

# Incentives and Self-Selection in Fostering Violence Levels in Conflicts

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## **Abstract**

In an approach broadening incentives to non-economic dimensions, we analytically investigate decentralized incentives, exogenous shocks and self-selection in explaining political violence in civil conflicts and wars. The focus is on the mechanisms that trigger individuals' decisions to: (1) join the combatants, (2) actually fight, or (3) donate resources to support the combatants.

In a game theory model of self-sorting into combatant vs. producer roles, we identify some main driving forces that trigger violent conflicts: opportunistic versus defence incentives, and coordination of producers and combatants through transfers and protection. We derive economic, demographic and ideological determinants of (1) the share of loot allocated to soldiers, (2) the relative size of the army, and (3) the soldiers' fighting intensity. Beyond a rich comparative statics, three increasingly violent types of society emerge from the analysis that may explain violence escalation. We discuss the existence and uniqueness of the conflict equilibrium and efficiency issues.

**Keywords:** violent conflict, looting, psychological trauma, personal damages, war, self-selection, incentives.

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# 1. Introduction

## 1.1. Theory

This is an attempt to tackle the complexity of the development of violence in a conflict context from the point of view of individual agents within one given society. We start from an existing war/conflict context and disaggregate a society in order to study analytically the level of violence generated by individual choices. We do not deal with ethnic polarization, strategic interactions of opponents, grievances and electoral outcomes.

This paper analytically investigates the triggers of violent conflict in communities that face a belligerent opponent. It does so by studying the self-sorting of agents into combatant or producer roles and by describing the potential stakeholders' scheme of war-related incentives. Three kinds of incentive are introduced—monetary gain, personal protection motives, and ideological/psychological tendencies—within a game theoretical framework. We examine several mechanisms that explain decisions by individual agents to (1) join the combatants, (2) actually fight or even kill, or (3) donate resources to support the combatants. We first discuss the theoretical literature. Then, we turn to empirical evidence in the applied literature.

### 1.1.1. Activity choice

The classic theoretical literature on violent conflict has tended to present individual participation in violent conflict as a decision of labour allocation akin to the way it is traditionally modelled in general equilibrium theory. In this perspective of activity choice between 'productive activities' and 'extortion activities through violence', the distinction often boils down to specifying different technology or reward functions, on the one hand, and discussing the role of markets, transfers and other economic incentives, on the other. In particular, the technology for violent activities is frequently characterized by a 'contest success function' that determines the outcome of the violent conflict and the rewards that can be obtained from it—generally nothing when the agent has been defeated. It seems fair to say that in this perspective the incentives driving the involvement of individuals in

violent activities are mostly seen as directly economically grounded in the respective activities, and often assumed to be measurable in terms of monetary gain.<sup>1</sup>

In Hirshleifer (1991), this kind of setting is used to explore the trade-offs between spontaneous activities ('anarchy') and the emergence of hierarchy, which suggests allowing for controlling, with some kind of social planner or leader. Various institutional environments have been explored for these questions, such as the possibilities of despotism where a leader grab all surplus (Usher, 1989), whether or not property rights can be enforced (Skaperdas, 1992), and whether different intertemporal fighting strategies are available (Bueno de Mesquita, 2013).<sup>2</sup>

Note, however, that in all these approaches this is a share of time that is modelled as allocated to alternative activities rather than individuals devoting themselves fully to war or to production. This modelling approach implies that discrete changes, or discrete costs for individuals, in the two alternative situations, where only for personal safety, are ironed out from the model. Note also that in this literature the general equilibrium perspective is emphasized, although such a setting may break down if full participation in fighting is reached in high violence contexts or, conversely, if combatants do not receive any share of the loot, a typical feature of modern armies under low violence. These two features may correspond to corner solutions, often overlooked in typical general equilibrium models.

One might expect models of anarchy and limited property rights to generate links between low income and violence. The theoretical connection between income and armed civil conflict, however, is not so clear cut (Blattman and Miguel, 2010). For example, Esteban and Ray (2008) suggest that 'The rich within a group can supply the resources for conflict, while the poor supply conflict labor.' In a contest model such as ours, the booty is a direct function of the rival group's resources. Hence, on the one hand, when the

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<sup>1</sup> Becker (1968) presented a seminal article where violent activities (crime, in this case) were analysed in a framework similar to other economic activities. Hirshleifer (1988, 1989, 1991), Garfinkel (1990) and Skaperdas (1992) developed models in which labour allocation between production vs. violent activities, often with opposing side, was confronted through a contest function. In Skaperdas (1992), agents decide on the allocation of financial resources for production or arms, whereas in Grossman (1991), peasants decide how to allocate their labour time among production, soldiering and insurrection.

<sup>2</sup> Grossman (1991) is an additional indicator in this vein of models where individual decisions to participate in violence can be related to income distribution and depend on the technologies of violence and production.

possibility of looting by the opponent is considered, the greater the national wealth, the more there is to fight over; thus, in standard formulations, the greater the equilibrium effort devoted to fighting instead of producing.<sup>3</sup> Moreover, larger resources may imply more generous financial support for combatants. Lower resources, however, may also make production less individually attractive than fighting if the resource scarcity is related to lower labour productivity. The complexity of the way shocks affect the returns and costs of conflict in general equilibrium has also been examined in Bò and Bò (2011).

### **1.1.2. Other incentives**

However, some social scientists have long considered that the individual incentives to take part in violent fighting are more varied than mere monetary gain.

Whereas in the classic crime-economics literature (Becker, 1968) agents are motivated by pure greed, several recent political-economic studies emphasize the ideology-oriented motivation of agents who join the soldiering population. Others allude to the possibility of an endogenous relationship between greed and grievance (Cuesta and Murshed, 2008; Regan and Norton, 2005).

From households' point of view, one important dimension of violent conflict is personal safety, in terms of being free from physical violence or damage, health hazards related to war and property losses. Grossman and Kim (1995) may be seen as an early attempt to incorporate safety considerations by distinguishing offensive weapons and fortifications in a general equilibrium model. This may generate more subtle results such as poor agents who may be better off under less secure claims to property. A similar motive is that of citizens who choose to arm themselves for protection from robbery or for self-defence (Boyd, Jalal and Kim, 2006. Lavie and Muller (2011) specifically introduce survival incentives jointly with monetary incentives in a rational equilibrium of combatant fighting decisions.

More generally, and as advocated by Rabin (2002), integrating psychological and economic motives should better allow researchers to understand the mechanisms of violent events, as in many economic problems. This has been a long tradition in economic analyses of violence, including the use of violence as a psychological weapon

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<sup>3</sup> See also Garfinkel and Skaperdas (2007) and Grossman (1991).

in terror strategies (for example, in Azam and Hoeffler, 2002). Such a standpoint implies taking seriously the psychological damage of suffering or exerting violence.

A different psychological dimension is that of ideological motives. Some people may be pushed to violence by ideological enthusiasm or, conversely, be deterred from it by ideological reasons such as pacifism. Ideology can also be manipulated by politicians and leaders in order to foster hate and violence (see, for example, Glaeser, 2005, 2006). Conflict-inhibiting norms comprise another kind of ideological feature that may vary considerably or not at all in a given population (Lesson and Coyne, 20??**AV**). However, the role of ideological heterogeneity does not seem to have been much studied in the theoretical literature about violent conflicts.

### **1.1.3. Support and enlistment**

If combatants can be financially motivated to participate in fighting, the source of funding must be specified. It may be direct individual looting, although this seems to be more likely in guerrillas and informal armies than in organized armies. More common perhaps is centrally organized, and even planned, looting, which suggests a role for a military leader. Looting in relation to army recruitment is studied by Blain and Pallage (2008). Another possibility is that the combatants may be directly financially supported by the population—for example through food delivery, shelter and supply of other consumable goods. Furthermore, specifying as a personal benefit a booty share of the leader serves as a defence budget for the army. So, the leader may also contribute some weapons supply, for example.

The way combatants can be incentivized or recruited has been the object of recent theoretical investigations. For example, Weinstein (2005) points out the importance of the importance for leaders to make credible promises about the private rewards to combatants. Such a mechanism could allow the leader to select the most ideologically motivated combatants, which brings us back to ideological motives. Gates (2002) proposes a simple model in which geography, ethnicity and ideology contribute to military success and deterrence from defection.

On the whole, most current theories on violent conflict tend to bypass the issues of enlistment of fighters and short-term financing of paramilitary organizations. As

discussed in Blattman and Miguel (2010), collective action problems are pervasive and not resolved theoretically. Some solutions have been attempted in the literature, such as emphasizing the leader's ideological charisma (Roemer, 1985), but in a rather ad hoc way. Introducing individual ideological and self-protection motives, as well as financial incentives generated by producers and the leader, will much alleviate these issues in our model.

Kuran (1989) proposes a model where people hide their desire for change as long as the opposition seems weak. In a model of mass mobilization, Koster, Lindelauf, Linder, and Owen (2008) show how ideological support can depend upon the perceptions of the actual state of support. These discussions raise the question of incorporating some measure of the size of participation into violent conflict models and interacting it with original incentives, such as survival or ideology. This is a route we follow with a central role of the relative proportion of producers and combatants in the logic of the equilibrium.

Bloch and Rao (2002) analyse how violence can be used as an incentive instrument for dowry violence in India. This approach could be extended to recruitment purposes in that it affects damage to agents who decide not to participate.

Ultimately, as highlighted in Besley and Persson (2010), under risk of severe conflict, the state's capacity to raise revenue affects the state's stability in various ways. This capacity contributes to stabilizing the fighting force through financial incentives. In our model, the taxation of producers to fund combatants, and more or less voluntary donations from the population to combatants, appear as major mechanisms for sustainable fighting.

Let us now examine how some of these theoretical ideas are supported by the empirical literature.

## **1.2. Empirical Literature**

### **1.2.1. Activity choice**

The dichotomy and the interaction between productive and violent activities are also salient in empirical work. For example, at country level, trade and peace have been found

to be correlated, although asymmetric information issues may generate higher probability of war in countries more open to global trade, as found in Martin, Mayer, and Thoenig (2008).

At a microeconomic level, arbitrages between productive and fighting activities have been found—for example, in India’s Naxalite conflict by Van den Ende (2011), among many other authors and contexts. Note that general income shocks may have ambiguous effects, as found in Columbia for price shocks by Dube and Vargas (2013), since they may both raise the opportunity cost of fighting and its reward from looting.

A related result that has been found in most contexts is that internal war is robustly correlated with low per capita incomes, as stated in Blattman and Miguel (2010), even though the causal relationship between both variables is generally unclear.

### **1.2.2. Other incentives**

The traumatic impact of exposure to violence is well known and can last over long periods, as shown by Kim and Lee (2012) in the case of the Korean War. Heterogeneous psychological sensitivity to violent events can be related to the way cultural norms deal with conflicts. For example, in Qing China, Kung and Ma (2014) found that counties with stronger Confucianism norms were less affected psychologically by producers’ rebellion.

In general, there is no reason to believe that any individual would feel the same as another about his participation in violence. Besides, tastes for violence or revolt have been found to be heterogeneous in European populations. Recent political-economic studies emphasize the ideology-oriented motivation of agents who join soldiering groups. Using data gathered from newspaper reports, Chen (2010) finds that areas of high baseline religiosity experienced more social violence than more secular areas in the aftermath of the Indonesian financial crisis. Krueger and Maleckova (2003) claim that terrorists’ primary motive lies in the mobilization of passionate support for their cause, and the cultivation of feelings of indignity or frustration, rather than poverty and education, which play minor roles. Others allude to the possibility of an endogenous relationship between greed and grievance (Cuesta and Murshed, 2008; Regan and Norton, 2005).

### 1.2.3. Enlistment and support

Lichbach (1994, 1995) illustrates how successful social movements offer selective material incentives to joiners. Popkin (1979, 1988) finds that political entrepreneurs developed mechanisms to directly reward a producer rebellion in Vietnam. Weinstein (2007) discusses how rebel fighters in Mozambique, Sierra Leone and Peru were remunerated by being allowed to loot civilian property and traffic in drugs. Material incentives may also be non-pecuniary. Where violence against civilians is commonplace, joining an armed group has often been a path to relative safety.<sup>4</sup> The prestige associated with martial success may also be valued for itself.

Military operations are expensive. The ability of local leaders to initiate a war is strongly linked to their ability to recruit enough soldiers and provide them with sufficient economic resources. Collier and Hoeffler (1998) suggest that net costs during a conflict may often be compensated by future expected earnings, possibly provided by civilian supporters. Recent evidence from Africa, for example, suggests that various fighting armies obtained most of their economic resources internally. Collier, Hoeffler, and Rohner (2009), using country-level panel data to investigate the determinants of civil war in the past 45 years, report evidence in support of a feasibility hypothesis: where a rebellion is financially and militarily feasible, it will occur. More specifically, armies often finance themselves by organized looting and fundraising.

Gates (2002) examines the organizational structure of rebel groups to understand patterns of recruitment and allegiance. He discusses how geography, ethnicity and ideology may play important roles in military recruitment and success. Labour and production opportunities may also affect combatant recruitment and violence, as Van den Ende (2011) finds empirically.

Nonetheless, most current theories of violent conflict tend to bypass the issues of enlistment of fighters and of short-term financing of paramilitary organizations. This is problematic, it having been argued that conflicting groups do not act in a unitary fashion and that leaders' decisions regarding collective action explain soldiers' enlistment and predations against civilians (Keefer, 2010). The current paper fills this gap.

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<sup>4</sup> See Goodwin (2006), Kalyvas and Kocher (2007), Lichbach (1995) and Mason and Krane (1989).



In modern civil conflicts, recruitment methods vary from being qualified to being abducted. It seems, however, that many armies allow combatants to choose the nature of their involvement. Humphreys and Weinstein (2008), using a unique data set that contained interviews with ex-combatants in post-war Sierra Leone, report: “In a response to a question about one’s reason for participation, 70 percent of CDF fighters reported joining because they supported the group political goals while about 10 percent of RUF recruits identified ideology as a motivation” (p. 438). Rather than protecting themselves by buying guns, as Doepke and Eisfeldt (2007) suggest, citizens may find it better to fund security specialists.

### **1.3. The Gaps We Fill**

Reflecting on the past theoretical and applied literatures has revealed several major missing theoretical elements. We fill these research gaps in this paper.

Firstly, most models in the literature deal with general equilibrium allocations, sometimes determined by a social planner or by a despot, or individualistic or ‘anarchic’ perspectives, this being convenient to model the incentive system of combatants in a simple and tractable fashion. However, in many violent conflicts—notably civil wars and rebellions—support by the local population is what allows the sustainability of insurrections and fighting. Indeed, combatants may find shelter, food and other support directly from local supporting households, rather than or complementarily to what they receive from the state or their hierarchy. Beyond being empirically and practically relevant, this is theoretically important, as opening such a financing channel may comprehensively change the balance and the nature of the global equilibrium. Such extension has not yet been explored, to our knowledge, and we endeavour to do it.

Secondly, the typical theoretical models in the literature allow for no or limited agent heterogeneity. This is not only an issue for deriving results in terms of distribution of effects on populations (for example, inequality analyses); it is also problematic if the fighting participation rate is a decisive component of the contest success function, which it should be in most situations. We deal with this lacuna by incorporating two sources of heterogeneity that allow for subtle and varied results. The first is the individual heterogeneity in productivity, which we also alternatively specify as individual

production shocks. The second is a novel notion of the ideological (or, alternatively, psychological) heterogeneity of individuals, which contributes to an explanation of how the sorting between producers and combatants takes place. We study how the relationship between these two kinds of heterogeneity affects the theoretical equilibrium.

Thirdly, little or no attention has been devoted to corner solutions, and associated discrete changes, in conflict models. We investigate two kinds of corner: complete specialization of labour supply in the fighting sector on the one hand, and zero looting reward allocated to combatants on the other. Corner solutions are important because they yield qualitatively different properties and comparative statics. The occurrence of corner solutions may also be more likely when dealing with violent and dramatic conflict situations that may correspond to large displacement of equilibrium situations or large changes in parameter levels.

Finally, many results in the literature correspond to monotonic comparative statics effects, which may limit the capacity of such models to describe complex environments. The rich set of innovative features in our model allows us to go beyond these unattractive theoretical rigidities. We perform this task through several means. Firstly, we incorporate new effects and new outcome variables (both kinds related to joint income, damage limitation and ideological/psychological motives). Secondly, we mix transitions between variation regimes and comparative statics results, which enables us to generate more flexible and varied results.

## **1.4. Our Modelling Strategy**

Three kinds of incentive affect the decision making of agents during a conflict: damage protection, income and ideology.

### **1.4.1. Personal damages**

Direct effects of a violent conflict are death, physical injury, mutilation, psychological shock and property damage. We address those effects as personal damages concerns that contribute to the utility function of agents, and as such shape their behaviour. We allow agents to influence the damages they suffer by two types of decision.

Firstly, agents can choose their activity sector. It is assumed that civilians and soldiers experience different damage patterns—for example, in traditional wars, soldiers generally face a higher probability of getting killed, while this may not be the case in civil wars. We assume that agents know in advance what the relative risk level of each sector is, and respond to that. We also allow for soldiers to change their mind and defect (avoid fighting).

Secondly, a civilian may transfer (or donate) resources to the fighting sector in order to improve their own personal security. We assume higher donations have a general effect and an individual effect on civilians' survival.

Finally, we assume a direct negative monotonous relationship between the winning probability of a party (or contest function) and the damages suffered by its members, at least in their perception of the situation.

#### **1.4.2. Ideology**

Whereas in the classical crime-economics literature (Becker, 1968) agents are motivated by pure greed, political scientists, sociologists and historians also emphasize the ideology-oriented motivation of agents (ref). As described before, those notions are also well supported empirically.

Participation in fighting, and possibly killing, as part of an army involves psychological and moral costs. Our model addresses this point by introducing an ideological component into the agent's decision-making process. We assume that ideological orientation may determine the psychological non-monetary payoff of the fighting members of the army. However, such an effect may be ambiguous. Even more, fighting may generate two simultaneous while opposite effects: a drop in utility due to the psychological cost of killing, and a rise in utility from the psychological reward for protecting members of the community or serving an ideal. Either way, we can describe these phenomena as generating a positive effect of ideology—on a certain ideology spectrum—on utility. On the ideological spectrum, highly ideological agents are expected to derive higher positive utility from favourable fighting and lower disutility due to the psychological cost of killing. It is possible to examine how a leader could exploit this

sensitivity to ideology through a propaganda strategy, as in Glaeser (2005). However, we resist this in this paper to avoid complicating too much an already intricate model.

### **1.4.3. Payoffs**

We assume that when choosing a sector, agents face a trade-off between producing goods and looting. By doing so, we differ from the classic ‘guns versus butter’ models based on time allocation by individuals instead of sector choices. Besides being more realistic, our specification allows the allocation of resources based on agents’ comparative advantages (production vs. fighting).

Production and looting are modelled in a conventional way: production is a function of an individual productivity parameter, whereas looting is increasing in the winning probability and in available resources. Booty may also not be taken literally. This term may indeed cover any direct financial incentive to engage in fighting that can be related to a given global potential benefit. In addition, we assume direct transfers from civilians to soldiers that affect the disposal income of both without changing the aggregate income.

### **1.4.4. The analytical framework**

Our point of departure is an existing conflict situation. Thus, we avoid questions of bargaining failure between opponents as discussed by Jackson and Morelli (2010), for example, in order to focus on internal society mechanisms that trigger an escalation of violence. We construct a rational equilibrium framework that shows how violent conflict emerges from agents’ interactions and self-sorting among producer and combatant roles. As a leading example, we consider the reciprocal relationship between an army and supportive producers.

In this framework, we analyse the effect of internal factors (productivity shocks, inequality and ideology) and external factors (relative economic resources, opponents’ military strength) on the level of violence and its incentives. Our results suggest that the various factors may be grouped into three channels of incentives that could be used as instruments by leaders, governments and other political decision-makers: (1) ideological heterogeneity across agents that stimulate/inhibit participation in violence; (2) financial rewards for combatants that may be allowed to share a prize obtained during a violent

contest (looting); and (3) cash transfers from producers to combatants (in the form of taxation or voluntary donations). Using the leader strategy as an organizing device, we elicit three different variation regimes that correspond to three qualitatively distinct types of society: defence, militarized and predator.

#### **1.4.5. Main findings**

The model first predicts that (1) the most ideologically radical agents join the combatants, while the others support them from outside; and (2) labour productivity in the production sector affects the intensity of the war being fought.

However, while a permanent negative aggregate shock amplifies war intensity, a transitory negative aggregate shock mitigates it. This result originates in the sequence of the events: a permanent shock affects agent behaviour before self-selection. When work becomes less productive, more agents prefer to join the more financially rewarding army. In contrast, once the roles are chosen, soldiers rely partly on producers' donations. In the absence of such donations, some soldiers may choose not to fight. Under these conditions, a transitory negative shock decreases the producers' income and, in turn, their donations, perhaps ultimately diminishing the population of fighting soldiers by promoting defection.

We also show how larger inequality between groups raises the likelihood of a large-scale conflict. Finally, we establish a link between the institutional fundraising mechanism and the size and cohesion of the army.

Before turning to the sequence of stages in the model, we set forth the map of the rest of this paper. Section 2 describes the benchmark model and its outcomes. Section 3 introduces heterogeneity via the individual shocks, presents the results with endogenized donations, and introduces an additional aggregate shock. Finally, section 4 concludes.

## **2. The Benchmark Model**

In our model, a society faces a potential opponent under a constraint of scarce resources. Leaders, soldiers and non-fighting supporters are affected, on the one hand, by the redistribution outcomes of the war and, on the other hand, by personal damage risk and ideological/psychological shocks. The model suggests that the number of combatants and the strength of their commitment are subject to decisions made by the agents at various stages along the warpath. Before the fighting begins, agents choose their level of involvement and their position—fighting/non-fighting—on the basis of the outcomes they expect; subsequently, non-fighting civilians decide how much to contribute to the war effort. Finally, soldiers decide whether or not actually to fight. This decision-making sequence shapes not only the individual’s destiny, but also the overall outcome of the conflict.

We consider two main agents, schematically called ‘soldiers’ and ‘producers’. Soldiers participate in fighting and looting; producers produce goods and finance the soldiering population.<sup>5</sup> Therefore, the two sources of material resources in this economy, production and looting, correspond to people’s choices between these activities. The model uses several motivational axes that combine monetary rewards, personal damages that individuals may suffer, and psychological/ideological return to their actions. The third agent in the model is the leader, who is assumed to be selfish and money-oriented.

While ideology and personal defence needs are assumed to be determined exogenously, their effect on agents enables the leader to adjust his expenditure on incentives. As in Weinstein (2005), the leader’s management of resource endowment may shape the character and the conduct of the combatant group.

## **2.1. General setting**

### **2.1.1. The technology of conflict**

Consider two contending parties, A and B. We denote their relative power—that is, the number of soldiers belonging to each party—as  $S_A$  and  $S_B$ . For any combination of

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<sup>5</sup> Boyd et al. (2006) use a similar dichotomy to study a general equilibrium environment in which the only activity of interest is armed robbery. The agents choose to be citizens or robbers and to purchase handguns or not. By arming, citizens can protect themselves from robbery. The government chooses the intensity of police efforts to arrest would-be robbers and to arrest citizens who arm for self-defence.

relative fighting power, the parties have an expected winning (or losing) probability. We define A's probability of winning as  $\omega(S_A, S_B)$ . Typically,  $\omega$  increases in  $S_A$  and decreases in  $S_B$ . Following Hirshleifer (1989), we use a functional form such that the probability of winning depends on the ratio of fighting power between the parties,  $\frac{S_A}{S_B}$ .

$$\omega(S_A, S_B) = \frac{S_A}{S_B} \quad (1)$$

The parties compete over a resource of monetary size  $\pi_B$ . Assume continuous fighting with constant looting of resources, with the value of booty acquired by party A given by:

$$L = L(\pi_B, \omega) = \pi_B \omega, \quad (2)$$

where  $L$  is increasing in both  $\pi_B$  and  $\omega$ .

### 2.1.2. Fighting force and labour force

Each party comprises a unit mass of utility maximizing agents. Our analysis focuses on the sorting decisions of members of A:<sup>6</sup> an agent may decide to join the fighting force (become a soldier) or commit to production by remaining a producer. We define the share of soldiers in society A as  $S^A$  and the share of producers as  $P^A = 1 - S^A$ .

Agents' utility is based on payoffs, personal damages and an idiosyncratic ideological element. For expositional purposes, we assume that the utility function is additively decomposable into three corresponding components:

$$U_i(\pi_i, H_i, I_i) = \pi_i + \log(H_i) + I(\psi_i, k_i), \quad (3)$$

where  $\pi_i$  is agent  $i$ 's disposable income,  $H_i$  is his health (or property) status (see below), and  $I(\psi_i, k_i)$  is a function of his ideological parameter  $\psi_i$  in combination with his fighting status ( $k_i = 1$  if fighting,  $k_i = 0$  otherwise).

Based on their ideological views, their utility is affected differently when they fight: stronger ideology is linked to higher utility (actually, smaller disutility); higher health

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<sup>6</sup> We consider the entire parameterization of rival group B as exogenously given. For example, this could be justified when group B represents the incumbent government and group A the rebels. Alternatively, group B may represent another society in potential conflict with the first. In that case, the model becomes symmetrical, involving possibly simultaneous decisions. This, however, goes beyond the scope of the current paper.

status increases utility with diminishing returns ( $\frac{\partial^2 U_i}{\partial^2 H_i} < 0$ ). As we show below, the two sectors differ in personal damages, disposal income and ideological rewards.

### 2.1.3. Income

The two primary sources of income in the economy are production and looting. The value of production is the direct income of the producers. We assume that producers make use of a constant-returns-to-scale technology. We define the product of agent  $i$  as  $\theta_i$ . Although we start our analysis with a constant productivity parameter ( $\theta_i = \theta$ , for all  $i$ ), later we allow for individual productivity shocks. In that case, individual productivity heterogeneity is drawn as  $\theta_i$  from a distribution  $F_\theta$  that satisfies  $E[\theta_i] = \theta$ .<sup>7</sup>

We allow some sort of transfers or taxation, which we term a ‘donation’, that producers may transfer willingly or unwillingly. In either case, the disposal income of a producer  $i$  is given by:

$$\pi_{P,i} = \theta_i - D_i, \quad (4)$$

Where  $D_i$  is the lump sum donation by producer  $i$  to the soldiers.

A soldier’s income is based on donations and looting. We assume that producers’ donations are directly and uniformly transferred to the fighting soldiers.<sup>8</sup> The amount of money that a fighting soldier  $j$  gets is denoted  $d_j$ . This amount depends on the size of donations and the number of donors relative to the number of recipients. Namely,  $d_j = \frac{1}{p_{kSA}} \sum_i D_i$ , where  $p_k$  is the probability of a soldier fighting.

The leader decides and announces the rule by which he and the fighting soldiers share the booty.<sup>9</sup> The shares are denoted by  $\{a_L, a_S\}$ , where  $a_L, a_S$  are the shares of the leader and the soldiers respectively, and  $a_L + a_S = 1$ .<sup>10</sup>

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<sup>7</sup> For simplicity, we omit possible non-zero correlations between the share of producers in the population and productivity. To reflect the direct effect of war-induced destruction on output,  $\theta$  could be multiplied by  $\omega$  to account for the sides’ relative strength. In the current model, however, we prefer the chosen specification because we consider group B to be passive.

<sup>8</sup> One example is when those donations are in the form of food and shelter supply directly from the civilians to the soldiers.

<sup>9</sup> This sharing mechanism is the outcome of a typical Laffer curve maximization by the leader. We assume that the leader’s share is the financing arm and other related conflict costs.

<sup>10</sup> A possible extension of the model may allow an allocation of booty to the producers as well (for example, through the redistribution of land). The current functional form, however, makes such allocation undesirable for the leader under any set of parameters.



This leads to the following expressions of disposal income:

$$\left\{ \begin{array}{l} \pi_L = (1 - a_S) L^B \\ \pi_{S,i} = k_i \left( \frac{a_S L^B}{S_A^F} + d \right) \\ \pi_{P,i} = \theta_i - D_i \end{array} \right\}, \quad (5)$$

where  $\pi_L, \pi_S, \pi_P$  are the incomes of the leader, a soldier and a producer, respectively.

#### 2.1.4. Personal damages

A destructive outcome of a violent conflict is a threat to the lives, health or properties of soldiers as well as civilians. We summarize these considerations by mentioning health status only to simplify the exposition. Let  $H_p$  and  $H_S$  represent the health status of producers and soldiers respectively. Health status, which varies with agents' type and actions, is assumed to increase with the probability of winning,  $\omega$ . The larger the fighting force relative to the opposing fighting force, the lower the health damage.

Let  $h_p$  and  $h_S$  denote the respective health parameters of producers and soldiers (to use corner solutions:  $0 < h_p \leq \frac{1}{S_B}$ ). Assuming that soldiers are at greater risk of personal damage than producers, we let  $\xi > 0$  be the parameter of excess damage among soldiers relative to producers. This corresponds to the case in which soldiers are targeted more than civilians. In this respect, the empirical literature provides mixed evidence regarding the relative risk of soldiers vs. civilians (see the Sierra Leone case in Humphreys and Weinstein, 2008). As a consequence, we also address the case of  $\xi < 0$  (that is, soldiers are safer than producers) later in the analysis. Changing the sign of  $\xi$  does not have a qualitative effect on the results, while it provides several interesting new insights. The respective health statuses are given by:

$$\begin{aligned} H_p &= \omega h_p, \\ H_S &= \omega(1 - \xi)h_p. \end{aligned} \quad (6)$$

#### 2.1.5. Ideology

Assume that agents are ex-ante identical, except for their ideological beliefs. Let  $\psi_i$  denote the ideological beliefs of agent  $i$  with an ideological support for war associated with higher  $\psi_i$ . Without derogating from generality, we assume that actual fighting generates disutility. Hence, we assume that  $\psi_i$  is drawn from distribution  $F_\psi$  with density

$f_\psi$  strictly positive, and  $\psi_i$  is bounded above by 0. We assign  $\psi_i = 0$  for the most extreme supporter of war. An agent experiences his idiosyncratic ideological beliefs,  $\psi_i$ , as a cost only if he actually participates in the fighting. This is a simple way of accounting for the psychological trauma of getting physically involved in killing, for example. Hence, this cost component is such that  $I(\psi_i, k_i) = \psi_i < 0$  if agent  $i$  is a soldier who fights ( $k_i = 1$ ) and  $I(\psi_i, k_i) = 0$  otherwise (producers and defecting soldiers).

### **2.1.6. Sequence of events**

The sequence of events in the model is as follows. First the leader announces a rule for sharing the loot ( $a_L, a_S$ ); then agents choose to become combatants or producers, after which productivity shocks ( $\theta_i$ ) take place (aggregate and/or individual) and the producers transfer their donations to the soldiers. Lastly, soldiers decide on fighting ( $k = 1/0$ ), and the booty is distributed.

### **2.1.7. Information**

The individual ideology parameter ( $\psi_i$ ) and the individual productivity shock ( $\theta_i$ ) are private information, while all agents are assumed to know the distribution of these independent parameters. The aggregate shock becomes public knowledge immediately after it occurs. Finally, all decisions made are also common knowledge.

### **2.1.8. Equilibrium concept**

The equilibrium results from the optimal sequential decisions made by the players. The model is solved backwards: we start with the final decision of the model—fighting of soldiers—and move backwards to previous decisions that make it possible: donations and sorting. Then, we solve the initial booty allocation announcement by the leader. At each stage, agents choose actions that maximize their expected utility based on the anticipated other players' response.

## **2.2. The Mechanisms**

We begin the analysis by using a benchmark model with constant productivity ( $\theta_i = \theta, \text{ for all } i$ ) and exogenous and homogeneous donations ( $D_i = D$  given, as for the example of a poll tax). This case is interesting on several grounds.

### 2.2.1. Decision to fight

A soldier chooses to fight if  $E(U_S|k=1) \geq E(U_S|k=0)$ . Recall that when a soldier defects, his income becomes null while his health status increases and becomes equal to that of a producer ( $H_P$ ). However, as the disposal income of producers is non-negative, clearly an agent ex-post prefers to become a producer rather than to join the army and defect. Since, in the benchmark model, productivity and donations are constant and certain at the sorting time, all agents who do decide to become soldiers will fight in the end.

### 2.2.2. Individual choice of activity type

Agent  $i$  joins the army if his expected utility is higher by making this choice ( $E[U_S] > E[U_P]$ ).<sup>11</sup> We rewrite<sup>12</sup> and plug ( $p_k = 1$ ) to get the marginal inequality:

$$\psi_i > \theta - \frac{D}{s^*} - a_S^* \frac{\pi_B}{s_B} + \chi, \quad (7)$$

where  $\chi = -\log(1 - \xi)$ , is a parameter that captures the utility gap due to the two populations' different personal damages.

We denote  $\underline{\psi} = \theta - \frac{D}{s^*} - a_S^* \frac{\pi_B}{s_B} + \chi$ , the reservation ideology level, which corresponds to an agent indifferent between being a soldier and a producer.<sup>13</sup> Above this ideological

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<sup>11</sup> For a non-fighting soldier ( $k_i = 0$ ), we obtain equivalently  $0 > \theta - D$ . This condition, however, is never satisfied because we assume that  $D$  satisfies a liquidity constraint ( $D \leq \theta$  a priori; then,  $D \leq \theta$  a posteriori, since productivity here is constant even when it is heterogeneous, and can then be perfectly predicted). Consequently, in this benchmark model, all soldiers fight:  $p_k^* = 1$ .

<sup>12</sup>  $\left(\frac{a_S L^*}{s^*} + D_i \frac{P^*}{s^*} + \psi_i\right) + \log(\omega(1 - k_i \xi) h_P) > \theta - D + \log(\omega h_P)$ .

<sup>13</sup> Recall that  $\psi_i^F = \chi - a_S \frac{\pi_B}{s_B} - D \frac{P^*}{s^*}$ . Then,  $\underline{\psi} = \theta - \frac{D}{s^*} - a_S^* \frac{\pi_B}{s_B} + \chi$  implies  $\underline{\psi} = \psi_i^F + \theta - D$ . Again, using the liquidity constraint assumption, we obtain  $\underline{\psi} > \psi_i^F$ , which ensures that all soldiers fight. Interestingly, there is a subpopulation among the peasants that would fight if forced to become soldiers; the remaining peasants would not fight even if forcibly conscripted.

threshold ( $0 > \psi_i > \underline{\psi}$ ), all agents join the army and eventually fight. Accordingly, the proportion of soldiers in group A is given by:

$$S^* = Prob(\psi_i \geq \underline{\psi}) = 1 - F_\psi[\underline{\psi}]. \quad (8)$$

The role of ideology in the model reflects the ways in which propaganda, hate and/or fundamentalism may contribute to violent conflicts. Ideology, however, is not the only determinant of individual involvement in violence. Several factors play a role in the self-selection process: greed for looting, the relative odds of winning the contest, donations to soldiers, relative health status and comparative income when remaining non-combatant.

### 2.2.3. Booty allocation

Lastly, we solve the first step of the model in which the leader allocates the booty to maximize his revenue:

$$\max_{a_S} E[(1 - a_S)L] \text{ s.t.: } 0 \leq a_S \leq 1. \quad (9)$$

By offering incentives to soldiers, the leader controls the relative size of the soldiering and civilian populations. We first consider the interior case ( $a_S \neq 0, S^* \neq 1$ ), for which the interior F.O.C. is  $(1 - a_S) \frac{\pi_B}{S_B} \frac{\partial S^*}{\partial a_S} - \frac{\pi_B}{S_B} S^* = 0$ . Then, since  $\frac{\partial S^*}{\partial a_S} > 0$  under our assumptions, we obtain  $a_S^* = 1 - \frac{S^*}{\left(\frac{\partial S^*}{\partial a_S}\right)}$ . Hence, the interior solution to the benchmark

model is given by:

$$\left\{ \begin{array}{l} a_S^* = 1 - \frac{S^*}{\left(\frac{\partial S^*}{\partial a_S}\right)} \\ S^* = 1 - F_\psi \left[ \theta - \frac{D}{S^*} - a_S^* \frac{\pi_B}{S_B} + \chi \right] \\ D_i = D \\ p_k = 1 \end{array} \right\}. \quad (10)$$

Note that, under positive donations, the size of the soldiering population is always positive. Without donations, a corner solution of no army may occur. In the absence of other financial incentives ( $a_S^* = 0$ ), donations would have an unambiguously positive effect on the size of the soldiering population.

### 2.2.4. Functional-form solution

To simplify the following calculations, we assign a uniform distribution to ideological beliefs:  $\psi_i \sim U[\psi_0, 0]$ , where  $\psi_0$  is a fixed negative parameter.<sup>14</sup> As a result, the share of soldiers in equilibrium is:

$$S^* = 1 - F_\psi [\underline{\psi}] = \frac{\psi}{\psi_0} = \frac{z - \frac{D}{S^*} - a_S v}{\psi_0} \quad (11)$$

where  $z \equiv \theta + \chi$  represents the shadow income of producers. We denote  $v \equiv \frac{\pi_B}{S_B}$  the ‘opportunity value’ of looting. Parameter  $z$  captures the two main advantages of being a producer: producing income and suffering lower personal damage. The combined parameter  $v$ , which is the marginal looting value, captures the potential of looting. Under the uniform distribution assumption, system (12) has a closed-form solution.

### 2.2.5. The three types of society

Our model yields three distinct war scenarios in which the incentives for fighting and the composition of society vary: a defence society, a militarized society, and a predator society.<sup>15</sup> They arise from considering corner solutions for the incentive parameter,  $a_S$ , and the size of the army,  $S$ .

#### i. Defence society

The defence society corresponds to a moderate version of conflict. In this society, the leader does not incentivize agents to join the army by sharing the loot with them ( $a_S^* = 0$ ). The sole income of soldiers comes from donations. The existence of an army stems from producers’ demand for security. In the benchmark model, donations are exogenous; however, they will later be made endogenous. As a result, the conflict level, measured by  $S^*$ , is relatively low ( $S^* = \frac{z - \sqrt{z^2 - 4D\psi_0}}{2\psi_0}$ ). A defence society  $S^*$  does not depend on looting—that is, on the opponent’s wealth—because soldiers receive no booty. The army size is increasing in donation and ideological beliefs and decreasing in the shadow

<sup>14</sup> Since we specify that ideology spreads on a negative scale, participation in fighting implies a non-positive psychological effect on utility. In certain contexts, however, some soldiers may experience a positive utility effect due to fighting (for example, suicide fighters). We address this issue in Lavie and Muller (2011).

<sup>15</sup> See detailed proofs in the Appendix.

income of producers. Notably, the opportunity value of looting ( $v$  – the ratio of the opponent’s strength to his resources) does not affect the army size.

The condition for a defence society is:

$$v \leq \sqrt{z^2 - 4D\psi_0}. \quad (14)$$

The intuition is that when the opportunity value is low enough, the incentives based on it are not effective for military mobilization. The right-hand side term in (14) reflects the benefits of being a producer: production income and health status. Since  $\psi_0 < 0$ , donations also make the above inequality hold. Lastly, a defence society is more likely when the ideological beliefs of the population are weak (low negative  $\psi_0$ ): without ideological heterogeneity, it would be harder for the leader to mobilize soldiers who would have no direct financial incentive. **AV**

## ii. Militarized society

A society becomes a militarized society when the opportunity value exceeds the threshold:  $\sqrt{z^2 - 4D\psi_0} < v$ .

As the opportunity value of looting increases, the leader finds it optimal to share the booty with the soldiers in order to encourage higher enlistment. That is, a militarized society is a society with increasing incentives to soldiers and reduced production. A militarized society implies higher conflict level, as measured by  $S$ , than a defence society. The size of the army is given by:

$$S^* = \frac{z-v}{2\psi_0}. \quad (15)$$

The number of soldiers increases according to the opportunity value and the ideological beliefs, and decreases in the shadow income of producers. In contrast to the defence society, the army size is not dependent on donations, which clearly shows that fundamentally distinct incentives are at work: greed now utterly dominates the protection motives. The booty allocation rule is  $a_S^* = \frac{z^2 - v^2 - 4D\psi_0}{2v(z-v)}$ . In this case, the leader's allocation rule fully offsets the effect of donations on soldiers. Similarly,  $a_S^*$  decreases with ideology, as it enables the leader to mobilize soldiers with lower monetary incentives.

### iii. Predator society

Finally, a society becomes a predator society when the opportunity value of war is so high that the entire population decides to join the army. This means that the economy is no longer productive; all income is obtained via looting. Conflict intensity is maximal ( $S^* = 1$ ). The condition for a predator society is given by:  $z - 2\psi_0 \leq v$ .

Note that, given the formulation of  $a_S^* = \frac{z-D-\psi_0}{v}$ , one must still take donations into account even if there are no donations in the predator society. This is because the leader must also account for the possibility of donations in the marginally close military society if he wishes to retain all agents as soldiers.

### 2.2.6. Determinants of society-switching

Once this taxonomy of societies is understood, the question of what triggers a shift from one society to another makes full sense. To discuss regime determination, we first recap a few synthetic parameters that capture the useful information. We use the term opportunity value,  $v$ , to denote the potential looting level of B by A per soldier of B. This parameter accounts for the size of the booty, discounted by the strength of the opponent. It is connected to the return to activity in the conflict, as in the studies by Hirshleifer and Grossman. Next, we define the producer shadow income,  $z = \theta - \log(1 - \xi) > 0$ . This parameter adjusts the producers' income for the producers' lower personal damage.

Parameter  $D$  is here the fixed level of donation by producers. Parameter  $\psi_0$  ( $< 0$ ) corresponds to the most severe psychological loss occasioned by fighting. We denote  $B_{DM} = \sqrt{z^2 - 4D\psi_0}$ , the defence society/militarized society bound (the DM bound) for the opportunity value parameter. Finally, we denote  $B_{MP} = z - 2\psi_0$ , the militarized society/predator society bound (the PM bound) for the opportunity value parameter.

The passage from one type of society to another may be described in terms of a shift in the relative level of the opportunity value parameter across one of the above bounds, as Figure 1 shows. This feature can be illustrated in terms of conflict escalation. Given fixed levels of  $\psi_0, D, \xi$  and  $\theta_i$ , the conflict size rises as the opponent's lootable wealth,  $\pi_B$ , increases and as its strength,  $S_B$ , decreases. For example, violence may be spurred by a transitory military weakness in an opponent, or by a positive shock to the opponent's lootable income. For example, a hike in the price of a local natural resource may trigger a local society to shift from a defensive phase to a military or even a predatory phase.

**Figure 1 to be included here**

Since we have made explicit the bounds of the society types in terms of the synthesized parameters  $v, B_{DM}$  and  $B_{MP}$ , we can analyse the switches in society type not only with respect to variations in  $v$  but also with respect to the intermediate parameter  $z$ , and even with respect to the basic parameters  $(\psi_0, D, h_P, \xi, \pi_B, S_B, \theta_i, \theta)$  by using the derivation chain rule.<sup>16</sup> That is to say, fixing the opportunity value, the signs of the society-bound derivatives allow us to investigate how parameter changes affect the society type.

Thus, provided the other parameters are fixed, we find that the partial derivatives of  $B_{DM}$  with respect to  $-\psi_0, D$  and  $\theta_i$  are positive. Since an upward shift in  $B_{DM}$  corresponds respectively to a more moderate population, more supportive producers (or harsher tax collectors, with higher  $D$ ) and an increase in productivity (with higher  $\theta$ ), it results in an extended domain for the defence society. In contrast, fewer resources (such as several

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<sup>16</sup>  $\frac{\partial \pi_B / S_B}{\partial S_B} = -\frac{\pi_B}{(S_B)^2} < 0$ ;  $\frac{\partial \pi_B / S_B}{\partial \pi_B} = \frac{1}{S_B} > 0$ ;  $\frac{\partial z}{\partial \theta_i} = 1 > 0$ ;  $\frac{\partial z}{\partial \xi} = \frac{1}{1-\xi} > 0$ ;  $\frac{\partial B_{DM}}{\partial D} = -\frac{2\psi_0}{\sqrt{z^2-4D\psi_0}} > 0$ ;  $\frac{\partial B_{DM}}{\partial \psi_0} = -\frac{2D}{\sqrt{z^2-4D\psi_0}} < 0$ ;  $\frac{\partial B_{DM}}{\partial z} = \frac{z}{\sqrt{z^2-4D\psi_0}} > 0$ ;  $\frac{\partial B_{MP}}{\partial z} = 1 > 0$ ;  $\frac{\partial B_{MP}}{\partial \psi_0} = -2 < 0$ .



consecutive years of drought) may trigger such a decline in productivity as to push the society towards a more violent regime.

There seems to be an obvious potential for political manipulation. For example, a hate campaign may yield an increase in the ideological parameter ( $\psi_0$ ) and trigger a shift from a defence society into first a military society, and then even perhaps into a predatory society. Conversely, better education and advocacy of peace may shift a society in the opposite direction.

Similar results are found for the corresponding partial derivatives of  $B_{MP}$ , with the difference that the derivative with respect to  $D$  is zero. As fixed donation  $D$  increases, the society tends to move from a militarized to a defence type, whereas the likelihood of a shift to a militarized society is no different from that to a predatory society (because the donation fully crowds out the allocation of booty).

### 2.2.7. Within-society comparative statics

Not all parameter changes are large enough to trigger changes in society type. Within each society type, parameter changes imply society-specific variations in the size of the soldiering population and the allocation of booty. As a consequence, they determine changes in state variables (total output, total looting, mortality, etc.), which can be easily calculated using the chain rule from the results for the synthetic parameters  $z$ ,  $B_{DM}$ , and  $v$ .

We first discuss the variations in  $S$  and  $a_S$  that structure and largely summarize the behaviour of the whole system. In a defence society, the incentive parameter  $a_S^* = 0$  is constant. In contrast, the number of soldiers,  $S^*$ , is sensitive to changes in parameters, even with a defence society. Firstly, the size of the army increases with donations ( $\frac{\partial S^*}{\partial D} > 0$ ) since when producers donate more, the indifference point between becoming a soldier and becoming a producer shifts in favour of the army.

Similarly, greater mortality risk and weaker ideological beliefs diminish army size ( $\frac{\partial S^*}{\partial z} < 0$ ;  $\frac{\partial S^*}{\partial \psi_0} > 0$ ). Lastly, since no booty is allocated to soldiers, it does not affect the army size ( $\frac{\partial S^*}{\partial v} = 0$ ).<sup>17</sup> In a defence society, higher donation per producer, lower producer

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<sup>17</sup>  $\frac{\partial S^*}{\partial D} = \frac{1}{\sqrt{z^2 - 4D\psi_0}} > 0$  and  $\frac{\partial S^*}{\partial z} = \frac{\sqrt{z^2 - 4D\psi_0} - z}{2\psi_0\sqrt{z^2 - 4D\psi_0}} < 0$ ;  $\frac{\partial S^*}{\partial \psi_0} = \frac{D}{\psi_0\sqrt{z^2 - 4D\psi_0}} - \frac{(z - \sqrt{z^2 - 4D\psi_0})}{2\psi_0^2} > 0$  and  $\frac{\partial S^*}{\partial v} = 0$ .

shadow income and stronger ideological beliefs all yield higher soldier enlistment. The opportunity value has no effect.

In a militarized society, the incentive instrument  $a_s^*$  varies conversely to  $D$  and  $-\psi_0$  ( $\frac{\partial a^*}{\partial D} < 0; \frac{\partial a^*}{\partial \psi_0} > 0$ ). Leader maximization yields an optimal army size that is manifested in a specific threshold of  $\psi$ . The larger donations crowd out the booty needed for incentivizing soldiers. In that case, while army size is independent of donations,  $a_s^*$  decreases with  $D$ . A less ideological population makes it harder to attract soldiers (and also makes a militarized society less likely). In response, the leader can allocate a smaller share of the booty to soldiers because the return to booty allocation has decreased. Correspondingly, the size of the army drops as society becomes less radical ( $\frac{\partial S^*}{\partial \psi_0} > 0$ ).

However, variations with respect to the opportunity value  $v$  and shadow income  $z$  are more complicated:  $\frac{\partial a^*}{\partial v}$  and  $\frac{\partial a^*}{\partial z}$  have ambiguous signs.<sup>18</sup> Whereas  $\frac{\partial a^*}{\partial v}$  is positive close to  $B_{DM}$ , it can be either negative or positive near  $B_{MP}$ , depending on the levels of  $D$  and  $z$ .<sup>19</sup> On the one hand,  $\frac{\partial a^*}{\partial z} = \frac{(v-z)^2 + 4D\psi_0}{2v(v-z)^2}$  changes sign when  $v$  varies in the society type. This derivative is negative next to  $B_{DM}$  and becomes positive with higher opportunity value.<sup>20</sup> That is, when the opportunity value of war is high enough, the return for soldiers increases and the leader offers soldiers a larger share of the booty when the producers' shadow income grows. When the opportunity value is small, the leader may find it too expensive to incentivize agents to become soldiers; hence, a higher shadow income for producers is not matched with larger incentives to soldiers.

As above, variations in the number of soldiers with respect to the main parameters are not ambiguous; they correspond to the directions found in the defence society. Now, in addition, the donation level per producer has no effect of the number of soldiers due to

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<sup>18</sup>  $\frac{\partial a^*}{\partial D} = \frac{4\psi_0}{2v(v-z)} < 0; \frac{\partial a^*}{\partial \psi_0} = \frac{4D}{2v(v-z)} > 0$  and  $\frac{\partial a^*}{\partial v} = \frac{(z^2 - 4D\psi_0)(\frac{v-z}{2}) - \frac{z}{2}v^2}{v^2(v-z)^2}$ .

<sup>19</sup>  $\left(\frac{\partial a^*}{\partial v} \Big|_{v = z - 2\psi_0}\right) = \frac{(z^2 - 4D\psi_0)(\frac{v-z}{2}) - \frac{z}{2}v^2}{v^2(v-z)^2} = \frac{(2\psi_0(-zD + \psi_0(4D - z)))}{v^2(v-z)^2}$ ,

while  $\left(\frac{\partial a^*}{\partial v} \Big|_{v = \sqrt{z^2 - 4D\psi_0}}\right) = \frac{(z^2 - 4D\psi_0)(v-z)}{v^2(v-z)^2} > 0$ .

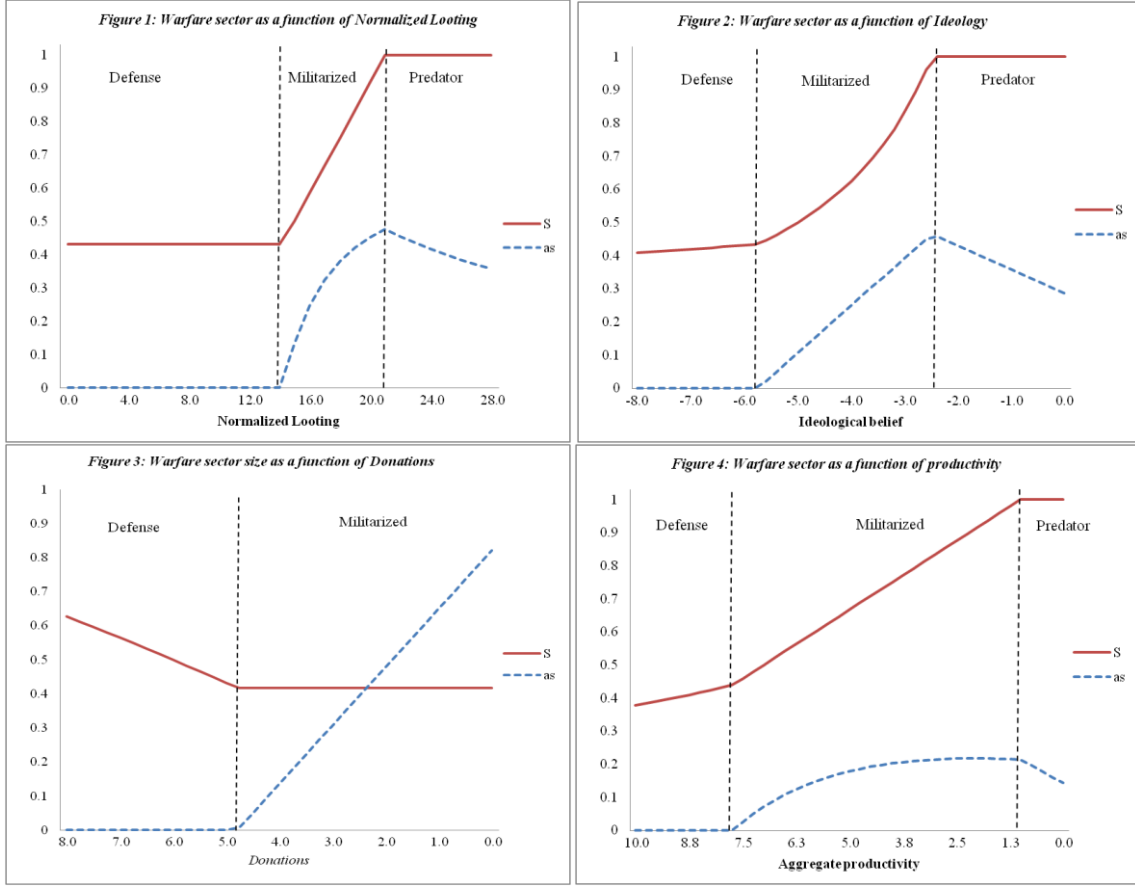
<sup>20</sup> Indeed, for  $v = \sqrt{z^2 - 4D\psi_0} : (v - z)^2 + 4D\psi_0 < 0$ , while for  $v = z - 2\psi_0 : (v - z)^2 + 4D\psi_0 > 0$ .

the compensation offered by a new positive effect of the normative looting:  $\frac{\partial S^*}{\partial D} = 0$ ;  $\frac{\partial S^*}{\partial z} < 0$ ;  $\frac{\partial S^*}{\partial \psi_0} > 0$ ;  $\frac{\partial S^*}{\partial v} > 0$ .

Finally, in a predator society, the army size,  $S^* = 1$ , is constant. In this case, the variations in incentive instrument  $a^*$  with respect to the synthetic parameters have clear directions:  $\frac{\partial a^*}{\partial D} < 0$ ;  $\frac{\partial a^*}{\partial \psi_0} < 0$ ;  $\frac{\partial a^*}{\partial v} < 0$  and  $\frac{\partial a^*}{\partial z} > 0$ . For example,  $\frac{\partial a^*}{\partial v} = -\frac{a^*}{v} < 0$ .

To summarize this discussion, the changes in the soldiering population,  $S^*$ , and the soldiers' share in looting,  $a_s$ , across and within society types are illustrated in Figures 2–5. Note in these figures that the aggregate productivity on the x-axis corresponds to parameter  $\theta$ , with an inverse x-axis, while the x-axis for the ideological parameter,  $\psi_0$ , is correctly oriented as this parameter is negative. We discuss the variations of the state variable of interest in the next subsection.

## Figures 2 to 5: A few fundamental variations



### 2.2.8. Variations in aggregate outcome variables

Our model provides rich predictions for many aggregate state variables in group A: production, looting, psychological damage and global welfare.

#### Global production

The total product of the society A economy is given by  $Q = (1 - S)\theta$  and increases commensurate with the proportion of producers. Hence, any parameter that raises soldier enlistment is also associated with a lower global production level: larger donations, stronger ideological support and higher opportunity value. Conversely, parameter  $\chi$ , the excess personal damage of soldiers, is positively related to total product. Finally, the direct effects and the indirect effects (through  $\frac{\partial S^*}{\partial \theta} < 0$ ) of the productivity parameter ( $\theta$ ) are consistent. Namely, higher productivity always results in a higher aggregate product.

## Personal damage

The disutility effect of personal damage is expressed by  $\log(H_P)$  and  $\log(H_S)$  for each producer and soldier, respectively. Both terms decrease with  $S$ , because producers and soldiers are individually safer when group A's military force grows.

Aggregate personal damages are also affected by the composition of the society. Since we assume that personal damage is higher for a soldier than for a producer ( $\chi > 0$ ), a larger army implies that relatively more agents are sorted into riskier roles.

Overall personal damages are given by:  $(1 - S)\log(H_P) + S\log(H_S)$ . Substituting terms, we obtain  $(1 - S)\log(h_P\omega) + S\log((1 - \xi)h_P\omega) = \log(h_P) - \log(S_B) + \log(S) - S\chi$ . Deriving with respect to  $S$  yields:  $(\log(h_P) - \log(S_B) + \log(h_S) - S\chi)' = \frac{1}{S} - \chi$ .

Thus, overall personal damages increase with  $S$ , for  $S > \frac{1}{\chi}$ , and otherwise decrease. The overall effect is driven by the decreasing marginal return of fighting soldiers (in respect to overall welfare from health/property). When the army is small, each additional soldier improves the total health/property component of welfare. However, once the fighting population is large enough, the marginal effect of each additional soldier becomes negative. Minimizing global personal damage is attained with an intermediate army size. This is a new insight, not yet found in the literature. Provided  $\chi > \frac{1}{S}$ , personal damage rises with  $\psi_0, D$  and  $\pi_B$  and falls with  $S_B$  and  $\theta_t$ . As the excess risk to each soldier,  $\chi$ , escalates, fewer soldiers enter the army, while their damage rate is higher.

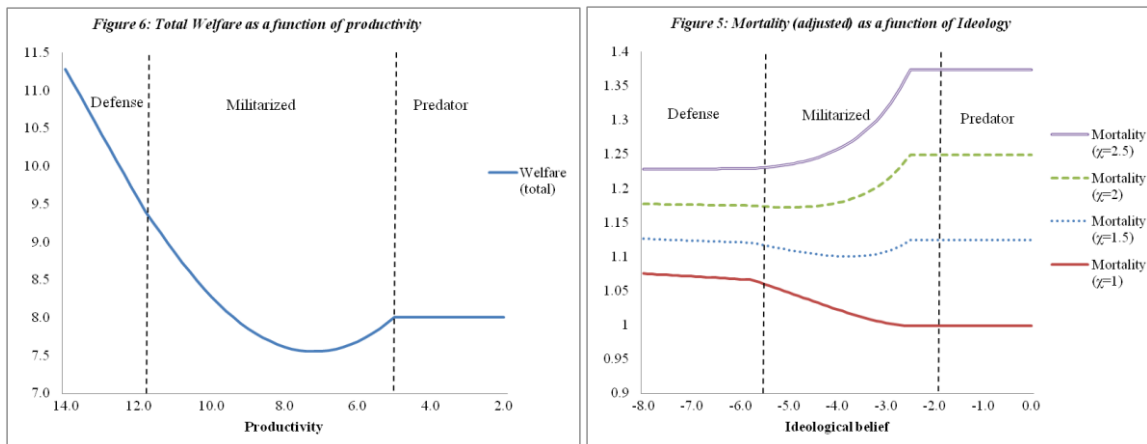
## Looting

Given that total looting is expressed by  $\pi_B\omega$ , it increases monotonously with  $S$ . Higher opponent income increases total looting (by allowing more lootable goods and fostering a larger number of soldiers in society A), while a larger opponent army decreases looting (by strengthening the opponent's defences and inducing fewer soldiers). Moreover, total looting increases with  $\psi_0$  and  $D$  that generate a higher enlistment of soldiers, and decreases with  $\theta$  and  $\chi$  that raise the proportion of producers.

## Psychological/ideological damage

As mentioned above, soldiers undergo a psychological shock when they fight, related to their idiosyncratic ideological parameters. As more agents become soldiers, this cost, aggregated to society at large, tends to increase. Under the uniform distribution assumption, the average psychological cost of fighting soldiers is equal to  $-\frac{1}{2}\underline{\psi}$ . Therefore, the total psychological cost covered by the society is  $-\frac{S^*}{2}\underline{\psi}$ . After several substitutions of terms, we have  $-\frac{\psi_0}{2}S^{*2}$ . As a result, the aggregate psychological cost increases with  $D$  and  $\pi_B$  and decreases with  $\theta$ ,  $S_B$  and  $\chi$ . The effect of ideology is more mixed. In a defence society and in a military society, stronger distribution support of ideological beliefs (through higher  $\psi_0$ ) ultimately yields higher aggregate psychological costs because producers tend to be less ideologically reluctant to enlist.<sup>21</sup> In a predator society, in contrast,  $S$  is constant, and therefore any increase in  $\psi_0$  would cut the aggregate psychological costs down.

**Figures 6 and 7: Variations in global outcomes**



## Welfare analysis

Three sub-populations must be considered in calculating aggregate welfare: private, army and leader. The combined surplus will be considered the welfare of society, which is equal to:

<sup>21</sup> See Appendix 1.5 for proofs for this entire section.

$$(14) \quad \Sigma U = \log\left(\frac{h_P}{S_B} S\right) - S\chi + (1 - S)\theta_i + \frac{\psi_0}{2} S^2 + vS,$$

which we can decompose into:

- The utility of (loss due to) personal damage:  $\log\left(\frac{h_P}{S_B} S\right) - S\chi$
- The utility gain from production:  $(1 - S)\theta_i$
- The psychological utility of (loss due to) fighting:  $\frac{\psi_0}{2} S^2$
- The utility gain from looting:  $vS$  to change as  $a_P vS$

In a military society, we can rewrite (14) into  $\Sigma U = \log\frac{h_P}{S_B} + \log\left(\frac{z-v}{2\psi_0}\right) + \lambda\theta_i - \frac{3(z-v)^2}{8\psi_0}$ .

Deriving with respect to the ideology-bound parameter, we obtain  $\frac{\partial \Sigma U}{\partial \psi_0} = -\frac{1}{\psi_0} + \frac{3(z-v)^2}{8\psi_0^2} > 0$ .

Stronger ideological beliefs increase the overall welfare. This result argues in favour of the use of propaganda by leaders as a way of mitigating war-induced damage rather than as a pure instrument. This, it seems to us, is consistent with typical outcomes in some official reinterpretations of history, sometimes post-war.

The effect on welfare of soldiers' excess personal damage is expressed as  $\frac{\partial \Sigma U}{\partial \chi} < 0$ . Even though a higher rate of personal damage to soldiers reduces the overall number of soldiers, it still has a negative effect on welfare anyway (by increasing overall damage). The opposite would be the case if being a soldier could improve one's personal security relative to being a producer ( $\chi < 0$ ). In this event, more agents would become soldiers and the overall outcome would be a positive externality through a strong army that better protects the entire population from personal damage and generates additional income from looting.

### *The effect of productivity on welfare*

If  $\frac{3}{\psi_0} < -1$ , then  $\frac{\partial \Sigma U}{\partial \theta} > 0$  always. That is, if the population's ideology is radical enough in favour of violence, higher productivity will always increase global welfare. In all other cases,  $\frac{\partial \Sigma U}{\partial \theta_i} \leq 0$  if  $\frac{2\psi_0}{3} \left(1 + \sqrt{1 + \frac{3}{\psi_0}}\right) \leq z - v \leq \frac{2\psi_0}{3} \left(1 - \sqrt{1 + \frac{3}{\psi_0}}\right)$ , and  $\frac{\partial \Sigma U}{\partial \theta_i} > 0$  otherwise.

That is, if the society is insufficiently radical in its ideology, leading to a relatively small  $S$ , there is an interval of values for  $z - v$  for which improving productivity reduces global welfare. The reason is that the upturn in productivity shifts more agents into the

productive labour force. As a result, looting decreases and personal damages rise, culminating in a decline in total welfare. Some of the discussed variations in aggregate personal damages and total welfare, across and within society types, are illustrated in Figures 6 and 7.

### 2.2.9. The ex-post motivation for war

Our model assumes that a war is already taking place. Still, it is of interest to examine the agents' ex-post (that is, post-sorting) motivations to go to war. This is important because if war is not advantageous to individuals ex-post, then violence-halting mechanisms should be easier to implement.

So far, we have compared the utility levels of agents in various roles under war. Now we compare each agent's utility between two states: war and peace. Clearly, if an agent derives enough disutility from personal damages (for example, far beyond the level that we specified), he will never prefer war, even though other agents may still choose to join the army. If this is not the case, in terms of the model, a producer will prefer war if and only if his utility level is definitively greater in war:  $U_{P,i}(war) = \theta_i - D_i + \log(H_P) > U_{P,i}(peace) = \theta_i - D_i$ . Producers always prefer peace due to donations and the personal damage war implies. Since they receive no booty, war affects them only negatively.

A soldier, in turn, prefers war under the following condition:  $U_{S,i}(war) = k_i \left( \frac{a_S L^B}{S^A} + d + \psi_i \right) + \log(H_S) > U_{S,i}(peace) = d$  (or, alternatively, zero if producers refuse to donate when there is no war). Soldiers may benefit from war because of looting, but they may incur higher personal damage and pay a psychological-ideological cost for their fighting.

In a defence society, the booty channel is blocked; therefore, soldiers in such a society should oppose war (as is the case in most conventional armies). Indeed, war would inflict only additional survival and psychological costs on them. Under such conditions, soldiers in an army will prefer war only if their ideological orientation is so strong that they can benefit from fighting (in our terms –  $\psi_i > 0$ ). In the case where producers donate only in wartime, implying that combatants are not paid in peacetime, soldiers have an additional motive to prefer war.



In a predator society, the only source of compensation for the population (all of whom are soldiers) is looting; therefore, war is supported by all. By construction, in a predator society, the psychological cost of war is lower (for everyone) than the expected benefits.

In a military society, soldiers may show mixed support of war on the basis of the intensity of their ideology—that is, the least ideologically oriented would tend not to support war.

Producers can foster war in two ways: by joining the army and by donating to fighters. Thus far, we have considered donations to be exogenously set. This specification has several drawbacks. Firstly, it downplays the role of the population in actively supporting war. Secondly, with fixed donations some producers may donate amounts that are suboptimal in terms of their personal damage prospects. Indeed, they might buy better protection from soldiers by adjusting their donation level to their own situation. Thirdly, in the military society, fixed donations are of little interest for the analysis because ultimately they are fully transferred to the leader. These elements motivate our endogenizing of donations in the next section.

### **3. The Model with Flexible Donations and Heterogeneous Productivity**

We now assume that producers individually set their donation level after the sorting phase. On the basis of this additional decision, the following subsections introduce an additional dimension of heterogeneity on productivity. This is done using different sequencing scenarios. Firstly, we analyse the case of individual productivity shocks that take place after the sorting decision; then, we study the effect of individual productivity shocks (or ‘heterogeneity’) that precede the sorting decision. Lastly, we examine the effect of an aggregate shock at the post-sorting stage.

#### **3.1. Endogenous Donations and Protection Motives**

Assume that donations are now based on producers’ voluntary contributions and result in a public good: better security. However, as in other public good models, agents

tend to free-ride in the absence of a private incentive.<sup>22</sup> By incentivizing soldiers to fight through direct donations to them, the producers reduce their personal damage, as the relative power of society  $A$  grows.

Specifically, we assume that producers can affect their relative prospects of personal damage by donating to soldiers in the following way:

$$H_{P_i} = ((1 + D_i)^\beta \omega h_p) \quad (15)$$

where  $\beta \in (1, \infty)$  is a parameter that describes a possible nonlinearity in the effect of  $i$ 's donation,  $D_i$ , on one's own personal damage. Note that producers still take the aggregate level of  $D$  as given because the effect of their own donation,  $D_i$ , on the aggregate donation,  $D$ , is negligible. The maximization of  $U_{P_i}$  with respect to  $D_i$  yields the following F.O.C:  $\frac{\partial U_{P_i}}{\partial D_i} = -1 + \frac{\beta}{1+D_i} = 0$ . The second-order conditions are satisfied by the values that we allowed for parameter  $\beta$ . Then, in the absence of a liquidity constraint, the optimal donation level is given by:

$$D_i = \beta - 1. \quad (16)$$

If, instead, individual donations have only a weak effect on personal security (in the sense that  $\beta \leq 1$ ), then nobody would donate and one would end up with  $D = 0$ , a special case in the benchmark model. Nevertheless, when  $\beta$  is large enough, donations are set optimally to be positive, although subject to the liquidity constraint.

Note that introducing endogenous donations without simultaneously introducing production heterogeneity would not change the qualitative outcome of the benchmark model. In both cases, donation size is constant among all producers and is common knowledge before the sorting process. Still, due to the protection motive, some minor differences would arise in the utility levels and the personal damages of producers, which would improve both in such an endogenous donation case.

### 3.2. Individual Productivity Shocks

We now allow for personal productivity shocks that take place after the agents are sorted into soldier and producer populations.<sup>23</sup> Assume that agent  $i$ 's productivity is drawn from a uniform distribution:  $\theta_i \sim U[0, 2\tilde{\theta}]$  and  $2\tilde{\theta} \geq \beta - 1$ . Then, all agents who satisfy

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<sup>22</sup> See Moore (1995) for an extended review of the topic.

<sup>23</sup> The alternative sequencing option is discussed in the next subsection.

$\theta_i \geq \beta - 1$  can implement the aforementioned optimal donation level ( $D_i = \beta - 1$ ), whereas the others are bounded by their liquidity constraint and therefore donate sub-optimally ( $D_i = \theta_i$ ). If so, the donation vector is:

$$D_i = \begin{cases} \theta_i & \text{if } \theta_i < \beta - 1 \\ \beta - 1 & \text{else} \end{cases} \quad (17)$$

Correspondingly, the mean donation per producer is given by:

$$D^* = E[D_i] = \left(\frac{\beta-1}{2}\right)\left(\frac{\beta-1}{2\tilde{\theta}}\right) + (\beta-1)\left(1 - \frac{\beta-1}{2\tilde{\theta}}\right) = (\beta-1) - \frac{(\beta-1)^2}{4\tilde{\theta}} \quad (18)$$

Due to the liquidity constraint, the proportion  $1 - \max\{\min\{1, \frac{\beta-1}{2\tilde{\theta}}\}, 0\}$  among the producers can yield the optimal donation of  $D_i = \beta - 1$ . To concentrate on the most interesting case, we now set  $\tilde{\theta}$  high enough so that  $\beta - 1 \leq 2\tilde{\theta}$ . In that case, not all producers pay  $\beta - 1$ , and we are back to another occurrence of the benchmark case. Hence, given  $E[\theta_i | \theta_i < \beta - 1] = (\beta - 1)/2$ , we obtain the result. Note also that introducing a subsistence minimum as an additional constraint would not change the logic of the model.

We denote the money transfer per fighting soldier in equilibrium by  $d^*$ . Given  $P^* = 1 - S^*$  and  $p_k^* S^*$ , in equilibrium this level is a constant number equal to  $d^* = \left((\beta - 1) - \frac{(\beta-1)^2}{4\tilde{\theta}}\right) \frac{P^*}{p_k^* S^*}$ . As stated above, producers' utility is given by  $U_{P_i} = \theta_i - D_i + \log H_{P_i}$ .

Thus, a producer's expected utility may be rewritten into:

$$E[U_{P_i}] = \tilde{\theta} - D^* + E[\log(\omega h_p)] + \hat{\beta}, \quad (19)$$

where  $\hat{\beta}(\beta, \tilde{\theta}) \equiv \beta E[\log(1 + D_i)] = \beta \left( \left(\frac{\beta-1}{2\tilde{\theta}}\right) (\log(\beta) - 1) + \log(\beta) \right)$  (see proof in Appendix 1.2).

To solve the model, we need to rewrite the decision equations. The fighting decision remains the same, since at this stage there is no uncertainty about the total donations to soldiers, specified in (18) and correctly anticipated by the agents at the sorting stage. Therefore, we still obtain  $p_k^* = 1$  as in the benchmark model. The sorting decision rule, determined by comparing  $E(U_{P,i})$  and

$E(U_{S,i}) = a_S^* \frac{\pi_B}{S_B} + \frac{D^* P^*}{S^*} + \psi_i + \log(\omega(1 - \xi)h_p)$ , becomes:

$$\psi_i \geq \tilde{\theta} - \frac{D^*}{S^*} - a_S^* \frac{\pi_B}{S_B} + \chi + \hat{\beta}. \quad (20)$$

Accordingly, we define  $\underline{\psi}(D^*) = \tilde{\theta} - \frac{D^*}{S^*} - a_S^* \frac{\pi_B}{S_B} + \chi + \beta$  as the ideology level of indifference among types. We denote the expected shadow income of producers:  $\hat{z} = \tilde{\theta} + \chi + \beta$ . The rest of the solution method follows the one used in the benchmark case, using  $\hat{z}$  instead of  $z$ . As a result, we obtain:

- i. Defence society: for  $v \leq \sqrt{\hat{z}^2 - 4D^*\psi_0}$ , we have  $a_S^* = 0$  and  $S^* = \frac{\hat{z} - \sqrt{\hat{z}^2 - 4D^*\psi_0}}{2\psi_0}$ .
- ii. Militarized society: for  $\sqrt{\hat{z}^2 - 4D^*\psi_0} < v < \hat{z} - 2\psi_0$ , we have  $S^* = \frac{\hat{z} - v}{2\psi_0}$  and  $a_S^* = \frac{\hat{z}^2 - v^2 - 4D^*\psi_0}{2v(\hat{z} - v)}$ .
- iii. Predator society: for  $\hat{z} - 2\psi_0 \leq v$ , we have  $a_S^* = \frac{\hat{z} - D^* - \psi_0}{v}$  and  $S^* = 1$ .

By construction, even at an equal donation level, the producers gain more utility in this case than in the benchmark case due to the improved personal damage induced by the private donation channel. Then, ceteris paribus,  $\hat{z} > z$ , which implies stronger support for the defence society ( $\sqrt{\hat{z}^2 - 4D^*\psi_0} > \sqrt{z^2 - 4D^*\psi_0}$ ). Furthermore, within the defence society, the size of the warring population decreases. The driving force for this result is the higher shadow income of producers, which makes it less attractive to join the army. The same holds for the militarized society: upper-bound support increases ( $\hat{z} - 2\psi_0 > z - 2\psi_0$ ) and the soldiering population wanes.

Another interesting difference concerns the mortality rate: due to the personal productivity shock, some producers (the richer ones) enjoy lower personal damage than others (the poorer ones). Additional volatility increases the share of producers who donate optimally and decreases that of producers who donate all their income; as a result, the aggregate donation falls. Hence, larger positive personal shocks are correlated with higher war intensity. The proportion of soldiers, stimulated by higher donations, grows in both the defence society and the militarized society. Personal damage inequality among producers also increases.

### 3.3. Income Heterogeneity

An alternative way of interpreting  $\theta_i$  is to define it as a parameter that describes income heterogeneity, a parameter that corresponds to productivity shocks preceding the

sorting stage. Assume that the agents differ in both their ideological views and their productivity and that the two attributes are non-correlated. For simplicity, assign half of the population to  $\theta_i = 2\tilde{\theta}$  and the other half to  $\theta_i = 0$ . In this case, producers who have no income do not donate and have a lower probability of survival. The others, who enjoy high productivity, donate  $D_i = \beta - 1$  and enjoy better personal security. The respective expected utilities of these producers are given by:

$$\left\{ \begin{array}{l} U_{P_i}^{(\theta_i=0)} = \log H_{P_i} \\ U_{P_i}^{(\theta_i=2\tilde{\theta})} = 2\tilde{\theta} - (\beta - 1) + \beta \log \beta + \log H_{P_i} \end{array} \right\}. \quad (21)$$

Agents' sorting decisions are differentiated by income. For low income agents, the enlistment threshold is given by:

$$\psi_{i,(\theta_i=0)} \geq -d^* - a_s^* \frac{\pi_B}{s_B} + \chi. \quad (22)$$

The corresponding threshold for high income agents is given by:

$$\psi_{i,(\theta_i=2\tilde{\theta})} \geq 2\tilde{\theta} - (\beta - 1) + \beta \log \beta - d^* - a_s^* v + \chi. \quad (23)$$

Finally, the average donation per fighting soldier is given by:

$$d^* = \frac{(\beta-1)(1-s^*)}{2s^*} = \frac{0.5(\beta-1)}{s^*} - 0.5(\beta - 1). \quad (24)$$

Then, the respective indifference ideology levels between the types are:

$$\left\{ \begin{array}{l} \underline{\psi}(\theta_i = 0) = -\frac{0.5(\beta-1)}{s^*} + 0.5(\beta - 1) - a_s^* v + \chi \\ \underline{\psi}(\theta_i = 2\tilde{\theta}) = -\frac{0.5(\beta-1)}{s^*} + 0.5(\beta - 1) + 2\tilde{\theta} - (\beta - 1) + \beta \log \beta - a_s^* v + \chi \end{array} \right\}. \quad (25)$$

High income agents are less likely to enlist than low income agents due to  $\underline{\psi}(\theta_i = 2\tilde{\theta}) - \underline{\psi}(\theta_i = 0) = 2\tilde{\theta} - \beta - 1 + \beta \log \beta > 0$ . In addition, it is possible that  $\underline{\psi}(\theta_i = 2\tilde{\theta}) > 0$ , yielding a situation in which only the poor are enlisted while all the rich remain as producers and finance the soldiering population by making donations.

A final implication is the ideological composition of the army. While low income soldiers are motivated by financial incentives, high income agents are incentivized through ideology: the mean ideological level of low income soldiers is  $\frac{\underline{\psi}(\theta_i=0)}{2}$ , whereas the mean ideological intensity of high income soldiers is  $\frac{\underline{\psi}(\theta_i=2\tilde{\theta})}{2}$ . Again,  $\frac{\underline{\psi}(\theta_i=2\tilde{\theta})}{2} > \frac{\underline{\psi}(\theta_i=0)}{2}$ . This implies that, on average, wealthy soldiers are more ideologically inclined to fight

than poor soldiers. In other words, high income soldiers are overrepresented among the most ideologically intensive segment of the soldier population.<sup>24</sup>

In the case of a warfare-supporting ideology correlated with low income, as in Bonacich (1972), the composition of the army versus the productive sector may become even more polarized with respect to income. Thus, higher income dispersion is likely to be correlated with larger differences in the ideological entry threshold between poor and rich agents. When economic inequality increases, more poor agents join the soldiering population while fewer rich agents do so. This results in a more efficient specialization of production: high productivity agents remain in the production sector and the less productive ones serve as soldiers. The cost of this income-based sorting is a weaker level of ideological conviction among soldiers. In the terms of our model, this may make soldiers more reluctant to fight.

Let us now consider the possibility of positive or negative general correlations between income and ideology dimensions. With the above specification, a positive correlation could, for example, be expressed by comparing conditional means:  $E(\psi|\theta = 0) \geq E(\psi|\theta = 2\tilde{\theta})$ . However, a more general notion of correlation in that case is that of first order dominance of one conditional distribution function on the other:  $F_\psi(z|\theta = 0) \leq F_\psi(z|\theta = 2\tilde{\theta})$ , for all  $z$ . Negative correlation corresponds to changing the sign of the inequality.

The population of initially liquidity constrained peasants, for whom the income is zero, will partly enrol as soldiers up to a number  $S_1$ ; the other subpopulation of peasants, with a high income, will contribute to the army up to a number  $S_2$ . Recovering the above thresholds, we have  $S_1 = 1 - F_\psi(\alpha_1|\theta = 0)$  and  $S_2 = 1 - F_\psi(\alpha_2|\theta = 2\tilde{\theta})$ , where  $0 > \alpha_2 \geq \alpha_1$ .

Therefore,  $F_\psi(\alpha_2|\theta = 0) \geq F_\psi(\alpha_1|\theta = 0)$ .

As a consequence,

$$S_2 - S_1 = F_\psi(\alpha_1|\theta = 0) - F_\psi(\alpha_2|\theta = 2\tilde{\theta}) \geq F_\psi(\alpha_2|\theta = 0) - F_\psi(\alpha_2|\theta = 2\tilde{\theta}) \geq 0,$$

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<sup>24</sup> This result is supported by empirical evidence regarding the socioeconomic background of suicide attackers. On average, such assailants tend to come from relatively affluent families and have above average educational attainment (Benmelech and Berrebi, 2007).

if income and ideology are negatively correlated in the sense of first order stochastic dominance. On the other hand,

$$S_1 - S_2 = F_\psi(\alpha_2|\theta = 2\tilde{\theta}) - F_\psi(\alpha_1|\theta = 0) \geq F_\psi(\alpha_2|\theta = 2\tilde{\theta}) - F_\psi(\alpha_2|\theta = 0) \geq 0,$$

if income and ideology are positively correlated in the sense of first order stochastic dominance.

These results can be generalized to an arbitrary number of income levels by using the same comparison method. That is, the poor (respectively, the rich) are more likely to enrol if income and ideology are systematically positively (respectively, negatively) related in the first order dominance sense.

### **3.4. Aggregate Productivity Shocks**

#### **Defection**

The rapid or gradual process of defection which is the main result of a negative aggregated productivity shock is of major interest in the study of violent conflict. Defections serve as an endogenous channel of violence reduction. It may be a relatively peaceful mechanism that reduces the military tension between hostile groups. Our model suggests that defection may occur when an aggregate negative shock hits society. In this section, we study productivity shock, but another plausible possibility is ideological shock. If agents experience a sizeable decrease in their ideological beliefs (due to new sources of information, misconduct of the leader and propaganda of the opposite side) it may result in a shift in their fighting decision.

### **3.5. Extensions: External War Funding**

The effect of external financial aid and external transfers on the recruitment and support to the army is a major concern in view of the empirical relevance of these transfers. To account for this possibility, we explore an extension of the model with an additional resource  $M$  made available to the leader as part of the resource allocation decision. Denote  $M \geq 0$  as the financial external support the group receives. In principle, external funding may originate for reasons other than the conflict situation, although

support for one's ethnic/national group in a conflict may be a major motivation for these transfers.

Correspondingly, the leader maximization becomes

$$\begin{aligned} \max_{a_S} E[(1 - a_S)(L^B + M)] \\ \text{s.t. } a_L + a_S = 1. \end{aligned} \quad (31)$$

The F.O.C in respect to  $a_S$  is now:  $(1 - a_S) \frac{\pi_B}{S_B} \frac{\partial S^*}{\partial a_S} - \left( \frac{\pi_B}{S_B} S^* + M \right) = 0$ . Provided  $\frac{\partial S^*}{\partial a_S} \neq 0$ , we get the optimal solution:

$$a_M^* = 1 - \frac{S^* + \frac{M}{S_B}}{\left( \frac{\partial S^*}{\partial a_S} \right)}. \quad (32)$$

The new  $a_M^*$  is lower than the benchmark  $a_S$ . This implies that a military society is less likely and a defence society is more likely than in the benchmark case. However, the effect on the share of soldiers ( $S$ ) can go either way. After plugging the solution in the sorting decision equation (using the uniform distribution simplification), we obtain the interior solution for  $S$  and  $a_M^*$ :

$$S^* = \frac{\left( z - a_M^* \frac{\pi_B}{S_B} \right) - \sqrt{\left( z - a_M^* \frac{\pi_B}{S_B} \right)^2 - (4D\psi_0 + a_M^* M)}}{2\psi_0}. \quad (33)$$

$$a_M^* = \frac{z^2 - \left( \frac{\pi_B}{S_B} \right)^2 - 4D\psi_0}{2 \frac{\pi_B}{S_B} \left( z - \frac{\pi_B}{S_B} \right) + 4\psi_0 M}. \quad (34)$$

These results indicate that external financing does not affect the defence society regime since the boundaries are equal to the benchmark model. In contrast, in the case of the military society, external financing depletes the share of the war income (loot and external transfers) that is allocated to the soldiers. Note that  $2 \frac{\pi_B}{S_B} \left( z - \frac{\pi_B}{S_B} \right) + 4\psi_0 M < 0$  whenever the military society condition is satisfied and hence  $\frac{\partial a_M^*}{\partial M} < 0$ . Moreover, the total number of soldiers is not affected by  $M$ . The leader exactly compensates for the increased global war income by reducing the soldiers' war income share. This means that the external funding benefits only the leader and does not change the recruitment propensity of soldiers or their fighting decision. What stands behind this result is the utility function of the leader: in our model, the leader maximizes only income. The leader allocates shares of the loot to soldiers only up to the point where the marginal return from any additional soldier is positive. In contrast to the producers, the leader does not experience



a liquidity constraint (since he is able to commit to future payment); hence the optimal contract he can offer without external support does not change when funds are available.

The same property holds for the predator society: external war funding benefits the leader but does not influence the decentralized incentives for soldiers since all of them already fight. To summarize, under external funding the three types of society are now given by:

- i. Defence society: for  $\frac{\pi_B}{S_B} \leq \sqrt{z^2 - 4D\psi_0}$ , we get  $a_S^* = 0$  and  $S^* = \frac{z - \sqrt{z^2 - 4D\psi_0}}{2\psi_0}$ .
- ii. Militarized society: for  $\sqrt{z^2 - 4D\psi_0} < \frac{\pi_B}{S_B} < z - 2\psi_0$ , we get  $S^* = \frac{z - \frac{\pi_B}{S_B}}{2\psi_0}$  and  $a_M^* = \frac{z^2 - \left(\frac{\pi_B}{S_B}\right)^2 - 4D\psi_0}{2\frac{\pi_B}{S_B}\left(z - \frac{\pi_B}{S_B}\right) + 4\psi_0 M}$ .
- iii. Predator society: for  $z - 2\psi_0 \leq \frac{\pi_B}{S_B}$ , we get  $a_S^* = \frac{z - D - \psi_0}{\frac{\pi_B}{S_B} - M}$  and  $S^* = 1$ .

## 4. Conclusion

The paper studies the emergence and nature of violent civil conflict. It proposes a rationale for violence that is based on three successive and intricate triggers: enlistment, donations to combatants and decisions to kill/fight.

This rationale is based on a model explaining the self-selection of agents into producers or combatant roles under the monitoring of a leader. An originality of the model is that it includes four individual incentive channels: (1) the ideology of agents that expresses their degree of support for their participation in violent behaviour; (2) the opportunity to loot the opponents; (3) monetary transfers from producers to combatants through the leader payroll policy for soldiers; and (4) the relative physical safety of the chosen roles under severe conflict. We particularly exhibit and study three triggers of violence: the financial support to combatants by these populations, the enlistment choice into an army and the decision to actually fight by soldiers.

The model characterizes three regimes that correspond to escalating steps along the violent path of a civil war. In a defence society, the army is small and is not directly incentivized by the leader into harming the opponent. In a militarized society, the warfare sector is larger, and the leader financially stimulates the army to commit violent crimes

and loot the opponent. Lastly, in a predator society, production vanishes and is replaced by all agents engaging in the warfare sector.

We pay special attention to the impact of ideology and productivity heterogeneities on the incentives scheme. As expected, the model predicts that those agents less averse to acting violently more easily join the army, while the more averse rather support it from outside. In addition, poor agents outnumber rich agents in enlistment while, among soldiers, the highly devoted more ideological agents turn out to be wealthier than the average soldier.

Labour productivity in the agricultural sector also affects the intensity of war. However, while a permanent negative aggregate shock promotes warfare, a transitory negative aggregate shock hampers it. This is because a permanent shock affects the behaviour of agents before their self-selection into combatants. With agriculture becoming permanently less productive, more agents will prefer to join the more financially promising army. However, once the roles have been chosen, soldiers may partly rely on producer donations. Without donations, some or all soldiers may choose not to fight—that is, to defect. Thus, a transitory shock decreases the income of producers who, as a result, donate less. In some cases, the outcome might be fewer fighting soldiers. It is remarkable that many of the model results can be expressed in terms of the synthesized parameters describing the shadow producer income and the opportunity value. This structure of the model implies that richer specifications of these two synthesized parameters would yield straightforward generalizations, allowing for more sophisticated incentive channels.

"Moreover, we have yet to develop persuasive arguments for non-traditional mechanisms—myopic or selfish leaders, for example, or the role of ideology and identity in reducing free-riding within armed groups. As a consequence, too little empirical work is motivated by (and explicitly derived from) formal models." (Blattman and Miguel)

Different outcomes may occur when the opponent is active. When the possibility of looting by the opponent is considered, the greater the national wealth, the more there is to

fight over; thus, in standard formulations, the greater the equilibrium effort devoted to fighting instead of producing.<sup>25</sup>

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<sup>25</sup> See also Garfinkel and Skaperdas (2007) and Grossman (1991).

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## Appendices

### Appendix 1.1: Proofs for the Benchmark Model

We solve the model backwards. We first write the solution for  $S^*$  as a function of  $a_S^*$  in (13). Then, we solve condition (12) to get the value of  $a_S^*$  for the interior solution.

Remember that:  $\underline{\psi} = \theta_t - \frac{D}{S^*} - a_S^* \frac{\pi_B}{S_B} + \chi$  and  $S^* = \frac{\Psi}{\psi_0}$  and  $S^*$  in  $[0, 1]$ . For brevity, we rewrite  $\underline{\psi}$ :  $\underline{\psi} = z - \frac{D}{S^*} - a_S^* \frac{\pi_B}{S_B}$ , where  $z = \theta + \chi$ .

Then, condition (13) corresponds to the quadratic equation for  $S^*$ :  $\psi_0 S^* = z - \frac{D}{S^*} - a_S^* \frac{\pi_B}{S_B}$ .

Solving for  $S^*$ , we have the unique positive root:  $S^* = \frac{(z - a_S^* \frac{\pi_B}{S_B}) - \sqrt{(z - a_S^* \frac{\pi_B}{S_B})^2 - 4D\psi_0}}{2\psi_0}$ , which is always positive as  $\psi_0$  is negative. For any positive  $D$ , the number of soldiers turns out to be positive.

Plugging  $S^*$  into condition (12), we obtain  $a_S^* = \frac{z^2 - (\frac{\pi_B}{S_B})^2 - 4D\psi_0}{2\frac{\pi_B}{S_B}(z - \frac{\pi_B}{S_B})}$ .

However,  $a_S$  has to belong to  $[0, 1]$ , which we now check. Solving for  $a_S^* \geq 0$ , we get the condition:  $\frac{\pi_B}{S_B} \geq \sqrt{z^2 - 4D\psi_0}$ .

Below such a threshold for  $\frac{\pi_B}{S_B}$ ,  $a_S^* = 0$  and accordingly:  $S^* = \frac{z - \sqrt{z^2 - 4D\psi_0}}{2\psi_0}$ . This constitutes our first corner solution that we denote ‘defence society’. Note that  $\frac{\pi_B}{S_B} \geq \sqrt{z^2 - 4D\psi_0}$

implies that  $a_S = 0$  whenever  $z \geq \frac{\pi_B}{S_B}$ . Otherwise, when  $\frac{\pi_B}{S_B}$  is large enough, we can examine if  $a_S^* < 1$ . This corresponds to  $\frac{z^2 - \left(\frac{\pi_B}{S_B}\right)^2 - 4D\psi_0}{2\frac{\pi_B}{S_B}\left(z - \frac{\pi_B}{S_B}\right)} < 1$ , which is satisfied regardless of how large  $\frac{\pi_B}{S_B}$  is.

Plugging the formula for  $a_S^*$  back into  $S^*$ , we get  $S^* = \frac{z - \frac{\pi_B}{S_B}}{2\psi_0}$  for the first corner solution.

However,  $S^*$  is bounded above by 1. Imposing it yields the condition for the minimal  $\frac{\pi_B}{S_B}$ , beyond which all agents would fight  $\frac{\pi_B}{S_B} \geq z - 2\psi_0$ , which corresponds to our second corner solution that we denote ‘predator society’. Replacing  $\frac{\pi_B}{S_B} = z - 2\psi_0$  in the formula for  $a_S^*$ , we get the following upper bound of  $a_S^*$ :  $\overline{a_S^*} = \frac{z - \psi_0 - D}{z - 2\psi_0}$ .

Indeed, the leader does not incentivize producers to become soldiers once  $S^* = 1$  is reached.

Remember that the maximization with respect to  $a_S$  is subject to  $S^* \leq 1$ . We can write the condition  $S^* \leq 1$  as:  $\psi_0 \leq \underline{\psi} = \theta - \frac{D}{1} - a_S^* \frac{\pi_B}{S_B} + \chi$  or, equivalently,  $a_S^* \leq \frac{z - \psi_0 - D}{\frac{\pi_B}{S_B}}$ .

Once  $\frac{\pi_B}{S_B}$  has reached  $z - 2\psi_0$ , all agents are soldiers and fight, and the relative share of the soldiers in the booty decreases with any other rise in  $\frac{\pi_B}{S_B}$ .



### Appendix 1.2

Recall that  $E[U_{P_i}] = \tilde{\theta} - E[D_i] + E[\log(\omega h_p)] + \beta E[\log(1 + D_i)]$ . We decompose the expression into:

$$\begin{aligned} E[D_i] &= \left(1 - \frac{\beta-1}{2\lambda\tilde{\theta}}\right) (\beta - 1) + \left(\frac{\beta-1}{2\lambda\tilde{\theta}}\right) E[\theta_i | \theta_i < \beta - 1] \\ &= \left(1 - \frac{\beta-1}{2\tilde{\theta}}\right) (\beta - 1) + \left(\frac{\beta-1}{2\tilde{\theta}}\right) \frac{(\beta-1)}{2} = (\beta - 1) - \frac{(\beta-1)^2}{4\tilde{\theta}} \text{ and} \\ E[\log(1 + D_i)] &= \left(1 - \frac{\beta-1}{2\tilde{\theta}}\right) \log(\beta) + \frac{\beta-1}{2\tilde{\theta}} E([\log(1 + \theta_i)] | \theta_i < \beta - 1) \end{aligned}$$

Since

$$E([\log(1 + \theta_i)] | \theta_i < \beta - 1) = \int_1^{\beta} \log(1 + \theta_i) d(1 + \theta_i) = \beta \log(\beta) - (\beta - 1),$$

we have:

$$\begin{aligned} E[\log(1 + D_i)] &= \left(1 - \frac{\beta-1}{2\tilde{\theta}}\right) \log(\beta) + \frac{\beta-1}{2\tilde{\theta}} (\beta \log(\beta) - (\beta - 1)) \\ &= \frac{(\beta-1)^2}{2\tilde{\theta}} \log\left(\frac{\beta}{e}\right) + \log(\beta) \end{aligned}$$

For brevity, we denote  $\hat{\beta}(\beta, \tilde{\theta}) = \beta E[\log(1 + D_i)] = \beta \left( \frac{(\beta-1)^2}{2\tilde{\theta}} \log\left(\frac{\beta}{e}\right) + \log(\beta) \right)$ .

Finally:  $E[U_{P_i}] = \tilde{\theta} - (\beta - 1) - \frac{(\beta-1)^2}{4\tilde{\theta}} + E[\log(\omega h_p)] + \hat{\beta}$ .

### Appendix 1.3

Using,  $D_i^*(\tilde{\theta}_I) > D_i^*(\tilde{\theta}_{II})$  and  $\psi_i^F(\tilde{\theta}_j) = a - b \frac{D_i^*(\tilde{\theta}_j)}{p_k(\tilde{\theta}_j)}$ , with obvious notations for  $a$  and  $b$ , we have  $p_k(\tilde{\theta}_j) = P \left[ \psi + b \frac{D_j^*(\tilde{\theta}_j)}{p_k(\tilde{\theta}_j)} > a \right]$ . Then, by implicit derivation of this expression, we obtain  $p_k' \left[ 1 + b \frac{p_k' D}{(p_k)^2} > a \right] = b \frac{p_k' D'}{p_k}$ . Since the right-hand side term of this expression is positive and the bracket in the left-hand side term is also positive, then  $p_k$  is necessarily increasing in  $\theta$ .

Let  $p_k(\theta) = P \left[ \psi + b \frac{D(\theta)}{p_k(\theta)} > a \right] = \int_{a - b \frac{D(\theta)}{p_k(\theta)}}^{+\infty} d\psi$ . By derivation, we obtain

$$p_k'(\theta) = - \left( a - b \frac{D(\theta)}{p_k(\theta)} \right) \left( -b \frac{[D(\theta)D'(\theta) - D(\theta)p_k'(\theta)]}{(p_k(\theta))^2} \right) \text{ which, after rearranging, can be written as}$$

$$p_k'(\theta) \left[ 1 + b \left( a - b \frac{D(\theta)}{p_k(\theta)} \right) \left( \frac{D(\theta)}{(p_k(\theta))^2} \right) \right] = - \left( a - b \frac{D(\theta)}{p_k(\theta)} \right) b \frac{D'(\theta)}{p_k(\theta)}$$

Since:  $a - b \frac{D(\theta)}{p_k(\theta)} > 0$  and  $b = P^*/S^* > 0$  in the case we consider, then

$\left[ 1 + b \left( a - b \frac{D(\theta)}{p_k(\theta)} \right) \left( \frac{D(\theta)}{(p_k(\theta))^2} \right) \right] > 0$ . Moreover,  $D'(\theta) > 0$  and  $p_k(\theta) > 0$ . Therefore,  $p_k'(\theta) > 0$ .

This implies  $\psi_i^F(\tilde{\theta}_{II}) > \psi_i^F(\tilde{\theta}_I)$ .