consumption, as shown by King, Plosser, and Rebelo (1988a).

Accordingly, the stand-in household solves the following maximization problem:

$$\text{Max}_{\{c_t, h_t, k_{t+1}\}} E_s \sum_{t=s}^{\infty} \beta (1 + \eta)^t \left[ \ln(c_t) - \kappa (1 - l_t)^{1+\frac{1}{\varphi}} \right]$$  \hspace{1cm} (1)$$

subject to the following constraints:

$$c_t + x_t = (1 - \tau_t^h) w_t h_t + r_t k_t - \tau_t^k (r_t - \delta) k_t + n i_t + \tau_t$$  \hspace{1cm} (2)$$

$$x_t = (1 + \eta)(1 + \gamma) k_{t+1} - (1 - \delta) k_t + \frac{\psi}{2} \left( \frac{x_t}{k_t - \delta} \right)^2 k_t$$  \hspace{1cm} (3)$$

$$1 = l_t + h_t$$  \hspace{1cm} (4)$$

$$h_t = h_t^{pr} + h_t^{pa}$$  \hspace{1cm} (5)$$

government policies

The objective function in (1) is the expected discounted value of a utility function in the CFE class, where $\beta > 0$ is the discount factor, $\eta$ is the working age population annual growth rate, $t$ a time index, $c_t$ detrended consumption per working age person, $h_t$ the fraction of available time the representative household allocates to work in the market, $\kappa > 0$ a parameter that controls the household’s valuation of consumption relative to leisure, and $\varphi$ the constant Frisch elasticity of aggregate labor supply.

Equation (2) is the household’s budget constraint, where $x_t$ is gross private domestic investment, $w_t$ the wage rate in terms of consumption per unit of the available time the
stand-in household devotes to work, $\tau^h_t$ the tax rate on labor income, $r_t$ the rental price of period $t$ private sector capital, $k_t$, $\tau^k_t$ the tax rate on income from that capital, $\delta$ the depreciation rate, $\tau_t$ lump-sum transfers (taxes if negative), and $ni_t$ net imports.

The variable $ni_t$, which captures the net exports component of aggregate demand, introduces an external-like sector with the minimalist approach proposed by Trabandt and Uhlig. As mentioned in the introduction, the inclusion of this variable mitigates the lack of correspondence between the otherwise closed economy model of this paper and the data for the U.S. economy, whose economic interactions with the rest of the world would have been considerably more challenging to model and parameterize explicitly. Introducing this admittedly crude adjustment seemed nevertheless particularly important for the goal of this paper, because a negative trade balance is the counterpart of the flow of income from foreign assets that households can devote to investment, one of the variables that will be used to make inferences about the Frisch elasticity values with the BCA methodology adopted for this paper. The empirical implementation of the model will take into account that in balanced growth the ratio of $ni_t$ to output should be characterized by a stationary stochastic process with unconditional mean $ni_y$.

It must be noted that, for similar measurement-related issues, CKM also included net exports in their model, except that they lumped it together with government consumption. It didn’t seem appropriate to maintain that consolidation in this paper, for whose purpose it is important to distinguish the responses of macroeconomic variables to anticipated government spending cuts from those induced by unanticipated external sector shocks.

Note that from a BCA perspective, the variables $ni_t$, $\tau^h_t$ and $\tau^k_t$ can be interpreted as wedges analogous to those in CKM, with stochastic properties discussed in section 5.2.

Equation (3) describes the evolution over time of the capital stock that the household rents to private firms which, for consistency with the NIPA methodology, excludes the public sector capital stock. This law of motion links the private capital stock available for
production at the beginning of a period, $k_t$, with the households’ investment decisions during that same period, $x_t$, and with the private capital stock that will be available at the beginning of the following period, $k_{t+1}$, after accounting for investment adjustment costs, whose magnitude is controlled by the parameter $\psi$.

In line with the treatment of macroeconomic aggregates introduced before, those in the law of motion (3) have also been detrended and are measured in units of the consumption good per working age person. In fact, the correction of the beginning-of-period $t+1$ capital stock by the gross growth rate factor $(1 + \eta)(1 + \gamma)$ is one of the adjustments needed to transform the original balanced growth economy into one without growth, but with the same quantitative properties in terms of impulse-responses and transitional dynamics.\(^6\)

Equation (4) states the time constraint that the stand-in household can distribute its total available time, normalized to 1, among non-market activities, $l_t$, (generically labeled as "leisure") and work in the marketplace, $h_t$.

Equation (5) states that the household can allocate the time it devotes to work between private sector firms, $h_{t}^{pr}$, and public sector agencies (inclusive of government-owned enterprises), $h_{t}^{pa}$. Note that for consistency with the standard treatment of labor input in the neoclassical growth model, the empirical counterpart of the variable $h_t$ is the fraction of time actually worked, not just paid. The data were therefore adjusted to omit the time for which workers were paid but not actually working, because they were on vacation, sick leave, etc.

The explicit distinction between the time households devote to work in the public and private sectors is uncommon, because the value added by both the private and public sectors is deemed the appropriate empirical counterpart of output in most models. This is not true for the model economy of this paper, in which all the value added is provided by the private sector, even if partly absorbed by government purchases not valued by households.

---

\(^6\)Recall that Greenwood, Hercowitz, and Rebelo have demonstrated that in this case the depreciation rate in (2) and (3) must be interpreted as the economic, rather than physical depreciation rate in the presence of underlying investment-specific technological progress.
Calibrating or estimating the relevant parameters of such an economy without taking into account the fraction of time that households work for government agencies could cause the model to overestimate the labor input absorbed by the private sector and, therefore, output, consumption, and investment.

4.2 Private Sector Firms’ Maximization Problem

There are two types of firms that produce output in the stationary economy without growth and without a government final good: private firms and government enterprises. As noted by Gomme and Rupert, the decisions of the latter are guided by administrative, rather than profit-maximizing considerations and are treated, therefore, as exogenous.

The behavior of private firms is instead modeled explicitly, an approach that requires one to be specific about the restrictions those firms face in the production of output.

The paper adopts the standard assumption that the model economy is populated by a large number of identical private firms that transform labor and capital inputs into output with a constant returns to scale technology that exhibits labor-augmenting technological progress and unitary elasticity of substitution between inputs. Under those conditions, the aggregate output of the model economy corresponds to that generated by a single representative firm endowed with a Cobb-Douglas production function:

$$y^{pr}_{t} = A e^{(1-\theta)z_t h_t^{\theta} (h^{pr}_t)^{1-\theta}},$$

where $y^{pr}_{t}$ is detrended output per working age person produced by private sector firms, $\theta$ the proportion of the remuneration to capital services in the private sector value added, and $z_t$ a stochastic technology level that introduces the fourth wedge considered for the particular implementation of the BCA methodology proposed in this paper. This technology level shifter corresponds conceptually to the efficiency wedge in CKM. The properties of the
stochastic process governing its evolution over time will be discussed in section 5.2.

The representative firm that stands for the large number of them making decisions in the economy solves, therefore, the following maximization problem:

\[
\max_{h^p, k_t} \left[ Ae^{(1-\theta)z_t} k_t^0 \left( h_t^p r_t \right)^{1-\theta} - w_t h^p_t - r_t k_t \right].
\]

Notice that in this economy, it is the stand-in household that makes the investment decisions.

4.3 Public Sector Policies

As mentioned in section 3.1, the allocation of resources by public sector entities is the result of complex social, political, and economic considerations, not aptly captured by the same profit- and utility-maximizing incentives faced by households and private sector firms. Given the difficulties in modeling explicitly the behavior underlying the economic decisions made by public sector agencies, the variables under their control will be exogenously determined.

4.3.1 Government Budget Constraint and the Sequester

Most studies that have taken into account general equilibrium effects when estimating or inferring the value of the Frisch elasticity from macroeconomic evidence, as those already mentioned by Chetty et al. (2013), Prescott (2004), Ríos-Rull et al., and Rogerson and Wallenius, assume a balanced government budget. To facilitate comparison with the results obtained in those studies, the paper adheres to that analytically and computationally convenient practice.

In particular, in the model economy the government absorption of output exclusively produced by the private sector, denoted \( g_{at} \), is equal every period to revenues from all sources.
minus transfer payments, as indicated by the following government budget constraint:

\[ ga_t = \tau_t w_t(h_{t}^{pr} + h_{t}^{pu}) - w_t h_{t}^{gc} + \tau_t (r_t - \delta) k_t + s_{t}^{ge} - \tau_t, \]  

(9)

where \( h_{t}^{pu} \) is equal to \( h_{t}^{gc} + h_{t}^{ge} \), with \( h_{t}^{gc} \) and \( h_{t}^{ge} \) representing the fraction of time the stand-in household works for government agencies and government-owned enterprises, respectively, and where \( s_{t}^{ge} \) denotes, for consistency with the NIPA methodology, surpluses (deficits, if negative) transferred by government-owned enterprises. In line with the treatment of variables corresponding to physical quantities discussed before, those of the same type in the government budget constraint are measured in units of the consumption good per working age population as well.

To avoid misunderstandings, note that the variable \( ga_t \) is conceptually different from the government consumption expenditure variable in CKM, which in the case of those authors includes value added by the government sector and, as mentioned earlier, net exports.

Moreover, for the purpose of the present paper it is convenient to interpret this variable as made up of a systematic, exogenous stochastic component, \( ega_t \), and of a non-systematic, deterministic component, \( pga_t \), whose relationship, after division by private sector output, can be formally represented as follows:

\[ \frac{ga_t}{y_{t}^{pr}} = \frac{ega_t}{y_{t}^{pr}} + \frac{pga_t}{y_{t}^{pr}}. \]  

(10)

In line with the historical developments described in section 2.1, the stochastic component \( ega_t \) is meant to capture the ups and downs of the government spending policy historically followed until the sequestration took place in 2013. The non-systematic, deterministic component \( pga_t \) is meant to capture the "policy regime change" of limited duration (from 2013 to 2021, to be precise) implied by the budget sequestration spending cuts.

For consistency with the balanced growth assumption, the stochastic component is pos-
tulated to evolve over time according to a stationary stochastic process with the following autoregressive representation:

\[
\ln \frac{e_g y_t}{y_{t-1}^{pr}} = (1 - \rho_g) \ln g_y + \rho_g \ln \frac{e_g y_{t-1}}{y_{t-1}^{pr}} + \sigma_g \varepsilon_t^g y,
\]

where \(g_y\) and \(\sigma_g\) are scalars, and \(\varepsilon_t^g y\) is a random variable with a standard normal distribution.

The policy component in (10), \(\frac{p_{gau}}{y_t^{pr}}\), is a placeholder that will be replaced by the spending cuts in the third column of Table 1 in the quantitative implementation of the model, with the practical effect of shifting down the government absorption of private sector output relative to the level implied by the exogenous component \(\frac{e_g y_t}{y_t^{pr}}\).

### 4.3.2 Public Sector Labor Demand

In line with the pattern of the previous stochastic process, the general government and government enterprises’ demand for labor services is also assumed to be autocorrelated, with the following representation:

\[
\ln h_{t}^{pu} = (1 - \rho_{hpu}) \ln h_{t}^{pu} + \rho_{hpu} \ln h_{t-1}^{pu} + \sigma_{hpu} \varepsilon_t^{hpu}
\]

where \(h_{ss}^{pu}\) and \(\sigma_{hpu}\) are scalars and \(\varepsilon_t^{hpu}\) is a random variable characterized by a standard normal distribution.

### 4.3.3 Government Enterprises Value Added

The value added by government enterprises, \(va_{t}^{ge}\), included in the business rather than the government sector of NIPA, should grow at the same rate as private sector output along a balanced growth path. Therefore, it is natural to postulate that the evolution of this variable
over time is determined by the following stochastic processes:

\[ \ln \frac{v \alpha^e_t}{y^e_t} = \ln vy + \sigma_{ge} \varepsilon^e_t \]  

(13)

where \( vy \) and \( \sigma_{ge} \) are scalars, and \( \varepsilon^e_t \) is a random variable characterized by a standard normal distribution.

4.3.4 Resource Constraint

For the purpose of subsequent analysis, it is useful to make explicit the resource constraint that results from consolidating the household’s budget constraint (2) with the government budget constraint (9), after taking into account that, for consistency with the NIPA methodology, output in the model economy originates in private sector firms according to (7) and in government-owned enterprises according to (13):

\[
c_t + x_t = \left[ 1 + \frac{\nu \alpha^e_t}{y^e_t} - \frac{\alpha_t}{y^e_t} + \frac{n_t}{y^e_t} \right] A e^{(1-\theta)z^e} k^0_t (h^e_t)^{1-\theta}.
\]

4.4 Model Calibration

As it should be apparent from the preceding section, the model economy involves a fairly large number of parameters. Attempting to estimate all of them with available statistical tools at an acceptable level of precision is doomed to failure given the limited available data, at most 37 annual observations, from 1977 to 2013, for the aggregate variables of interest. Therefore, it seemed wise to calibrate as many parameter values as possible with the widely accepted quantitative discipline imposed by the requirement that the steady state economic relationships between variables and/or parameters predicted by the model economy should match those prevailing in the actual economy, on average, over fairly long periods of time.

The parameters of the model economy whose values were set with a calibration approach
are listed in Table 2. Whenever the calibrated values involved the use of historical averages, they correspond to the period 1997-2007. Observations during and after the Great Recession were deliberately omitted, on the grounds that the large changes that many macroeconomic variables experienced during that unusually deep contraction were persistent, but not permanent, and didn’t have an everlasting impact, therefore, in the long run trends of the actual economy.

Missing from Table 2 are model parameters that can only be inferred from the high frequency movements of the economic variables under their influence, by definition absent from steady state relationships. Parameters of this type fall in three groups: 1) the coefficients of stationary stochastic processes that drop out from the model equations in steady state, 2) the parameter $\psi$ controlling the scale of the investment adjustment costs and the aggregate Frisch elasticity, $\varphi$, and 3) parameters whose steady state values depend on the latter.

The parameters in the first group were estimated with the techniques discussed in the next section. As to those in the second group, the investment adjustment costs parameter $\psi$ is set equal to 17, by the same logic and references invoked by Christiano, Eichenbaum, and Rebelo (2011).

Recall that the goal of the paper is to establish which of the rather diverse Frisch elasticity values claimed to be empirically relevant in existing studies can best account for the dynamics of macroeconomic variables induced by the budget sequestration. To that end, the paper considers the following five values, representative of those advocated by some and disputed by others in the literature:

$$0.5, 1, 1.9, 2.5, \text{ and } 3.$$

The first Frisch elasticity value stands for the point estimate of 0.52 for that parameter obtained by Smets and Wouters (2007) in a study pioneering the estimation of dynamic stochastic general equilibrium models with Bayesian techniques.\(^7\) The value of 1.0 is suggested

\(^7\)See endnote 30 of Chetty et al. (2013) clarifying that the value of 1.92 reported by Smets and Wouters
by the survey evidence on the response of labor supply to a large wealth shock examined by Kimball and Shapiro (2008). The value of 1.9 has been proposed by Hall (2009) in a study that includes a labor wedge. The value of 3 has been inferred by Prescott (2004) from the study on labor income tax differences across countries mentioned earlier. Finally, the value of 2.5 in between the last two was added to the list for completeness.

As to the parameters in the third group, they include those that are implied by steady state relationships that depend, precisely, on the values of the Frisch elasticity. That is the case of the utility function parameter $\kappa$, whose value is reset for each of the five Frisch elasticity values considered, exploiting the steady-state version of the intratemporal first order necessary condition implied by the household’s maximization problem.

5 Sizing up the Frisch Elasticity Values Most Consistent with the U.S. Budget Sequestration Spending Cuts Evidence

5.1 Overview

The first and second steps of the adapted BCA approach implemented in this paper are the same as in CKM. The first step represents the model in a state space form, suitable for estimating with maximum likelihood techniques unobserved state variables and the unknown parameters of the stochastic processes controlling their evolution over time.

The second step proceeds with the maximum likelihood estimation of the parameter values and state variables just mentioned, using the relevant data from 1977, the first year for which they were available with enough detail to apply the Gomme-Rupert measurement approach, until 2012, the year before the sequester took effect. Given that the likelihood of

\[
&\text{is actually the reciprocal of the aggregate Frisch elasticity.}
\]
the data is conditional on parameter values, the estimation had to be repeated for each of
the five Frisch elasticity values listed in section 4.4.

The third and last step, which departs from CKM, is critical for the purposes of this
paper. Recall that CKM exploit the state-space representation of the model to recover the
wedges that replicate the data exactly at each point in time and then feed them one by
one in the model economy to measure the marginal effects of each wedge on macroeconomic
variables. In this paper, what is fed into the model, for each Frisch elasticity value considered
and corresponding parameter estimates from the previous step, is the sequence of spending
cuts implied by sequestration.

In principle, each Frisch elasticity value will be associated with a different set of estimated
state variables as of the beginning of 2013 and a different configuration of wedges that
replicate the data for that year exactly. Knowledge of the realized state variables and
wedges makes it possible to compute the value of the likelihood function for each of the
Frisch elasticity values considered. By design of the underlying statistical tool, the higher
the likelihood value, the more likely is the corresponding Frisch elasticity value to account
for the joint performance of macroeconomic variables during 2013.

5.2 Technical Details

5.2.1 State-Space Representation

The first step in implementing the adapted BCA approach is to represent the model in a
state-space form, which is accomplished as usual, by specifying transition equations that
govern the evolution of state variables over time and measurement equations that define the
mapping between the states and the relevant observed data.

In dynamic stochastic general equilibrium models, the link between observables and state
variables in the measurement equations is provided by the equilibrium decisions rules which,
as already anticipated, this paper computes with the standard practice of approximating
the true decision rules with a first order Taylor expansion around the non-stochastic steady state. This ensures a linear mapping between state variables and observables.

With the further assumption that the transition from one state to the other is governed by a linear Markov process, the state-state representation of the model economy of this paper can be formalized by the transition equation:

\[ S_t = T S_{t-1} + Q \omega_t, \]  \hspace{1cm} (14)

and the measurement equation:

\[ Y_t = D S_{t-1} + C \omega_t. \]  \hspace{1cm} (15)

To see how the different pieces of the model economy fit together in the state-space representation above, it will be helpful to spell out more fully the elements of the vectors and matrices in equations (14) and (15), starting with the 7x1 vector \( S_t \) of end-of-period state variables in the transition equation,

\[ S_t = [k_{t+1} - k_{ss}, \ln \left( \frac{eg_t}{y_t} \right) - \ln gy, \ln h^{pu}_t - \ln h^{pu}_{ss}, z_t - z_{ss}, \frac{n_t}{y_t} - n_i y, \tau_t h - \tau_{ss} h, \tau_t k - \tau_{ss} k]' \]

where a subindex ”ss” identifies the steady state value of the period \( t \) variable immediately to the left.\(^8\)

\(^8\)For consistency with the timing convention adopted in the law of motion of capital (3), the capital stock at the end of period \( t \) is denoted in the vector \( S_t \) as the beginning of period \( t+1 \) capital stock, \( k_{t+1} \).
Consider next the $7 \times 7$ matrix $T$:

$$
T = \begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} & T_{17} \\
0 & \rho_{g_a} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_{h_p} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_z & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_{n_i} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_{r_h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_{r_k}
\end{bmatrix},
$$

where the first row of this matrix is simply the result of replacing in the law of motion for the private capital stock, (3), the equilibrium decision rule for investment, $x_t$. The second and third rows of the matrix simply replicate the stochastic processes in equations (11) and (12), respectively. The rest of the rows of this matrix represent the wedges, expressed in terms of ratios to private sector output when appropriate, as stochastic Markovian processes that depend only on their own past. Interactions between these processes were ruled out by assumption, for the same reasons given earlier: the limited data available would have prevented the reliable estimation of the large number of parameters implied by a less parsimonious specification.\(^9\)

The elements of the $7 \times 1$ vector of exogenous shocks $\omega_t$ are as follows:

$$
\omega_t = \begin{bmatrix}
\varepsilon_{g_y} \\
\varepsilon_{h_p} \\
\varepsilon_{g_e} \\
\varepsilon_z \\
\varepsilon_{n_i} \\
\varepsilon_{r_h} \\
\varepsilon_{r_k}
\end{bmatrix},
$$

where the first three elements correspond to the innovations identified in equations (11), (12), and (13), and the remaining elements capture the innovations to the four wedges $z_t$, $n_i$, $r_h$, and $r_k$. The variance-covariance matrix of this vector, $E[w_t w_t']$, is denoted by $\Sigma$ and

\(^9\)It is not clear, in any case, that the interactions would be significant, as they are not statistically different from zero in CKM.
characterized by the following elements:

$$\sum = \begin{bmatrix} \Sigma_{11} & 0_{3x4} \\ 0_{4x3} & \Sigma_{22} \end{bmatrix},$$

where $\Sigma_{11}$ is a $3x3$ identity submatrix, and $\Sigma_{22}$ a $4x4$ submatrix, with diagonal elements equal to 1 and possibly non-zero off-diagonal elements. This specification assumes that the stochastic process for the government absorption of private sector output, characterized by equation (11), as well as that for the public sector labor input, characterized by equation (11), are orthogonal to all the others, whereas the innovations to the wedges are allowed to be correlated with each other.

Fully spelled out, the $7x7$ matrix $Q$ is given by

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & Q_{15} & Q_{16} & Q_{17} \\ \sigma_{gy} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{hpu} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ni} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{kh} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{tk} \end{bmatrix},$$

where the elements of the first row are coefficients implied by the linearized equilibrium decision rule for the capital stock and the rest of the elements just capture the standard deviations of all the exogenous stochastic processes in the model.

In the measurement equation, the $7x1$ column vector $Y_t$ contains the observable variables:

$$Y_t = [y_{i}^{pr} - y_{ss}^{pr}, c_t - c_{ss}, x_t - x_{ss}, h_t^{pr} - h_{ss}^{pr}, \ln \frac{e^{g_{yt}}}{y_t^{pr}} - \ln g_y, \ln h_t^{pu} - \ln h_{ss}^{pu}, \ln \frac{v_{q_{et}}}{y_t^{pr}} - \ln v_y]^\prime,$$
where again a subindex “ss” identifies the steady state value of the corresponding variable.

It is worth to clarifying at this point a potential confusion created by the inclusion of the element $\ln \frac{eg_{t}}{y_{t}} - \ln gy$ in the vector of observables $Y_{t}$. Strictly speaking, the variable directly observable in the data is $ga_{t}$, not the individual components identified in equation (10). However, as this equation makes apparent, in the absence of the temporary policy regime component $pga_{t}$, the stochastic component $ega_{t}$ is equal to $ga_{t}$ and therefore, observable as well. This equality holds, therefore, between 1997 and 2012, before the budget sequestration was triggered. When it breaks down in 2013, $ega_{t}$ is no longer observable but it can be inferred from the data and the spending cuts for that year implied by the legislation that enacted the budget sequestration. In particular, without spending cuts in 2013, the observation $ga_{2013}$ would have been higher by $sc_{2013}$, the amount by which the sequestration would lower government spending that year, as per the CBO estimates in Table 1. Thus, $ega_{2013}$ can be inferred from the equality $ega_{2013} = ga_{2013} + sc_{2013}$ implied by equation (10).

The $7\times 7$ matrix $D$ can be rewritten as

$$D = \begin{bmatrix} \mathbb{D}_{4\times7} \\ 0 & \rho_{ga} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{hpu} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{hpu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{hpu} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{hpu} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{hpu} \end{bmatrix},$$

where the elements $\mathbb{D}_{ij}$ of the $4\times 7$ submatrix $\mathbb{D}$ consist of the coefficients of the linearized equilibrium decision rules for the endogenous variables in the vector $Y_{t}$, the element $\rho_{ga}$ restates in matrix notation the first term of equation (11), and the element $\rho_{hpu}$ restates that of equation (12).
Finally, the 7x7 matrix $C$ is given by

$$
C = \begin{bmatrix}
\sigma_{gy} & 0 & 0 \\
0 & \sigma_{hpu} & 0 & 0 \\
0 & 0 & \sigma_{vy} \\
\end{bmatrix}
$$

where the elements $C_{ij}$ of the 4x7 submatrix $C$ are obtained from the equilibrium decision rules and the last three rows restate the second term in equations (11), (12), and (13).

Having made explicit the mapping between the model economy in section 4 and its state-space representation in this one, it is possible to proceed with the second step to estimate the unknown state variables and parameters of the model.

### 5.2.2 Estimation

The parameters not listed in Table 2 were estimated using all the data available up to the year 2012, before the temporary policy regime change represented by the budget sequestration spending cuts took effect. It seemed reasonable to include the observations during and after the Great Recession because that contraction, by most accounts, was characterized by the virulent manifestation of several frictions. Those observations might contain, therefore, information particularly useful for estimating the parameters of the stochastic processes of the wedges meant to summarily capture those frictions in the model. For consistency, all not calibrated parameters, including those of the stochastic process (11) for the government absorption of private sector output, and those of the stochastic process (12) for the public sector labor input, were estimated therefore with data for the period 1977-2012.

Given the linear structure of the model, the estimation of the unknown parameters and state variables can be accomplished with a straightforward application of the Kalman filter. Following standard practice, whenever required by the corresponding algorithm, the initial
values of the state variables were set equal to their steady state values.

The resulting sets of estimates of the state variables, autocorrelation coefficients, and relevant variances and covariances, one for each of the five Frisch elasticity values considered, were assumed to characterize the joint distribution of the stochastic variables that will enter in the calculation of the likelihood of the data in the subsequent step of the modified BCA methodology proposed in this paper.

Before proceeding to the last step of that adaptation, it is useful to note that, once parameter values and state variables have been revealed by the appropriate estimation procedure, the realization of the innovations to the wedges that replicate the data exactly in any year over the period 1977-2012 could be recovered from equation (15), which implies:

$$\omega_t = C^{-1}Y_t - C^{-1}DS_{t-1}.$$  

### 5.2.3 Incorporating The Budget Sequestration Spending Cuts

Notice that the decisions economic agents started to make once they became aware that the budget sequestration would materialize were influenced not only by the 2013 prescribed spending cuts, but also the subsequent ones through 2021. The equilibrium decision rules of the previous step are no longer valid, therefore, because they depend only on the previous period state variables. Thus, the new decision rules need to be recomputed with an algorithm that takes into account their dependence on non-stochastic policy regime changes that will be in effect in the future. Juillard (2006) suggested the general principle behind such an algorithm in the context of perturbation methods: treat perfectly anticipated current and future deviations of a policy variable from its steady state value as exogenous deterministic state variables and approximate the decision rules around the steady state with standard perturbation methods.

In the case of the spending cuts under study, the algorithm involves adding nine deter-
ministic state variable, one for each of the years in the period 2013-2021 over which the spending cuts mandated by the Budget Control Act would remain in effect, and modifying the state-space representation of the model accordingly, as follows:

\[ S_t = T S_{t-1} + Q \omega_t + \mathcal{M} \Delta_t, \]  

(16)

\[ Y_t = D S_{t-1} + C \omega_t + \mathcal{B} \Delta_t, \]  

(17)

where \( \Delta_t \) is a 9x1 column vector whose elements capture the sequence of spending cuts and \( \mathcal{M} \) and \( \mathcal{B} \) are conformable matrices, with dimensions 7x9.

Notice that the matrices \( T, D, C, \) and \( Q \) are the same as those obtained in the estimation stage because, as argued in section 2.1, the Budget Control Act prescribed the budget sequestration for a limited period of time and the temporary spending cuts that it implied were assumed, accordingly, not to affect the steady-state equilibrium of the model. Thus, the terms of the decision rules involving state variables that were already present in the model do not change, an implication consistent with setting the relevant parameters, including the elements of the matrix \( Q \), equal to the estimates from the previous stage.

The effect of the budget sequestration spending cuts on the decision rules is captured additively, by the elements in \( \mathcal{M} \Delta_t \) and \( \mathcal{B} \Delta_t \), where \( \Delta_t \) represents the deviations of the sequence of current and future spending cuts from their steady state value. Given the temporary nature of the spending cuts, their steady state value is zero. Taking into account that the steady-state private sector output has been normalized to one, the elements of the vector \( \Delta_t \) are, therefore, the spending cuts themselves. Formally:

\[ \Delta_{2013} = \frac{1}{100} [0.24, 0.49, 0.53, 0.54, 0.52, 0.50, 0.48, 0.45, 0.43]' \]

It is worthwhile to recall also that the variable \( \ln eg_{2013} \) in the vector \( Y_t \) is not directly
observable in 2013, but can be inferred by adding to the government absorption of goods and services observed that year, $g_{a_{2013}}$, the spending cuts that the budget sequestration prescribed for that year—the first element of the vector $\Delta_{2013}$.

5.2.4 Gauging the Ability of Frisch Elasticity Values to Account for the Budget Sequestration Evidence

The methodology above was designed to assess which of the Frisch elasticity values under consideration best accounts for the sequestration evidence with the following steps:

1. The seven exogenous shocks realized in 2013 were recovered, for each of the five Frisch elasticity values considered, from the system of seven equations in seven unknown implied by (17):

$$\omega_{i,2013} = C_{i}^{-1}Y_{2013} - C_{i}^{-1}D_{i}S_{i,2012} - C_{i}^{-1}B_{i}\Delta_{2013},$$

where $i = 1, 2, 3, 4, 5$ indicates that the elements of the matrix or vector correspond to those associated with the particular Frisch elasticity value $i$. Recall that the vector of state variables $S_{i,2012}$ was obtained in the estimation stage.\(^\text{10}\)

2. The Gaussian multivariate probability distribution of the seven observable macroeconomic variables in the vector $Y_t$ induced by the state-space representation of the model was exploited to calculate the likelihood of the 2013 observations of those macroeconomic variables, conditional on each Frisch elasticity value considered. As indicated earlier, all distributional parameters relevant for this calculation were fixed at the values obtained in the estimation stage.\(^\text{11}\)

\(^{10}\)Given that $C$ is a square matrix, this step is generally feasible, except in the rare occasion in which this matrix happens to be singular.

\(^{11}\)More specifically, the value of likelihood function for the year 2013 can be computed quite straightforwardly, with the formula [13.4.1] on page 385 in Hamilton (1994), after exploiting the isomorphism between the dynamic system of equations (14) and (15) and the system $\xi_{t+1} = F\xi_t + G\omega_{t+1}, Y_t = A'x_t + H'\xi_t$, 

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3. Finally, the ability of each Frisch elasticity value to conform with the evidence was ranked by the decreasing order of the corresponding value of the likelihood function.

6 Findings

Table 3 reports, in decreasing order, the value of the log likelihood of the data computed in the final step above, along with the corresponding Frisch elasticity value. The table readily reveals that, by the well-established accepted likelihood metric, the information about the aggregate Frisch elasticity contained in macroeconomic variables under the effects of the budget sequestration spending cuts tend to favor low, rather than high, values for that parameter.

[insert Table 3 about here]

One reason to be skeptical of this finding is that it has been obtained under the assumption that the spending cuts implied by the sequester were fully credible. To establish its robustness, the likelihood was recalculated under the assumption that households and businesses made their decisions in 2013 projecting that the spending cuts actually implemented would end up being only half the size of those originally intended. The choice of this alternative spending cuts scenario was not totally arbitrary. It was suggested by the result in Hu and Zarazaga (2016), who reported that alternative scenario, rather than the full-size spending cuts scenario, as more likely to account for the budget sequestration evidence when inspected under the lens of a model without investment adjustment costs and with a different specification of wedges.

where \( \xi_{t+1} = [S_t - M \Delta_t \omega_{t+1}]', \) \( F = \begin{bmatrix} T & Q \\ 0 & 0 \end{bmatrix}, \) \( G = [0 \ I]', \) \( I \) is an identity matrix, \( A' = B, \) \( x_t = \Delta_t, \) and \( H' = [D \ C]. \) To avoid misunderstandings, note that in Hamilton’s book the matrix \( Q \) denotes the variance-covariance matrix of the state variables, while in the paper, that notation is reserved for the matrix of coefficients of the shocks in the transition equation.
The results for the alternative scenario, reproduced in Table 4 below, replicate nevertheless the same ordering of the Frisch elasticity values obtained for the full-size spending cuts.

[insert Table 4 about here]

It is fair to recognize that the case for low Frisch elasticity values seemingly supported by the preceding analysis can be questioned, because it is based on the admittedly limited evidence provided by the immediate response of macroeconomic variables to the budget sequestration. On the other hand, it is precisely the evidence from such rare events, in the case of this paper, a policy regime change triggered by almost unprecedented circumstances, that it is coveted as possessing the "controlled experiment" characteristics particularly desirable for obtaining readings of specific economic relationships or parameters cloaked in the multiple influences typically contaminating more complex data sets.

In any case, limited as that evidence might be, its reliability as a source of information about the Frisch elasticity, as extracted in this paper, is buttressed by the exhaustive analysis of a more comprehensive data set with state-of-the arts econometric techniques in Ríos-Rull et al. Those authors obtained point estimates for the Frisch elasticity parameter in the fairly low range of 0.35-0.70 in the presence of government spending shocks and in the just a notch wider range of 0.30-0.85 in their absence. Equally comforting is that Smets and Wouters also reported, as mentioned earlier, a low point estimate for the aggregate Frisch elasticity of 0.52.

Overall, then, there is no obvious reason not to include the low Frisch elasticity value of 0.5 most favored by the budget sequestration evidence in the list of candidates for calibrating that parameter in representative agent, general equilibrium models, studying issues or phenomena unrelated to that episode, but in which the aggregate intertemporal labor substitution elasticity still plays an important role. It seems legitimate to ask, nevertheless,
which of the aggregate Frisch elasticity values considered in this paper would have been favored by the business cycle statistics predicted by the model, according to the more casual metric typically used by the RBC literature.

6.1 Implications for Business Cycles

Table 5 reports conventional business cycle statistics for the actual and model economy for three representative values of the Frisch elasticity. The first column lists the statistics examined and the second one, their corresponding values for the actual economy, calculated with the standard procedure from HP-filtered annual data for the period 1977-2007. The calculations deliberately excluded the Great Recession years and its aftermath, to avoid contaminating the business cycle statistics with the abnormally large deviations from trend that most macroeconomic variables exhibited over that period.

The entries in the third to fifth columns report the average value of the corresponding statistics for the model economy, obtained by simulating it 2,000 times for 31 periods, using as input simultaneous random draws from the distributions characterizing the stochastic innovations of the seven shocks. The entries in the sixth to eighth column show the business cycle statistics predicted by the actual economy when only the same TFP shocks used in the previous simulations are kept active.

As inspection of the table reveals, the model economy replicates well the business cycle statistics of the actual one, when all of the seven shocks are active. Of course, this ought to be expected, because after all the estimation step in section 5.2.2 was meant to do exactly that, to fit the model to the data.

In any case, the row in Table 5 with the legend "Sum squared deviations" in the first column provides a metric that serves the purpose of assessing which of the several Frisch
elasticity values considered in the paper would have delivered the best overall performance, in terms of replicating the business cycle statistics of the actual economy. The metric is borrowed from the simulated method of moments literature and it is simply calculated by subtracting each standard deviation predicted by the model in column 2 of the table from its actual economy analog, squaring each of the resulting differences, and finally, adding them all up. By this standard, the Frisch elasticity value with the smallest distance is 1.9, as shown by the number in bold in the fourth column of the table. The same metric was larger for the other two elasticity values, 1.0 and 2.5, not included in the table. In other words, if the selection of parameter values in a study focused on the business cycle phenomenon were not guided by the calibration principle, but by the goal of replicating as closely as possible the subset of business cycle statistics of the actual economy by the proposed metric, the aggregate Frisch elasticity parameter should have been set to the value of 1.9.

Inspection of the last three columns of Table 5 reveal that even if the "estimated" value of the aggregate Frisch elasticity is four times larger than the one most favored by the calibration approach, the TFP shocks alone are not successful in replicating key features of the business cycle statistics. In particular, even the relatively large aggregate Frisch elasticity value of 1.9 reproduces the anomalies encountered in the early RBC literature that labor input fluctuates much less than in the data and, also counterfactually, less than labor productivity. Another well-known prediction of models in which the burden of economic fluctuations falls on TFP shocks is the strong positive correlation between labor input and labor productivity, in contrast with the almost zero correlation between these two variables in the actual economy, as documented for completeness in the last row of Table 5. To be fair, the anomaly reappears in the opposite direction, although somewhat more moderately, when the model economy is under the influence of all shocks.

In any case, notice that for the large value of 3.0 that minimizes the proposed business cycle metric, also the largest of those considered in the paper, the TFP shocks-only model
economy can account for about one-third of the labor input fluctuations, a non-negligible fraction but nevertheless short of identifying those shocks as the dominant source of economic fluctuations. A casual extrapolation of the increase in the standard deviation of labor input as aggregate Frisch elasticity rises, suggests that a value considerably higher than 3.0 would be needed to reverse this model prediction.

Interestingly, this result is reminiscent of the one strongly suggested by Ríos-Rull et al., who found that TFP shocks account for 15% of labor input fluctuations when the Frisch elasticity parameter is set equal to 1 in the model specification with the same constant Frisch elasticity utility function for the stand-in household adopted in this paper. The analogous contribution obtained for that same elasticity value in this paper (not reported in Table 5) is 20%. It is hard to attribute the documented modest contribution of TFP shocks to economic fluctuations to the particular utility function just mentioned, because Ríos-Rull et al. found that for the alternative Cobb-Douglas utility function specification adopted by Prescott (2004), TFP shocks in their model economy account at the most for a third of the labor input fluctuations with the Frisch elasticity set equal to 2. It turns out that for one of the values of the Frisch elasticity very close to the one considered in this paper, 1.9, the contribution of the TFP shocks to labor input volatility implied by the model economy in this paper is almost identical, 28%, as can be verified in Table 5.

Needless to say, the usual caveat applies that the finding under assessment may not be invariant to details of the model specification and choice of calibrated parameter values, such as the one controlling the magnitude of the investment adjustment costs. On the other hand, a variety of macroeconomic studies that respect the general equilibrium discipline adopted in this paper have reached a similar conclusion, that the aggregate Frisch elasticity seems to fall in the low end of the range considered empirically plausible in the literature. That seems certainly to be the case, as already noted, in the studies by Smets and Wouters and by Ríos-Rull et al., which estimated that parameter with a Bayesian econometric approach.
More significant perhaps is the similarity of the result of this paper with that of Chetty et al. (2013), one of the other studies that also exploited the evidence associated with fiscal policy regime changes to infer the value of the aggregate Frisch elasticity.

It is true, though, that this paper, as well as those by Smets and Wouters, and Ríos-Rull et al., adhere to the widely accepted practice of assuming that one single parameter in the utility function of the stand-in household can eventually pick up from the evidence the higher Frisch elasticity implied by the extensive margin of the labor supply decision. None of these papers addresses, therefore, the specification concerns discussed in the introduction, that is, that models that don’t treat the intensive and extensive margins of that decision separately may be badly missing the dynamics of labor markets and consistently underestimating the aggregate Frisch elasticity. That concern might be dismissed with the observation that the study by Chetty et al. (2013) explicitly included in the analysis both labor supply margins. But their model, borrowed from the one that Rogerson and Wallenius proposed for illustration purposes, is highly stylized and omits too many features of actual economies that, as suggested in the introduction of this paper, may not be innocuous for the results they reported. Thus, their study doesn’t completely rule out the possibility that one of the reasons why many other macroeconomic studies keep finding that the evidence favors low values for the aggregate Frisch elasticity may be that they fail to consider explicitly the intensive and extensive margins of the labor supply. The goal of this paper was not to address this intriguing possibility, but a corollary of its findings may well be that that conjecture deserves more attention than it has received until now.

In any case, it seems appropriate to close this section assessing the main result of the paper with a perspective that applies also to those of other studies that have summoned the courage to attempt to extract the value of the elusive aggregate Frisch elasticity with models in the general equilibrium tradition. Perhaps the nature of the task is analogous to that of detecting exoplanets from the almost imperceptible wobbles they induce in the
position of their host star as they orbit around it. It takes a great number of observations of such wobbles to conclude with some confidence that a distant planet is responsible for them. The analogy suggests that it will take several fiscal policy "experiments" such as those studied by Prescott, Chetty et al., (2013) and this paper, to reach more reliable conclusions about the magnitude of the aggregate Frisch elasticity from the "wobbles" they induce in macroeconomic variables. From this more detached viewpoint, the particular measure of the aggregate Frisch elasticity obtained in this paper is just one of the many that will be needed to reach an agreement on its magnitude. The measurement instruments will matter too. Along this line of thought, a more significant contribution of this paper may be perhaps to have suggested how to build, with elements borrowed from the BCA approach, one such instrument potentially useful to obtaining further measurements of that elasticity the next time the opportunity arises.

7 Concluding Remarks

The size of the marginal-utility-of-wealth-held-constant labor supply real wage elasticity at the aggregate level hasn’t been settled in the profession yet. Depending on the particular microeconomic or macroeconomic study that has attempted to measure it, this elasticity can be as low as 0.5 or as high as 3.0. This is an unfortunate state of affairs, because the differences between the end values in this range matter a lot for the outcomes of a variety of public policies, such as the fraction of tax cuts eventually self-financed through endogenous labor supply feedback effects, as well as for gaining insights into important macroeconomic phenomena, such as the nature of business cycles or the empirical relevance of liquidity traps.

For that reason, time and energy continue to be devoted to try to measure the aggregate Frisch elasticity from different angles. Fiscal policy regime changes with experiment-like features offer a fertile ground for those measurement efforts, because the transitional dy-
namics effects they induce in macroeconomic variables depend in part on the magnitude of the aggregate Frisch elasticity. That was precisely the motivation that led Chetty et al. (2013) to examine the evidence associated with a 1987 tax holiday in Iceland and a change in welfare benefits in Canada in the 1990s. After reading such evidence with the lens of the representative agent macroeconomic model proposed by Rogerson and Wallenius, they arrived at the recommendation that those models should be calibrated to match a Frisch elasticity of aggregate hours of 0.75. And it was by examining differences in labor income tax rates across countries that Prescott (2004) reached the conclusion that the aggregate Frisch elasticity should be calibrated to the much larger value of 3.0.

This paper couldn’t resist the temptation of exploiting, therefore, with the same measurement purpose, the evidence associated with a rather unique fiscal policy development with unusual features, the non-negligible temporary government spending cuts initiated in the U.S. by a 2013 mandated budget sequestration procedure.

As a first step in the process of teasing out the aggregate Frisch elasticity value most consistent with the budget sequestration evidence, the paper presented a chronology of events and previous findings that established, with considerable confidence, that households and businesses didn’t expect that the contingency that would trigger the spending cuts, prescribed in the 2011 Budget Control Act, would materialize until it actually did in 2013. The performance of macroeconomic variables in that year can be interpreted, therefore, as capturing the effect of exposing households and businesses to the "controlled experiment" of suddenly, as if without previous warning, reducing the government absorption of private sector output for about a decade.

The next step was to construct a measuring device suitable for extracting the information about the value of the Frisch elasticity revealed by that policy regime change. To that end, a representative agent model economy, within the general equilibrium, balanced growth paradigm, was built with a methodological approach inspired by the "event study" and
Business Cycle Accounting traditions.

Following a procedure entirely analogous to that in Chetty et al. (2013), the sequence of spending cuts implied by the budget sequestration "policy experiment" was fed into the model economy, with the goal of establishing which Frisch elasticity value, among several considered, generated model predictions more consistent with the performance of key macroeconomic variables—private sector output, consumption, investment, and government absorption of goods and services among them—for the year 2013.

This paper finds that, by the standards of the well-accepted metric provided by the value of the likelihood function, that evidence suggests that the aggregate Frisch elasticity ought to be calibrated, in representative household, general equilibrium models addressing other issues in which it plays a critical role, to values closer to the low end than to the high end of the 0.5-3.0 range of estimates that have been proposed as empirically plausible in the literature.

It is somewhat reassuring that this finding, despite differences in assumptions and model specifications, is consistent with that of other studies similarly inspired by fiscal policy changes with experimental features, such as the one by Chetty et al. just mentioned, as well as with the estimates of that parameter with econometric techniques obtained by Smets and Wouters and Ríos-Rull et al.

In closing this summary of the motivation, methodological approach, and results of the paper, it is worth forcefully reiterating that its main finding, that the aggregate Frisch elasticity is rather low, should not be taken as the last word on the subject. On the contrary, it will take many observations to narrow down the wide range of values of that elasticity currently deemed empirically plausible by different criteria. Returning to the analogy with the astronomical sciences suggested earlier, pinning down the value of that elusive elasticity from the movements it induces on macroeconomic variables may not be that different, after all, from the challenging task of detecting exoplanets from the miniscule wobbles they induce
on the trajectory of their host stars as they orbit around them. From this more detached viewpoint, the particular measure of the aggregate Frisch elasticity obtained in this paper is just one of many that will be needed to reach an agreement on its magnitude. The measurement instruments will matter too. On that count, this paper has shown how to build, with elements borrowed from the Business Cycle Accounting approach, one such instrument, easy to adapt to different economic environments and, for that reason, potentially useful to obtain further measurements of the aggregate Frisch elasticity when another rare opportunity to do so arises.
References


Table 1: Annual budget sequestration spending cuts

<table>
<thead>
<tr>
<th>Year</th>
<th>$ billion (*)</th>
<th>% of model economy output (**)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>35</td>
<td>0.24</td>
</tr>
<tr>
<td>2014</td>
<td>75</td>
<td>0.49</td>
</tr>
<tr>
<td>2015</td>
<td>85</td>
<td>0.53</td>
</tr>
<tr>
<td>2016</td>
<td>89</td>
<td>0.54</td>
</tr>
<tr>
<td>2017</td>
<td>90</td>
<td>0.52</td>
</tr>
<tr>
<td>2018</td>
<td>90</td>
<td>0.50</td>
</tr>
<tr>
<td>2019</td>
<td>89</td>
<td>0.48</td>
</tr>
<tr>
<td>2020</td>
<td>88</td>
<td>0.45</td>
</tr>
<tr>
<td>2021</td>
<td>87</td>
<td>0.43</td>
</tr>
</tbody>
</table>

(*) Congressional Budget Office (2013), p. 10 and Table 1-5, p. 27.
(**) Authors’ calculations.
Table 2: Calibrated parameters and corresponding values

<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Steady-State value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ (discount factor)</td>
<td>0.9546</td>
</tr>
<tr>
<td>$\eta$ (working-age annual population net growth rate)</td>
<td>0.0126</td>
</tr>
<tr>
<td>$\gamma$ (TFP annual net growth rate)</td>
<td>0.0078</td>
</tr>
<tr>
<td>$\delta$ (depreciation rate)</td>
<td>0.0621</td>
</tr>
<tr>
<td>$i$ (before-tax annual net rate of return on private capital)</td>
<td>0.0858</td>
</tr>
<tr>
<td>$y_{ss}^{pr}$ (steady-state private sector output)</td>
<td>1</td>
</tr>
<tr>
<td>$x/y^{pr}$ (investment-output ratio)</td>
<td>0.2121</td>
</tr>
<tr>
<td>$k/y^{pr}$ (private capital–private sector output ratio)</td>
<td>2.5681</td>
</tr>
<tr>
<td>$\theta$ (private capital income share)</td>
<td>0.38</td>
</tr>
<tr>
<td>$g_y$ (fraction of private sector output absorbed by general government)</td>
<td>0.0825</td>
</tr>
<tr>
<td>$v_y$ (government enterprises value added–private sector output ratio)</td>
<td>0.0156</td>
</tr>
<tr>
<td>$\sigma_{vy}$ (standard deviation of $v_y$)</td>
<td>0.0856</td>
</tr>
<tr>
<td>$n_{iy}$ (net exports–private sector output ratio)</td>
<td>0.026</td>
</tr>
<tr>
<td>$h_{ps}^{pr}$ (fraction of time worked in private sector)</td>
<td>0.21</td>
</tr>
<tr>
<td>$h_{ps}^{ma}$ (fraction of time worked in public sector)</td>
<td>0.03</td>
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<tr>
<td>$\tau_{ss}^{c}$ (capital income tax rate)</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau_{ss}^{l}$ (labor income tax rate)</td>
<td>0.23</td>
</tr>
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Table 3: Frisch elasticities and corresponding log likelihood value

<table>
<thead>
<tr>
<th>Frisch elasticity value ($\varphi$)</th>
<th>Log likelihood of observables ($Y_t$) in 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>11.063</td>
</tr>
<tr>
<td>1.0</td>
<td>11.032</td>
</tr>
<tr>
<td>1.9</td>
<td>10.996</td>
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<tr>
<td>2.5</td>
<td>10.980</td>
</tr>
<tr>
<td>3.0</td>
<td>10.969</td>
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</table>
Table 4: Frisch elasticities and corresponding log likelihood value for spending cuts half the size of those prescribed by budget sequestration

<table>
<thead>
<tr>
<th>Frisch elasticity value (φ)</th>
<th>Log likelihood of observables ($Y_t$) in 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>11.178</td>
</tr>
<tr>
<td>1.0</td>
<td>11.159</td>
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<tr>
<td>1.9</td>
<td>11.135</td>
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<tr>
<td>2.5</td>
<td>11.125</td>
</tr>
<tr>
<td>3.0</td>
<td>11.118</td>
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Table 5: Business Cycle Statistics, Actual and Predicted

<table>
<thead>
<tr>
<th>Model</th>
<th>Frisch elasticity ((\varphi))</th>
<th>Frisch elasticity ((\varphi))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All shocks</td>
<td>TFP shocks only</td>
</tr>
<tr>
<td>standard deviation (in %)</td>
<td>Data 0.5 1.9 3.0</td>
<td>0.5 1.9 3.0</td>
</tr>
<tr>
<td>(y^r_t)</td>
<td>2.17 1.72 1.95 2.05</td>
<td>1.28 1.48 1.55</td>
</tr>
<tr>
<td>(c_t)</td>
<td>1.82 1.33 1.25 1.23</td>
<td>0.65 0.72 0.74</td>
</tr>
<tr>
<td>(h^r_t)</td>
<td>2.02 1.80 2.08 2.20</td>
<td>0.25 0.56 0.68</td>
</tr>
<tr>
<td>(x_t)</td>
<td>6.75 5.97 6.89 7.24</td>
<td>3.64 4.37 4.63</td>
</tr>
<tr>
<td>(\frac{y^r_t}{h^r_t})</td>
<td>0.76 1.30 1.28 1.28</td>
<td>1.05 0.95 0.91</td>
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<tr>
<td>Sum squared deviations</td>
<td>– 1.39 0.66 0.89</td>
<td>15.00 9.51 7.87</td>
</tr>
<tr>
<td>(correlation(h^r_t, \frac{y^r_t}{h^r_t}))</td>
<td>0.01 -0.40 -0.38 -0.39</td>
<td>0.92 0.88 0.87</td>
</tr>
</tbody>
</table>