

The Economic Geography of the Public Sector: Theory and Evidence from Secondary Schools in Paris

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Abstract

This paper develops a quantifiable model of public service location to investigate the relation between urban structures and public services. The State decides on a location strategy, i.e. the number, location and capacity of public facilities, while anticipating how residential density, housing prices and locational characteristics will react. The State's objective is to maximize individual utility while minimizing the sum of fixed and variable facility costs. We prove the existence of an equilibrium for any given set of facility locations; and derive the cost minimizing set of locations using a gradient method inspired by the generalized Weiszfeld method. The model remains tractable thanks to the use of stochastic shocks to commuting decisions which yield a gravity equation for commuting flows. We show that the State can strategically locate facilities to increase population density, and thus reduce commuting and facility costs. The cost minimizing strategy is defined by a bell shaped relationship between school locations and distance to the center. In a counterfactual exercise, we then show that the provision of public services under a budget constraint generates agglomeration forces important to explain urban structures; and that a tighter public budget constraint will result in higher density in core places.

Keywords: Public service location, agglomeration, Weiszfeld method, commuting, gravity.

JEL classification: H11; R53; R41.

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1 Introduction

The New Economic Geography following Krugman (1991) shows that a core-periphery structure can emerge from agglomeration economies. Spatial spillovers of productivity or knowledge are a typical example of such agglomeration forces. This approach diverges from more traditional economic geography theories, such as the Central Place Theory developed by Christaller (1933), which insists on the provision of services to surrounding areas in order to explain the existence of cities. Public services are an example of the services offered by central places. Following these traditional theories, this paper extends the New Economic Geography perception of cities by arguing that the provision of public services under a budget constraint generates agglomeration forces which are important to explain urban structures.

The distribution of public services across space (partly) defines the quality of the public sector. Accessibility and efficiency of a public service is a function of the distance between individuals and the public service in question. To illustrate this, consider healthcare services. Buchmueller, Jacobson, and Wold (2006) and Nicholl, West, Goodacre, and Turner (2007) show that proximity to an hospital strongly influences the chances of recovery, or even survival, after an accident. Distance also impacts the public services' performance in the case of education services. Frenette (2006), for example, shows that proximity to university impacts the probability of attending university. Larger distances to education services can also be a source of income or ethnic segregation (see Burgess, McConnell, Propper, and Wilson, 2007; Söderström and Uusitalo, 2010). Moreover, larger distances to public services also have negative indirect effects. Longer daily commutes to school increases traffic which increases air pollution and eventually exposure to pollution. Recent medical research show that walking to school along busy streets reduces children cognitive capacity.¹ Liu, Ma, Liu, Han, Chuang, and Chuang (2015) show that commuting in general by foot or by car leads to a worst cardiovascular condition. However, reducing the distance between individuals and public services is very costly. The state incurs important costs when opening or closing a new public facility, such as a new school or a new hospital. Additionally, the choice of opening a new facility is a strong commitment as most public facilities remain fixed at a particular location for a long period of time. It follows that the decision of where to locate any new facility is of high importance.

Thisse and Wildasin (1992) present the first economic analysis of the public facility location problem in the context of an urban area. In their model, the location decisions of firms and households are a function of the location of the public facility. More recently, Berliant, Peng, and

¹See, among others, Alvarez-Pedrerol, Rivas, López-Vicente, Suades-González, Donaire-Gonzalez, Cirach, de Castro, Esnaola, Basagana, Dadvand, Nieuwenhuijsen, and Sunyer (2017).

Wang (2006) develop a model in which the level of provision, the number of facilities and their location is endogenously determined. Bellettini and Kempf (2013) present a political economy perspective which solves the facility location problem both when distance to facility is positively and negatively correlated to individual benefits. Compared to our paper, these analyses remain theoretical. Redding, Sturm, and Wolf (2011) constitute a rare empirical analysis of how the location of public facilities (airports) impact the distribution of economic activity. Not directly related to the facility location problem, Turnbull (1989) and Brueckner, Thisse, and Zenou (1999) present models in which local amenities and public goods play a central role in shaping the distribution of economic activity. Also, an important literature on this topic comes from Operations Research. In this field, the question is to solve various forms of the classical Fermat-Weber location problem². This body of literature takes the location of individuals (or costumers) as well as land prices as given; hence, ignoring the endogeneity of individuals' location decisions. However, this question of endogeneity is particularly relevant for public services as Fack and Grenet (2010) document it in the context of secondary schools in Paris.

In this paper, to analyze the relation between public services and urban structure, we develop a quantifiable model of public service location in which the State decides on a location strategy, i.e. the number, location and capacity of public facilities, while anticipating how residential density, housing prices and locational characteristics will react. The State's objective is to maximize the sum of individual utility while minimizing the sum of facility costs. Individual utility is affected by distance to public services and residential prices, among other factors. Facility costs are of two types. First, fixed facility costs refer to fixed cost associated to the functioning of a facility. These costs are assumed to be constant across location; and hence, do not depend on the size or the location of the facility. They typically contain basic infrastructure maintenance, general administration, etc ... Second, variable facility costs are costs associated to the facility size. In our model, we suppose that facility size affects performance negatively to fit the education literature as in our application we study secondary schools. However, the model could very well incorporate a positive relationship between size and performance.

We show that for any given facility location, a unique equilibrium exists. To solve for the cost minimizing location strategy, we use a gradient method inspired by the generalized Weiszfeld method as proposed by Weiszfeld (1937) and Iyigun and Ben-Israel (2010). Using probabilistic assignments, this gradient method allows us to solve the large scale and complex optimization problem at hand within a few iterations. It also permits the analysis of marginal cost minimizing

²For a review of the facility location problem literature in Operations Research, see ReVelle and Eiselt (2005) and Farahani and Hekmatfar (2009).

decisions.

We then apply our model to the case of the French “Collèges”, i.e. lower secondary schools, in a radius of 20km around the city center of Paris. We use data from 2010. In France, pupils attend lower secondary education between the ages of 11 and 15 on average. As an example of public facilities, French secondary schools offer several advantages. First, we can identify precisely the users of the public service as secondary education is compulsory in France until the age of 16. Second, as it is compulsory, we also know exactly how many seats the State must provide. Finally, a 2007 law relaxed greatly the constraint imposed by school districts by allowing pupils to attend schools outside of their districts under the only condition of seat availability.

After calibrating the model to fit the urban characteristics of Paris, we show that the State can strategically locate facilities to increase population density, and thus reduce commuting and facility costs. The cost minimizing strategy is defined by a bell shaped relationship between school locations and distance to the center. In a counterfactual exercise, we withdraw the two forces *a priori* generating a core-periphery structure, i.e. preference to locate centrally and larger availability of floor space in the center, and show that such a structure can emerge simply from the State’s cost minimizing behavior. Furthermore, we show that a tighter public budget constraint will result in higher density in core places. These results support the claim that the public sector plays an important role in shaping a core-periphery structure. Furthermore, these results link the public budget constraint to peripheral (or rural) decline.

The paper proceeds as follows. We describe our model and its main assumptions in section 2, and solve for the equilibrium and the cost minimizing solution in section 3. In section 4, we introduce the data. We estimate the model structurally and calibrate it in section 5. Before presenting the counterfactual analyses in Section 7, we describe the baseline cost minimizing strategy and compare it to the observed strategy in Section 6. In Section 8, we derive the marginal cost minimizing decision for the State knowing the observed strategy. Section 9 concludes.

2 Theoretical framework

In this section, we develop our quantifiable model of public facility location. The modelisation of individuals’ behavior and the land market is inspired by Ahlfeldt, Redding, Sturm, and Wolf (2015).

2.1 Individuals

The utility of individual o living in place i using a facility in site k is linear in an aggregate consumption index C_{iko} , such that: $U_{iko} = C_{iko}$. This consumption index depends on consumption of the single final good (c_{iko}), consumption of residential floor space (l_{iko}), the utility for residential amenities in i (B_i), the disutility from commuting from place i to place k ($d_{ik} \geq 1$), the disutility from commuting from place i to the central business district (CBD) ($D_i \geq 1$) and an idiosyncratic shock that is specific to the individual and varies with the individual's place of residence and service provision (z_{iko}). The disutility from commuting from place i to place k is model as an iceberg cost $d_{ik} = e^{\nu\rho_{ik}} \in [1, \infty)$, which increase with the euclidean distance ρ_{ik} between places i and k . The distance to the city center is also modeled as an iceberg cost $D_i = e^{\zeta\xi_i} \in [1, \infty)$, which increases with the euclidean distance to the center ξ_i . The use of euclidean distances is required to use our gradient optimization method, as explained in section 3. Here, we make the assumption that the private sector produces only in the CBD. The aggregate consumption index is assumed to take the following Cobb-Douglas form:

$$C_{iko} = z_{iko} \frac{B_i}{d_{ik} D_i} \left(\frac{c_{iko}}{\beta} \right)^\beta \left(\frac{l_{iko}}{1-\beta} \right)^{1-\beta}, \quad 0 < \beta < 1 \quad (1)$$

The idiosyncratic shock (z_{iko}) describes the heterogeneity in the utility that individuals derive from living in i and using public service in k . For each individual, this idiosyncratic component is drawn from an independent Fréchet distribution following McFadden (1974) and Eaton and Kortum (2002):

$$F(z_{iko}) = e^{-P_i z_{iko}^{-\epsilon}} \quad (2)$$

where $P_i > 0$ refers to the average utility from living in place i and $\epsilon > 1$ is the shape parameter that controls the dispersion of idiosyncratic utility.

After observing her realization of the idiosyncratic utility for each pair of residence and provision of public service, each individual chooses her place of residence and the place of her public service to maximize her utility. Combining our choice of the final good as numeraire ($p_i = p = 1$) with the first-order conditions for consumer equilibrium, we obtain the following demands for the final good and residential land for individual o at place i and using public service in place k .

$$c_{iko} = \beta w \quad (3)$$

$$l_{iko} = (1 - \beta) \frac{w}{Q_i} \quad (4)$$

where w is the wage received by the individuals in the CBD. Here, we make the standard assumption that rent is accrued by absentee landlords; and hence, not spent within the city. Substituting (3) and (4) into (1), we obtain the following indirect utility function:

$$U_{iko} = \frac{B_i z_{iko} w Q_i^{\beta-1}}{d_{ik} D_i}. \quad (5)$$

2.2 Individuals' location choices

We look at the combined location choices of individuals across the metropolitan area. Following Ahlfeldt et al. (2015), we first derive the probability that individuals choose a particular combination of residence and public service location:^{3 4}

$$\begin{aligned} \pi_{ik} &= Pr[u_{ik} \geq \max\{u_{ik}\}; \forall i, k] \\ &= \frac{P_i (d_{ik} D_i Q_i^{1-\beta})^{-\epsilon} (B_i)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon} \equiv \frac{\Phi_{ik}}{\Phi} \end{aligned} \quad (6)$$

Equation (6) shows that individuals sort across all combinations of residence and public service location depending on their idiosyncratic preferences and the characteristics of these locations. Residential locations with an higher value of P_i lead to higher positive draws of utility for the combination.

We can derive the probability that an individual decides to live in i out of all possible locations in the metropolitan area by summing Φ_{ik} across all public service locations.

$$\pi_{Ri} = \frac{\sum_{l=1}^K P_i (d_{il} D_i Q_i^{1-\beta})^{-\epsilon} (B_i)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon} \equiv \frac{\sum_{l=1}^K \Phi_{ik}}{\Phi} \quad (7)$$

The same can be done for the probability that an individual decides to use the public service in k out of all possible public service location choices:

$$\pi_{Sk} = \frac{\sum_{j=1}^I P_j (d_{jk} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon} \equiv \frac{\sum_{j=1}^I \Phi_{ik}}{\Phi} \quad (8)$$

³For full derivation, see sections A.1 and A.2 of the Appendix.

⁴With the index j , we refer to all places in I different from i . Similarly, with the index l , we refer to all places in K different from k .

Note that the number of people residing in i and using the service in k denoted H_{ik} , residing in i denoted H_{Ri} , and using the service in k denoted H_{Sk} can be obtain by multiplying these probability by the total number of individuals H :

$$H_{ik} = \pi_{ik} \times H, \quad H_{Ri} = \pi_{Ri} \times H, \quad H_{Sk} = \pi_{Sk} \times H \quad (9)$$

Finally, we consider the case of a metropolitan area in a wider economy, with population mobility across the two zones. This implies that the expected utility of moving to the metropolitan area must equal the reservation level of utility in the wider economy (\bar{U}):

$$\mathbb{E}[u] = \gamma \Phi^{1/\epsilon} = \gamma \left[\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_i w)^\epsilon \right]^{(1/\epsilon)} = \bar{U} \quad (10)$$

where $\gamma = \Gamma(\frac{\epsilon-1}{\epsilon})$ with $\Gamma(\cdot)$ being the Gamma function; and \mathbb{E} is the expectation operator and expectation is taken over the distribution of idiosyncratic utility.

2.3 State

The State decides on the number, location and capacity of public facilities while anticipating how residential density, housing prices and locational characteristics will react. For brevity, we refer to any State's combined choice as a *location strategy*. The State's objective is to maximize individual utility while minimizing the costs associated to providing public services which are fixed and variable facility costs. Formally, the State decides on a location strategy by choosing the coordinates c_k of the facilities in order to minimize the sum of the inverse of the individual utility and the sum of facility costs. Let us define the cost of providing public services in a particular location k as:

$$\Pi_k = \sum_{i=1}^I U_{ik}^{-1} H_{ik}^\omega + F \quad (11)$$

F refers to the fixed facility costs, constant across all locations by definition. Putting ω aside for now, $\sum_{i \in I} U_{ik}^{-1} H_{ik}$ is the sum of the utility costs associated to public facility k . It is the sum across all residential locations I of the product between the inverse of the representative individual utility and the number of commuters from location $i \in I$ to facility k . H_{ik} is as defined in (9). U_{ik} is $\frac{B_i w Q_i^{\beta-1}}{d_{ik} D_i}$. Minimizing U_{ik}^{-1} is equivalent to maximizing the utility of the H_{ik} individuals commuting from i to k as the shock z_{iko} is idiosyncratic and monotonically related to U_{ik} . Hence, higher values of U_{ik} lead to higher draws of individual utility.

To fully endogenize the State's location strategy, the model is based on bilateral commuting decisions. Hence, the capacity of a facility is an outcome of the model. Thus, it is difficult in that context to introduce a cost based on total facility capacity which would partly determine the facility's capacity. However, the model can easily incorporate a cost by link ik based on the commute size. Hence, if more people commute to a facility, it increases its size and thus affects its performance. In (11), ω is defined such that $\omega \geq 1$. It introduces this cost by artificially increasing the number of commuters from i to k . However, one could also want to model larger costs for smaller commutes. This could be done by imposing $\omega \leq 1$.

Summing over all $k \in K$ public facilities, the State's minimization problem is:

$$\min_{c_k, \dots, c_K} \Pi = \sum_{k=1}^K \sum_{i=1}^I U_{ik}^{-1} H_{ik}^\omega + K \times F \quad (12)$$

2.4 Land market

In our model, land market prices are determined by the individuals residential location choices as the state does not consume land. We assume that the observed floor price in the data $\mathbb{Q}_i = Q_i$. We further consider, as often done in the urban literature, that floor space L is provided by a competitive construction sector that uses geographic land G and capital M . We follow Ahlfeldt et al. (2015) and Epple, Gordon, and Sieg (2010), and assume that the production function takes the Cobb-Douglas form: $L_i = J_i^\mu K_i^{1-\mu}$. The corresponding dual cost function for floor space is then $\mathbb{Q}_i = \mu^{-\mu} (1-\mu)^{-(1-\mu)} \mathbb{P}^\mu \mathbb{R}_i^{1-\mu}$; where $\mathbb{Q}_i = Q_i$ is the price for floor space, \mathbb{P} is the common price for capital and \mathbb{R}_i is the price for geographic land. Since the price for capital is the same across all locations, the relationships between the quantities and the prices for geographical land and floor space can be expressed as:

$$L_i = \phi_i K_i^{1-\mu}, \quad (13)$$

$$\mathbb{Q}_i = \chi \mathbb{R}_i^{1-\mu}, \quad (14)$$

where $\phi_i = J_i^\mu$ determines the density of development and χ is a constant.

Residential land market clearing implies that the demand for residential floor space equals the supply of floor space in each location: L_i . This residential land market clearing condition can be written as:

$$\mathbb{E}[l_i] H_{Ri} = (1-\beta) \frac{w}{Q_i} H_{Ri} = L_i \quad (15)$$

2.5 Discussion of assumptions

To complete the description of our model, we first discuss two important assumptions which we impose in the baseline model but that we shall relax later on. These two assumptions relate to the monocentricity of cities. First, individuals utility decrease with the distance to the center D_i . Second, the available floor space L_i in i is a function of land space K_i which is exogenously given and the density of development ϕ_i which is calibrated using observed variables. Hence, more floor space is available in the center than outside. These two assumptions are first imposed to insure that more individuals locate close to the center. However, in the counterfactual exercise in Section 7.1, we will show that these assumptions are not necessary to obtain a core-periphery structure.

Additionally, note that the assumption that the State does not consume land does not imply that the State's location strategy has no impact on housing prices, as these prices are a function of the distance to the public services. This assumption implies however that the state does not impact housing prices by restricting the available floor space.

3 Equilibrium

We solve for the equilibrium of the model in two steps. First, we describe the optimal behavior of individuals conditional on a location strategy of the State. Second, we describe the method we employ to derive the cost minimizing location strategy.

3.1 Individuals' behavior conditional on a location strategy

Conditional on a location strategy, the equilibrium of the model is referenced by the vectors $\{H, \pi_R, \pi_S, Q\}$. The following expressions define the elements of the equilibrium:

$$\gamma \left[\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j w)^\epsilon \right]^{(1/\epsilon)} = \bar{U} \quad (16)$$

$$\pi_{Ri} = \frac{\sum_{l=1}^K P_i (d_{il} D_i Q_i^{1-\beta})^{-\epsilon} (B_i)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon} \quad (17)$$

$$\pi_{Sk} = \frac{\sum_{j=1}^I P_j (d_{jk} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon} \quad (18)$$

$$(1 - \theta_i) \phi_i K_i^{1-\mu} = \frac{(1 - \beta) w}{Q_i} H_{Ri} \quad (19)$$

Proposition 1: Assuming strictly positive, finite and exogenous characteristics ($P_i \in (0, \infty)$, $B_i \in (0, \infty)$, $K_i \in (0, \infty)$, $D_i \in (1, \infty)$, $d_{ik} \in (1, \infty) \times (1, \infty)$), there exist a unique general equilibrium vector $\{H, \pi_R, \pi_S, Q\}$.⁵

3.2 Deriving the cost minimizing strategy

Solving the minimization problem of the State is complex. The *brute-force search* approach which consists of systematically evaluating all location strategies is not suited as there are an infinity of possible locations within a circle of radius 20km around Paris city center. One could however discretize the State's problem by assuming that the State would only be able to build at a specific set of locations, e.g. at the center of blocks. On top of imposing a strong assumption on the possible locations, discretizing the State's problem does not really help solving the problem. In total with 1031 cells of $1km^2$ in a radius of 20km around Paris's city center, we would need to evaluate 2^{1031} possible strategies. One method to solve such discrete optimization problems has been proposed by Jia (2008). In essence, this method permits the derivation of an upper and a lower bound of the location strategy⁶. One can then evaluate all location strategies within these bounds and select the one that minimizes the cost function (or maximizes a profit function). However, this method has several drawbacks which make it impractical in the present location problem. First, it requires the definition of a mapping which must be an increasing function of the number of locations K ; and hence, the method is more suitable in case where facilities are complements and not substitutes. Second, it aims at finding the global optimum, and thus, is not suited to analyze marginally optimal choices. Finally, it does not guarantee that the lower and upper bound are close to each other. Hence, the number of strategies to be evaluated can still be large.

In this paper, we propose a more direct method to derive the optimal location strategy inspired by the generalized Weiszfeld method (Weiszfeld, 1937; Iyigun and Ben-Israel, 2010)⁷, which allows facilities to be located anywhere on the map. This gradient method enables us to solve directly for the optimal location strategy for a given number of facilities within few iterations. Additionally, this method has the advantage of being suitable for both finding the global optimum, but also to find the marginal optimal choices. This is due the fact that the

⁵See proof of equilibrium in the Appendix, section B.

⁶Here, we describe the method proposed by Jia (2008) in the context of the facility location problem. Originally, Jia (2008) studies the optimal location of retail stores such as Wal-mart.

⁷For an english version of Weiszfeld (1937), see Weiszfeld and Plastria (2009)

method exploits the separability of the State's minimization problem. In what follows, we present the proposed method and its convergence properties.⁸

For a given number of facility locations K and the associated overall fixed cost, we can reformulate the State minimization problem (12) as:

$$\min_{c_1, \dots, c_K} \Pi = \sum_{k=1}^K \sum_{i=1}^I \overline{U_{ik}^{-1}} H_{ik}^\omega d_{ik}, \quad \text{where } d_{ik} = e^{\nu \|x_i - c_k\|} \quad \text{and} \quad \overline{U_{ik}^{-1}} = \frac{D_i}{B_i w Q_i^{\beta-1}} \quad (20)$$

$\|x_i - c_k\|$ is the Euclidean distance between the individual locations x_i and the facilities or centers c_k . Taking for a moment $\overline{U_{ik}^{-1}} H_{ik}^\omega$ as a constant, note that (20) is a separable function of the centers, which allows us to decompose the multi-facility problem into multiple single facilities problem. Compared to a discrete optimization problem, we can then optimize over the coordinates of the facility locations; and therefore, use a standard gradient optimization method. The optimal location will be the one for which the gradient is equal to 0.

We can optimize over the coordinates of the centers using the separability of (20) as follows:

$$f(c_1, \dots, c_K) = \sum_{k=1}^K f_k(c_k), \quad \text{where } f_k(c_k) = \sum_{i=1}^I \overline{U_i^{-1}} H_{ik}^\omega e^{\nu \|x_i - c_k\|}, \quad k \in K \quad (21)$$

Setting the gradient of $f_k(c_k)$, $\nabla f_k(c_k)$, equal to 0, we get K mappings (one for each facility) $T_k : \mathbf{c} \rightarrow T_k(\mathbf{c})$:

$$T_k(\mathbf{c}) = \sum_{i=1}^I \left[\frac{e^{\nu \|x_i - c_k\|} \overline{U_i^{-1}} H_{ik}^\omega / \|x_i - c_k\|}{\sum_{j=1}^I e^{\nu \|x_i - c_k\|} \overline{U_i^{-1}} H_{ij}^\omega / \|x_i - c_k\|} \right] x_i, \quad \forall c_k \neq x_j. \quad (22)$$

The optimization procedure needs however to account for the changes in the individuals' utility and commuting decisions $\overline{U_i^{-1}} H_{ik}^\omega$ as they are functions of the distance d_{ik} . A natural iterative approach to solving (20) is then to fix the individuals' utility and commuting decisions $\overline{U_i^{-1}} H_{ik}^\omega$, and optimize (20) with respect to the centers using the gradient method above, then fixing the center, we can solve for the individuals' utility and commuting decisions simply using (5) and (6), etc. until the optimum is reached.

Proposition 2: Given individual location points $\{x_i : i \in (1, N)\}$, let $\{c_k : k \in (1, K)\}$ be arbitrary points, and π_{ik} the corresponding individuals' utility and commuting decisions given

⁸See Appendix, section C for a detailed derivation of the proof of the convergence of our method which follows closely the convergence proofs of the generalized Weiszfeld method.

by (5) and (6), then the condition:

$$\nabla f_k(c_k) = 0 \tag{23}$$

is necessary and sufficient for the points $\{c_1, \dots, c_K\}$ given by $T_k(\mathbf{c})$ to minimize $f(c_1, \dots, c_K)$ from (21).

Empirically, the iterative process cannot go endlessly. We stop it when the sum of the distance between the centers of the last two iterations is less than an arbitrarily given criteria. In our application, the iteration stops when the last move made by each center is less than 0.01m. Formally,

$$\sum_{k=1}^K \mathbb{1}[d(c_k^+, c_k) < 0.01] = 0, \tag{24}$$

where $d(c_k^+, c_k)$ is the distance between the last two iterations of a center’s location.

4 Data description

In this section, we describe the data used in the application, present basic urban characteristics of Paris and describe the French secondary education system.

Data source The core data employed in the application combines three main datasets: geo-localized individual data, geo-localized public service data and commuting data (from 2010). First, information about the location, capacity and fixed costs of the French secondary school system is provided by the French Ministry of Education. We know the exact coordinates of each establishment, as well as the number of students registered in each school. Second, geo-localized individual data is given by the 2010 gridded population data (“données carroyées”), which registers the number of person residing in squares of 200mx200m covering the whole country. The population by age group in each grid is also available. Third, data about the average commuting time to school is provided by the National Institute for Statistics and Economic Studies (INSEE).

Additionally, we obtain municipal income and further demographic data from the INSEE. The website “Meilleursagents.com” provided us with municipal housing price data. This data is based on the notary database “BIEN” and their own transaction records.

We create our units of analysis by designing a grid of 1 km^2 cells within a 20km radius from Paris city center, i.e. “Hotel de Ville”.⁹ This leads to 1031 cells. As all data described above is geo-localized, it is then straightforward to join it to our grid based on spatial overlay. Finally, we compute distances between each cell.

Urban characteristics of Paris Figures 1 and 2 provide empirical facts about the urban characteristics of Paris. As expected, we observe that residential density and housing prices decrease strongly as we move to the outskirts. More precisely, this pattern of residential density is observed only from 4 km of the center onwards. Within the first 4 km, residential density increases. The reason for this pattern is double. First, the very city center is occupied by wealthier individuals which are likely to own larger apartments. Second, the available residential space is strongly limited in the very center by natural factors (Seine river) and by the presence of public facilities (government facilities, museums, etc ...).

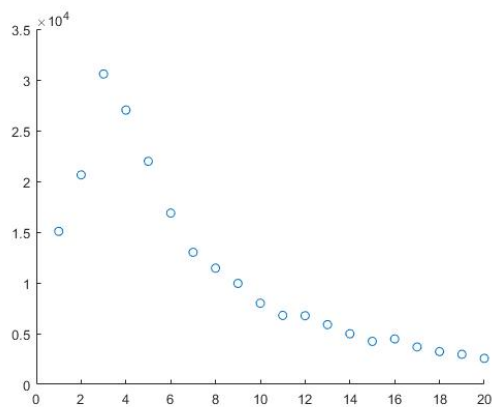


Figure 1: Residential density

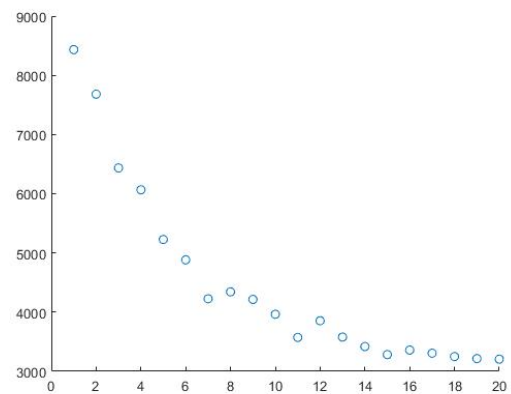


Figure 2: Avg. housing price

French secondary school system To empirically analyze the location of public services, it is useful to focus on each public service separately, apart from all others. Considering the location of all public services together without distinction might lead to bad spatial coverage for each particular service. In our application, we focus on the location of “Collège”, i.e. French lower secondary education. In France, secondary education is organized in two stages: the lower secondary education called “Collège” for pupils aged between 11 and 15, and the higher secondary education called “Lycée” for pupils aged between 15 and 18.¹⁰

Compared to other public services, studying the location of French secondary schools offers three advantages. First, as education is compulsory in France until 16 years old, we are able to

⁹The grid is created using the Geographic Information System ArcGIS.

¹⁰For simplicity, we shall refer from now on to the French “Collège” as secondary school.

identify easily and precisely the users of the public service under study. According to UNESCO statistics, the schooling rate in lower secondary education in France was 99,79% in 2010. We consider that all individuals of ages between 11 and 15 are users of secondary schools. Second, as it is compulsory, the State must provide a seat to all pupils aged between 11 and 15. Finally, the requirement to attend a school within a particular school district was significantly relaxed in a 2007 law. From 2007 onwards, available seats in any district could be accessed by pupils residing outside the district.

When looking at public schooling, one important consideration to keep in mind is that the private sector offers a competing service. In France, 17% of all pupils are attending a private establishment. For simplicity and clarity of the approach, our model assumes that no private education is offered and that only pupils going to a public establishment need to be offered a seat. This assumption should not harm the generality of the model as private schools do not really represent an alternative to public schools in France. 97% of all private schools are catholic schools. The choice to attend a private school is often made once and motivated by religious considerations. Hence, the decision to move from private to public schools (or the reverse) is rarely occurring.

A first look at the actual distribution of secondary schools provide clear and interesting patterns. Figure 3 illustrates the average school capacity as a function of distance to the city center in 2010. We observe that average capacity increases from 475 to 600 pupils as you move away from the center. Knowing the high density in the center, one could have expected a flatter or even negative relationship. Figure 4 displays the average number of schools as a function of distance to the city center. No clear pattern is observed which is surprising for the same reason as for the average capacity.

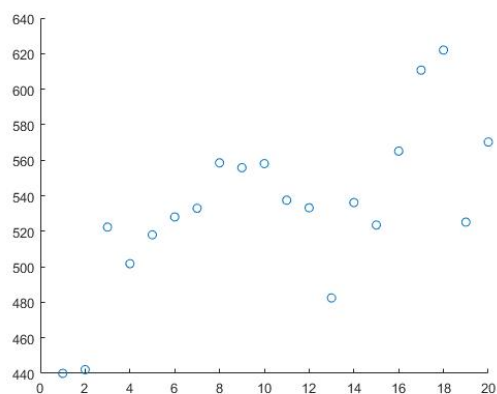


Figure 3: Secondary school seats

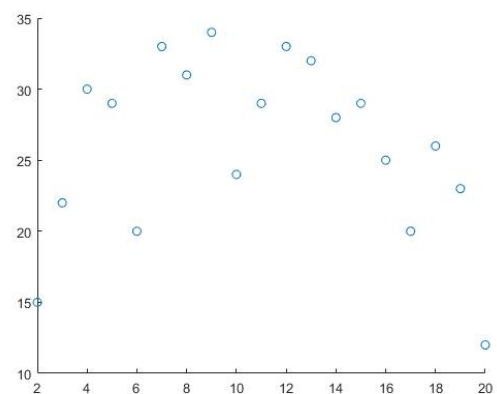
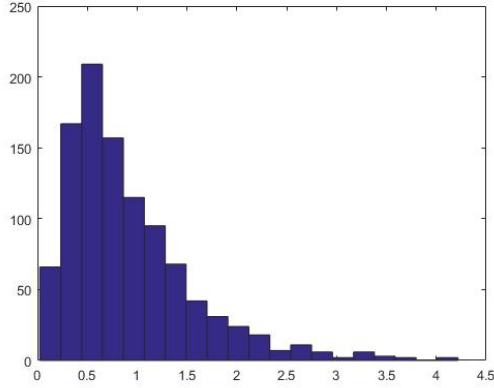


Figure 4: Number of schools

Figure 5 displays the average distance between pupils' residential locations and the nearest

secondary schools. On average, pupils are 917m away from the nearest school. Some variation is observed as some pupils are within a few meters of the school whereas others are at 4km from it.

Figure 5: Average distance to facility



5 Structural Analysis

In this section, we estimate the commuting parameters and fixed costs, calibrate the model for residential amenities and density of development, and evaluate the fitness of the model to the real world.

5.1 Estimation of commuting parameters

From the commuting shares (6), the model predicts a semi-log gravity equation for commuting flows between residence i and public service k :

$$\ln \pi_{ik} = -\nu \rho_{ik} + \omega_i + v_k \quad (25)$$

where ω_i are residence fixed effects capturing residence characteristics $\{B_i, T_i, D_i, Q_i\}$, v_k are public service fixed effects, and the denominator of (6) is captured by the fixed effects as it is a constant. The parameter ν is the semi-elasticity of commuting flows with respect to the euclidean distance. It is defined as $\nu = \kappa \epsilon$ where κ is the commuting cost parameter and ϵ is the heterogeneity parameter from the Fréchet distributed shock on individuals' utility.

To empirically retrieve the semi-elasticity of commuting flows, we use municipal data from the 346 municipalities and the 20 Parisian districts on which our grid of 1km² cells lies. For all municipalities/districts, we know the number of pupils commuting from one municipality/district

to another in 2010. This leads to $346 \times 346 = 119,716$ links. As bilateral distance data is at the cell level, we compute the euclidean distance between two municipalities by taking the average distance between all cells in a municipality to all cells in another one.

Table 1 displays the estimation of (25) using this data. In column I, we estimate a linear fixed effects model. We obtain a semi-elasticity of commuting of -0.15. Due to the log relationship, this model does not account for the commuting share equal to zero; and hence, is likely to underestimate the semi-elasticity of commuting. An approach which will model the zero and positive commutes jointly is needed. We do so in column II and III, which report the estimation of a Pseudo Poisson Maximum Likelihood (PPML) model and of a Negative Binomial (NB) model, respectively. We then obtain a semi-elasticity of commuting of -1.45 (-1.40) for the PPML (NB). This semi-elasticity is significantly larger than what Ahlfeldt et al. (2015) obtain (-0.07) when looking at residence-workplace commutes. This reflects the fact that parents generally prefer their kids not to commute long distances to go to school, as pupils (aged 11 to 15) might commute on their own or because parents need to drop them before commuting themselves to work.

Table 1: Gravity estimations of commutes

	I	II	III
	ln bilateral commuting probability in 2010	ln bilateral commuting probability in 2010	ln bilateral commuting probability in 2010
Great circle distance ($-\kappa\epsilon$)	-0.15*** (.004)	-1.45*** (0.012)	-1.40 (1.12)
Estimation	OLS	PPML	NB
Fixed effects	Yes	Yes	Yes
Observations	6,371	119,716	119,716
R^2	0.33	-	-

Notes: PPML stands for Pseudo Poisson Maximum Likelihood, and NB for Negative Binomial.

*** p<0.01, ** p<0.05, * p<0.1

5.2 Estimation of fixed facility costs

Fixed costs refer to the all costs of running a secondary school that are not size dependent. They contain basic infrastructure maintenance, general administration, library cost etc ... Formally, this implies that the overall cost C_k of school k is the sum of a fixed variable cost V times the

number of students in k and a fixed cost F . C_k can then be expressed as follows:

$$C_k = V \times S_k + F \quad (26)$$

Hence, to retrieve the fixed costs F empirically, we will estimate the variable costs V using data on all secondary schools in France. Data about total infrastructure costs of secondary schools, i.e. free of teacher costs which is a variable costs, is available at the department level (in French, “département”). Here, it is important to note that the costs of running a school also incorporates the costs of opening a school which is split across all facilities. Hence, using the number of secondary schools and their capacity in each department, we compute the average total cost of a school (C_d) and the average number of pupils in a school (V_d) in each department. Adding various population and income covariates and regional fixed effects, we can then estimate the following linear model:

$$C_d = \beta_1 V_d + \mathbf{X}'\beta + a_r + u_d \quad (27)$$

The results of estimating (27) are summarized in Table 2. Column I presents results without covariates or fixed effects, column II includes covariates and column III includes both covariates and fixed effects. Across all specifications, we observe a significant positive effect of the number of pupils on the total infrastructure costs of secondary schools of about EUR 1,272 per pupil. Using (26), this leads to a yearly fixed facility cost of EUR 102,855. In the analysis, we will start with this value as a benchmark before analyzing the sensitivity of our results to the magnitude of the fixed costs.

Table 2: Estimation of fixed costs

	Avg. secondary school costs	Avg. secondary school costs	Avg. secondary school costs
Pupils per school	1,499*** (209.6)	1,297*** (281.5)	1,272*** (322.9)
Covariates	NO	YES	YES
Fixed effects	NO	NO	YES
Observations	94	94	94
R^2	0.418	0.607	0.704

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

5.3 Calibration of variable facility costs

As seen in Section 2.3, facility size costs are introduced through an increase in commuting costs by links between i and k using ω in (12). The task is now to calibrate ω so that it fits the case of secondary schools in Paris. The literature shows that larger schools have a negative impact on pupils performance (Leithwood and Jantzi, 2009). This is the case in general but also in the special case of French schools (Afsa, 2014). Hence, we model ω as follows:

$$\omega = 1 + \xi \tag{28}$$

We take $\xi = 0.15$, as estimated by Afsa (2014) using lower secondary school pupil level data in a panel between 2006 and 2012 in France.

5.4 Calibration of remaining parameters and locational characteristics

Apart from the semi-elasticity of commuting flows estimated above, we set the values of the parameters from our model using standard sources in the literature. Table 3 presents the value and source of these parameters. We estimated semi-elasticity of commuting parameter in the case of public services as it is not estimated in the literature. Using the value of the shape parameter ϵ and the estimated commuting probability parameter ν , we can retrieve the commuting cost parameter $\kappa = \frac{1.45}{6.83} = 0.21$.

Table 3: Value and source of parameters

Parameter	Value	Source
ϵ	6.83	Ahlfeldt et al. (2015)
ζ	0.01	Ahlfeldt et al. (2015)
β	0.75	Davis and Ortalo-Magne (2011)
μ	0.72	Combes, Duranton, and Gobillon (2012)
ξ	0.15	Afsa (2014)

In what follows, using the structure of the model, we show that there is a one-to-one mapping between observed and unobserved location characteristics. We then derive the unobserved location characteristics representing residential amenities and density of development.

Let us define, and denote by a tilde, the composite variable representing residential amenities:

$$\tilde{B}_i = B_i T_i^{1/\epsilon} \tag{29}$$

First note that we can rewrite (17) using (16) as:

$$\pi_{Ri} = \left(\frac{\gamma}{\bar{U}} \right)^\epsilon \sum_{l=1}^K \frac{P_i(B_i w)^\epsilon}{(d_{il} D_i Q_i^{1-\beta})^\epsilon} \quad (30)$$

Given the parameters $\{ \beta, \epsilon \}$ and the observed data $\{ \mathbf{Q}, \mathbf{H}_R, \mathbf{D}, \mathbf{d}, \mathbf{A}, H, w \}$, we then can obtain a one-to-one mapping for $\tilde{\mathbf{B}}$:

$$\tilde{B}_i = B_i T_i^{1/\epsilon} = \frac{\bar{U}}{\gamma} \frac{D_i Q_i^{1-\beta}}{w} \left(\frac{H_{Ri}}{H \sum_{l=1}^K (d_{il})^\epsilon} \right)^{1/\epsilon} \quad (31)$$

Similarly, given the parameters $\{ \beta, \mu \}$ and the observed data $\{ \mathbf{Q}, \mathbf{H}_R, \mathbf{w}, \mathbf{K} \}$, we can show, using (15), that there is a unique mapping for the density of development $\tilde{\phi}$.

$$\tilde{\phi}_i = \frac{(1-\beta)w}{Q_i K_i^{1-\mu}} H_{Ri} \quad (32)$$

\tilde{B}_i is represented in Figure 6 as a function of distance to the city center. Similarly, $\tilde{\phi}_i$ is represented in Figure 7. Figure 6 shows that residential amenities are relatively constant across all locations. The variance increases away from the center has some locations compensate a larger distance to the center with higher local amenities. Interpreting the density of development is less straightforward as available floor space is a function of this density and available land space. However, as expected, we still observe that the density of development decreases with distance to the center.

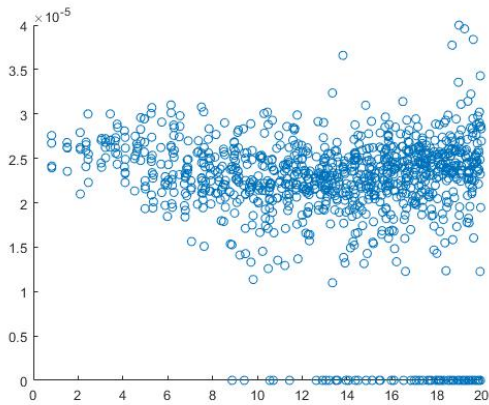


Figure 6: Residential amenities

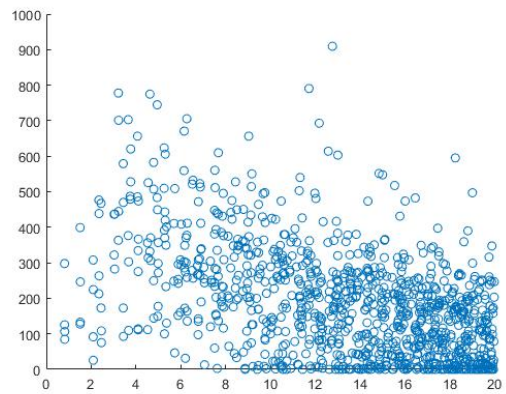


Figure 7: Density of development

6 The cost minimizing strategy

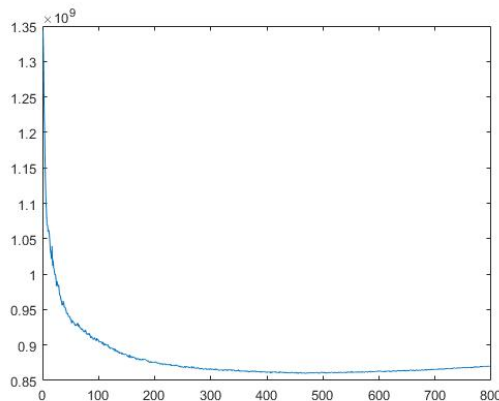
In this section, we first describe the cost minimizing strategy for the State; and second, compare it to the observed strategy.

6.1 Description of the cost minimizing strategy

The cost minimizing strategy characterizes the location, capacity and overall number of public facilities to minimize the associated utility and facility costs. We derive this strategy by deriving the cost minimizing location and capacity of facilities for each total number of facilities between 1 and 800.¹¹ In what follows, we will describe the overall number of facilities under the cost minimizing strategy, before analyzing the location and capacity of each facility under that strategy.

Figure 8 illustrates how the cost minimizing total number of facilities is obtained. The horizontal axis displays the number of facilities, while the vertical axis shows the associated minimized costs. It appears that overall costs are minimized when 438 facilities are built (and optimally located) within a 20km radius of the Hotel de Ville.

Figure 8: Cost minimizing number of facilities

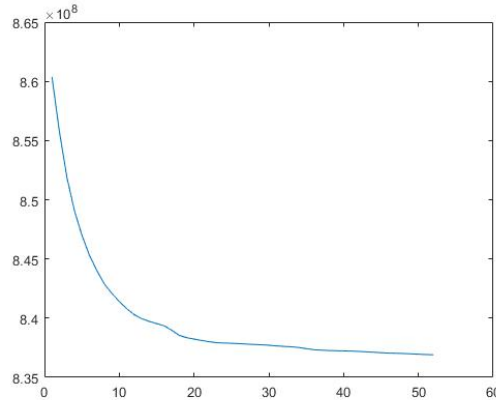


With $K = 438$, Figure 9 shows the costs obtained at each iteration of our method, which converges within 52 iterations. This means that the move made by each facility between iterations 51 and 52 is smaller than 0.01m. As one would expect, the cost reduction is very important in the first iterations whereas it becomes very small after 40 iterations.

Figure 10 shows how schools' location and distance to the center are related in the cost minimizing strategy. A clear bell shaped relationship is observed with around 5 facilities built in

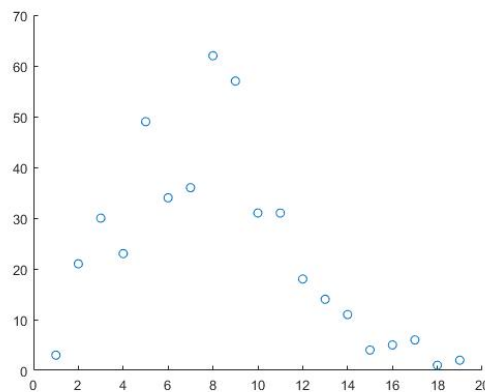
¹¹We stop at 800 as it is clear that the fixed costs dominate the cost function well below that point, as illustrated in Figures 8.

Figure 9: Convergence of the method with $K = 438$



the very center and in the outer periphery and above 30 facilities built at each distance between 6 and 12km. Interestingly, even though the model considers a radius of 20km around the city center, few schools are located at more than 14km from the center. Given that individuals can move freely, simple circle properties explain why locating a facility further away is more costly. The State is bound to provide services to any pupil aged 11 to 15; and hence, to “cover” all areas where pupils of that age range are located. Knowing that the area grows exponentially with the radius¹², costs will be smaller for the State if individuals are located closer to the center. Hence, by locating facilities closer to the center which in turns encourages individuals to locate centrally, the State minimizes the area to be covered; and hence, minimizes costs.

Figure 10: Cost minimizing locations

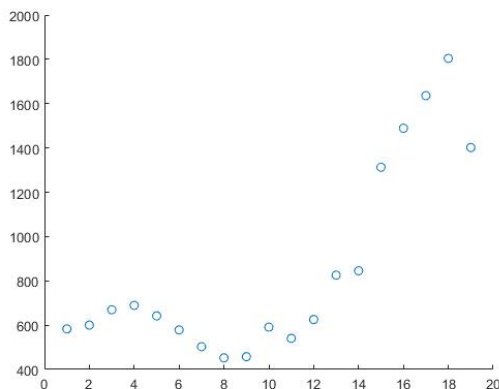


We now turn to the description of the average school capacity under the cost minimizing strategy. Figure 11 displays the average capacity as a function of distance to the center. Within 12 km of the center, the average capacity seems to be relatively stable around 600 pupils. This

¹²Remember that the area A of a circle of radius r is equal to πR^2 .

is due to the fact that increasing one facility more than the other increases more costs than distribution this increase across all locations, due to the shape of the variable costs (ω enters the cost function exponentially). The picture is different after 12 km of the center. The average capacity increases from around 900 at 13km to almost 2000 at 19km of the center. This can be explained using Figure 10. Cost are minimized if few large schools are built in the periphery. This follows from the fact that the State “increased” concentration in the center; and therefore, few pupils are left in the outer periphery. Hence, the fixed costs are relatively important relative to the number of pupils. It is then cost minimizing to build few large schools in the outer periphery.

Figure 11: Cost minimizing capacity



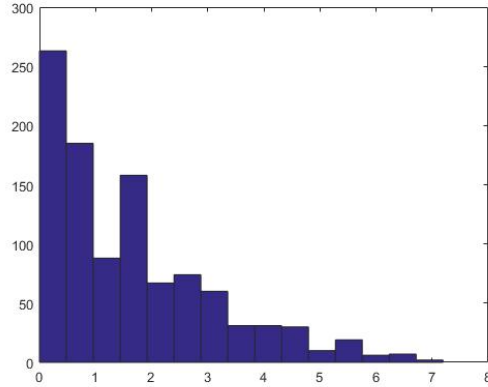
6.2 The observed and the cost minimizing strategies

In the reminder of this section, we will compare the cost minimizing strategy to the observed one. Understanding their differences can help deriving policy recommendations. The cost minimizing strategy is only slightly less costly than the observed one (about 1% less). The gains come mostly from a smaller sum of fixed facility costs due to the smaller number of schools (438 to 499).

These gains are only partly reduced due to an increased average distance between residential locations and the nearest facility (from 917m to 1.7km, Figure 12). Even though this result has to be taken with caution because of the discrete nature of the measure of residential locations, it is still a smaller average distance knowing that the observed average distance (measured in the same manner) is 916m (see Figure 5).

Another way to compare the two strategies is to look at the location of facilities and pupils in both. Figures 13 and 14 display the difference in pupils and number of schools, respectively, at each distance of the center. Both pupils and schools are located more in the close periphery

Figure 12: Distance to facilities in the cost minimizing strategy



and less in the outer periphery in the cost minimizing strategy. Once again, this is due to the State anticipating accurately the reaction of the individuals in their residential location choices.

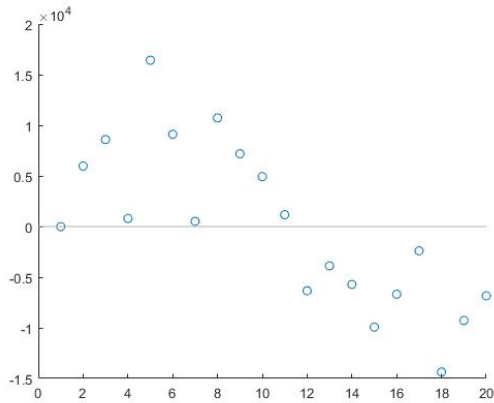


Figure 13: Difference in capacity

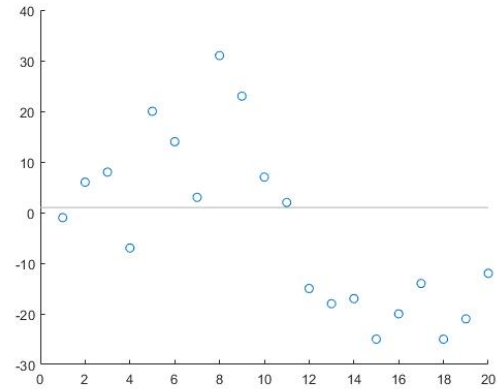


Figure 14: Difference in number of schools

The increased concentration in the close periphery can also be observed when looking at the difference in housing prices and residential density (Figure 15 and Figure 16, respectively) between the two strategies. The main displacement is observed on residential density. In the cost minimizing strategy, residential density is significantly higher in the city center and close periphery and lower in the outer periphery.

Finally, we compare the two strategies on the map. Figures 17 and 18 show the school locations across Paris' metropolitan area in the observed and cost minimizing strategies, respectively. The set of locations in the cost minimizing strategy appears much more "ordered". This simply follows from the fact that residential locations is measured in cells of 1km^2 . Hence, locating a facility in the middle of a cell minimizes the distance between the individuals residing in this cell and the facility. Interestingly, the cost minimizing strategy appears to locate facilities more

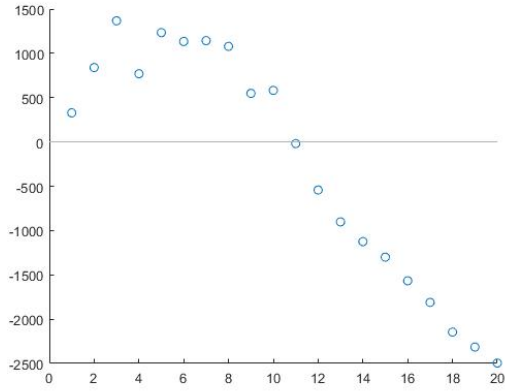


Figure 15: Difference in housing prices

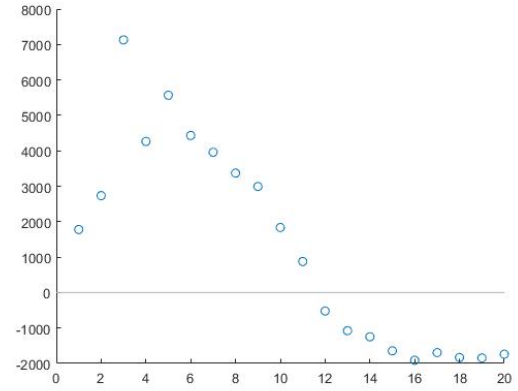


Figure 16: Difference in residential density

in a north-south axis. This reflects the fact that the State could concentrate pupils more in the north and south of Paris and less in the east and west; probably due to higher available space on the north-south axis. We also observe that some central areas do not receive facilities in any of the two strategies. This follows from the restriction on available floor space and locational characteristics. The area west of the center, for example, contains the Seine river, large boulevard and many public administrative buildings. Hence, as residential floor space is strongly limited, few pupils leave in those locations. It follows that it is relatively more costly for the State to build facilities in such locations.

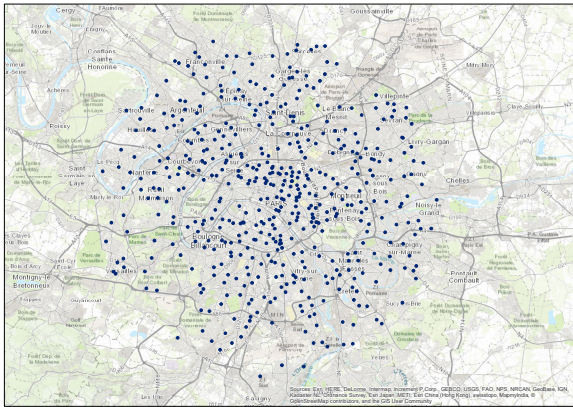


Figure 17: Observed school locations

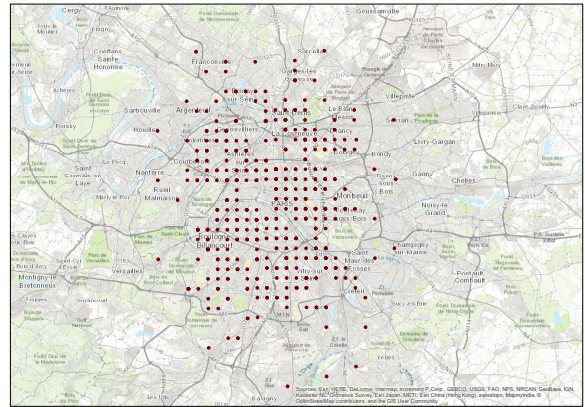


Figure 18: Cost minimizing school locations

7 Counterfactual analysis

In this section, we use counterfactual exercises to further analyze the relationship between the State minimization problem and the existence of agglomerations. First, we will withdraw the assumptions of monocentricity that we imposed in the model so far. We wish to see if the State

minimization problem leads to the formation of an agglomeration without other agglomeration forces. Second, we wish to see how shocks on facility costs and on the State’s valuation of individuals utility impact the spatial distribution of individuals.

7.1 State cost minimization and agglomeration

The description of the cost minimizing strategy reveals that the State’s attempt to provide public service at a minimum costs can lead to concentration of individuals in the city center. This finding contrasts with the findings of the “New Economic Geography” literature since Krugman (1991). In this literature, agglomerations are created so that agents can enjoy agglomeration economies such as knowledge or productivity spillovers. In our model, a core-periphery structure emerges, not because of positive spillovers across agents, but because higher concentration reduces commuting costs and the need for facilities which leads to smaller overall costs. The agglomeration forces arising from the State’s minimization problem which we detail in this paper are an additional way to explain the existence of agglomerations, and do not contradict standard explanations based on agglomeration spillovers.

To rigorously investigate whether the State’s cost minimization can lead to the emergence of a core-periphery structure, we simplify the model by removing the two forces which were *a priori* imposing this structure in our model: the preference to be close to the center and the higher amount of floor space available in the city center. Hence, we assume that individuals are indifferent in locating at any distance of the city center and that available floor space is everywhere equal to the average floor space available in a 20km radius of Paris. Formally, the aggregate consumption index becomes,

$$C_{iko} = z_{iko} \frac{B_i}{d_{ik}} \left(\frac{c_{iko}}{\beta} \right)^\beta \left(\frac{l_{iko}}{1-\beta} \right)^{1-\beta}, \quad 0 < \beta < 1 \quad (33)$$

and the available floor space L_i is now a constant at $\bar{L} = \frac{1}{I} \sum_{i \in I} L_i$.

Without these agglomeration forces, the State’s cost minimizing strategy is to build 470 facilities, i.e. slightly more than with these forces. This is due to the fact that without these forces, it is marginally more difficult to increase concentration in the city center for the State.

Under this cost minimizing strategy and without any direct agglomeration forces, a core-periphery structure appears as shown in Figures 19 and 20. This result confirms that the public sector plays an important role in shaping the spatial distribution of individuals.

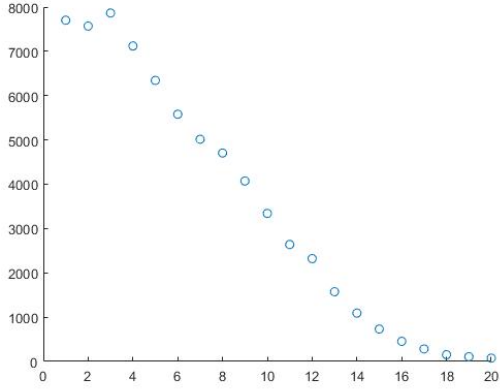


Figure 19: Housing prices without agglomeration forces

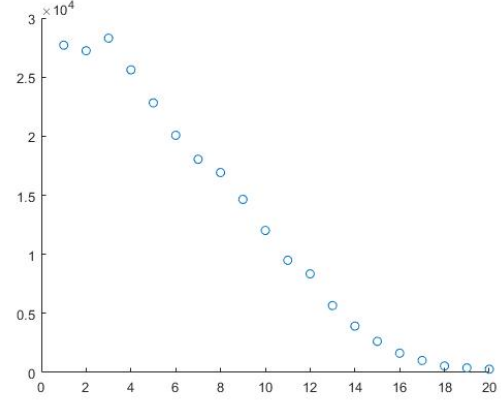


Figure 20: Residential density without agglomeration forces

7.2 Shocks to fixed facility costs and utility valuation

To investigate the impact of shocks in fixed costs and utility valuation, we introduce weights α_1 and α_2 in the State's cost minimization problem as follows:

$$\min_{c_k, \dots, c_K} \Pi = \sum_{k=1}^K \sum_{i=1}^I \alpha_1 U_{ik}^{-1} H_{ik}^\omega + \alpha_2 (K \times F) \quad (34)$$

In the baseline, we implicitly assumed $\alpha_1 = 1$ and $\alpha_2 = 1$. In the minimization problem, an increase in α_1 will give more weight to the individuals utility. Similarly, a relative increase in α_2 compared to α_1 leads to more weight given to facility costs.¹³¹⁴ More than how these shocks impact the overall number of facilities, we are interested in how they impact the distribution of individuals as a function of distance to the center.

Increase in fixed facility costs We suppose that fixed facility costs increase by 50%, i.e. $\alpha_2 = 1.5$, and derive the cost minimizing strategy as before. Such increase in the fixed facility costs can reflect an actual increase of the fixed costs but also the wish for the State to reduce its total spending. In that second scenario, the increase is equivalent to weighting more the facility costs relative to the individuals' utility. It appears that costs are minimized when 306 facilities are built at cost minimizing locations. Hence, an increase of 50% in the fixed facility costs leads

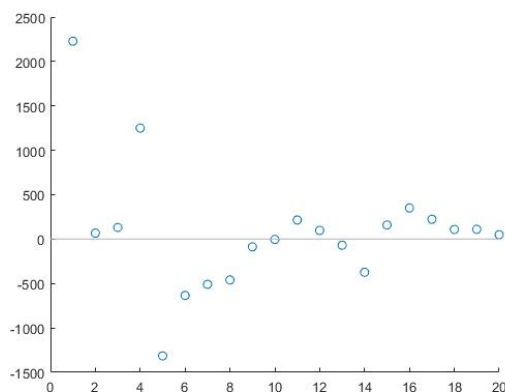
¹³For simplicity, we suppose that the facility variable cost remains the same. We could introduce α_3 as an additional exponent to ω to study how overall cost react to changes in facility costs. However, this is qualitatively equivalent to an increase in α_2 .

¹⁴It is clear from (34) that a decrease in α_1 is qualitatively equivalent to an increase in α_2 . However, note that due to the shape of the individuals utilities it is not quantitatively equivalent.

to a decrease in the number of facilities of 40% compared to the baseline cost minimization strategy.

An increase in the costs leads to an increased concentration of individuals in the city center as shown in 21. If fixed facility costs become relatively more important, the State will adopt a strategy where individuals locate more in the city center in order to save in commuting and fixed facility costs.

Figure 21: Difference in residential density with 50% increase in fixed facility costs

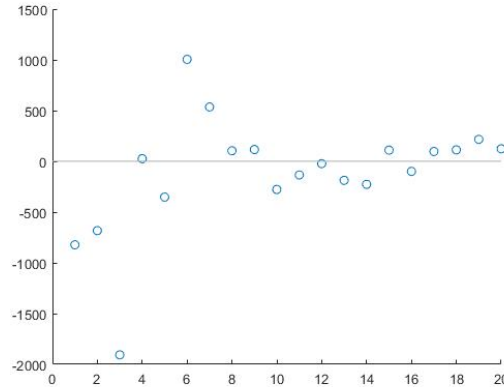


Increase in individual utilities We suppose that the State values individual utilities 50% more, i.e. $\alpha_1 = 1.5$, and derive the cost minimizing strategy as before. Such scenario could happen for many reasons for example if the State decided to make of prioritize education and allocate more funding to the school system. Following an increase in α_1 , it appears that costs are minimized when 685 facilities are built at cost minimizing locations. Hence, an increase of 50% in the valuation of individual utilities leads to an increase in the number of facilities of 64% compared to the baseline cost minimizing strategy.

In contrast to the impact of an increase in the costs, an increase in utility valuation leads to a decrease in concentration of individuals in the city center as shown in 22. If individual utilities become relatively more important, the State will adopt a strategy where individuals concentrate less in the city center which leads to higher levels of utility at the cost of higher commuting and fixed facility costs.

The analysis of these shocks is particularly interesting if linked to the question of rural decline. Representative of rural areas often accuse centralization reforms of the public services to increase rural decline; to which the central government often answers by reversing the causality, i.e. centralization reforms do not cause rural decline, it is rural decline that renders centralization reforms necessary. The findings here give support to the claim of the representative of rural

Figure 22: Difference in residential density with 50% increase in utility valuation



areas.¹⁵ If the State is in a complicated financial situation and wishes to reduce spending on public services, a cost minimizing solution is to locate facilities more in the center as to increase urban concentration to gain in commuting and facility costs. In turn, such strategy fosters rural decline *no matter* what the economic and demographic situation is in rural areas.

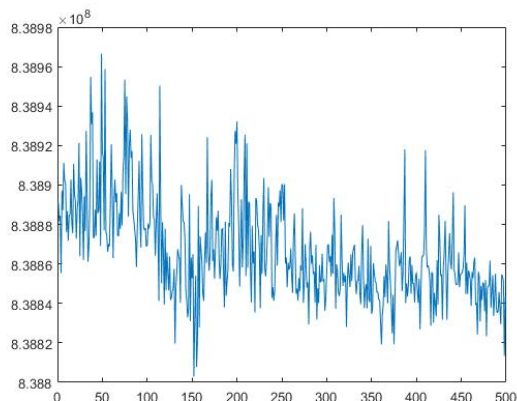
8 Marginal cost minimizing strategy

We call marginal strategy, the decision to open or close a facility given the observed location strategy. To derive this strategy, we need to compare the reduction in cost due to the closing of the marginally more costly facility to the reduction in cost due to the opening of a facility at the marginally cost minimizing location.

Cost of closing a facility To determine which facility is marginally more costly, a straightforward method is the brute-search method. We can alternatively “switch off” all 499 facilities within a 20km radius of Paris. The cost minimizing closure is the one that leads to the highest reduction of overall costs. Figure 23 displays the marginal cost of closing all facilities alternatively. The costs are minimized by switching off facility 152, which is the “Collège Blaise Pascal” in the very south of Paris. Closing this facility would result in an overall cost reduction of 0.01%. Interestingly, note that this facility is in the outskirts of Paris metropolitan area. In line with the results in section 6, closing a facility in the outskirts reduces commuting and fixed costs.

¹⁵These findings are also in line recent empirical analysis of the impact of centralization reforms on the distribution of economic activity. Egger, Koethenbueger, and Loumeau (2017) show, for example, that local merger reforms lead to higher concentration of economic activity in the center at the expense of the periphery.

Figure 23: Marginal cost of closing a facility (by facility)



Cost of opening a facility To derive the cost minimizing location where to open one more facility, we exploit one key advantage of the method employed to solve for the State’s minimization problem. The method uses the separability of the State’s problem to minimize costs. Hence, keeping all other locations constant, one can simply derive the cost minimizing location of one additional facility. At this location (4km south of the “Hotel de Ville”), opening a new facility will be overall less costly than opening it anywhere else. However, by opening a facility at this location, the State would increase overall cost by 0.004%.

Hence, it appears that the marginal cost minimizing decision for the State would be to close facility 152, which is the “Collège Blaise Pascal” in the south of Paris.

9 Conclusion

This paper provides a flexible framework in which to analyze the relation between public services and urban density. In our model, the State decides on a location strategy, i.e. the number, location and capacity of public facilities, while anticipating how residential density, housing prices and locational characteristics will react. The State’s objective is to maximize individuals utility while minimizing the sum of fixed and variable facility costs. Our modeling approach simplifies necessarily the observed world. One interesting research question that we could not answer in the context of this paper is to see how the presence of competing private facilities impacts the equilibrium and the relation between public services and urban density.

A key contribution of the paper is to relate budget constraint and agglomerations. We see that the State can strategically locate facilities to increase population density, and thus reduce commuting and facility costs. Furthermore, the more the State will want to reduce costs, i.e. weights the facility costs, the more it will increase density in the core places. On the other hand,

a looser budget constraint leads individual utility to be valued more. In turn, more facilities are built and urban density decreases. We also believe that the method proposed in this paper constitute an interesting contribution to the literature. At the cost of using euclidean distance, our gradient method offers great flexibility even when facing complex and large optimization problems.

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A Theory Appendix: Derivation of individual location choices

In appendix A, we detail the analytical derivation of individual location choices presented in the paper. For the sake of completeness, we sometimes repeat what we presented in the main part of the paper.

A.1 Residence and public service location choice

As the relation between the aggregate consumption index (1) and the idiosyncratic component of utility is monotonic, the distribution of utility of a individual living in i , working in j and using public service in k is also Fréchet distributed:

$$G_{ik} = Pr[U \leq u] = F\left(\frac{ud_{ik}D_iQ_i^{1-\beta}}{w}\right), \quad (35)$$

$$G_{ik} = e^{-\Phi_{ik}e^{-\epsilon}}, \quad \Phi_{ik} = P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(w)^\epsilon$$

We first derive the probability that individuals choose a particular combination of residence and public service location.

$$\begin{aligned} \pi_{ik} &= Pr[u_{ik} \geq \max\{u_{is}\}; \forall r, s] \\ &= \int_0^\infty \prod_{k \neq k} G_{is}(u) \left[\prod_{i \neq i} \prod_k G_{is}(u) \right] g_{ik}(u) du \\ &= \int_0^\infty \prod_{i=1}^s \prod_{k=1}^s \epsilon \Phi_{ik} u^{-(\epsilon+1)} e^{-\Phi_{is}u^{-\epsilon}} du \\ &= \int_0^\infty \epsilon \Phi_{ik} u^{-(\epsilon+1)} e^{-\Phi u^{-\epsilon}} du \end{aligned} \quad (36)$$

Noting that

$$\frac{d}{du} \left[-\frac{1}{\Phi} e^{-\Phi u^{-\epsilon}} \right] = \epsilon u^{-(\epsilon+1)} e^{-\Phi u^{-\epsilon}}, \quad (37)$$

we obtain the probability that a individual resides in i and use public service in k :

$$\pi_{ik} = \frac{P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon}{\sum_{i=1}^I \sum_{k=1}^K P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon} \equiv \frac{\Phi_{ik}}{\Phi}. \quad (38)$$

Equation (38) shows that individuals sort across all combination of residence and public service location depending on their idiosyncratic preferences and the characteristics of these locations. Residential locations with an higher value of P_i or public service locations with an higher value of B_i lead to higher positive draws of utility for the combination. To ensure the

tractability of the general equilibrium, and because we do not observed the required individuals characteristics in our data, we abstract from other dimensions of individuals heterogeneity.

We can derive the probability that an individual decides to live in i out of all possible locations in the metropolitan area by summing Φ_{is} across all public service locations.

$$\pi_{Ri} = \frac{\sum_{k=1}^K P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon}{\sum_{i=1}^I \sum_{k=1}^K P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon} \equiv \frac{\Phi_i}{\Phi} \quad (39)$$

The same can be done for the probability that an individual decides to use the public service in k out of all possible public service location choices:

$$\pi_{Sk} = \frac{\sum_{i=1}^I P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon}{\sum_{i=1}^I \sum_{k=1}^K P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(B_i)^\epsilon} \equiv \frac{\Phi_k}{\Phi} \quad (40)$$

A.2 Derivation: Distribution of Utility

Here, we detail the derivation of the distribution of utility across places in the metropolitan area and the wider economy.

As the relation between the aggregate consumption index ((1)) and the idiosyncratic component of utility is monotonic, the distribution of utility of a individual living in i , working in j and using public service in k is also Fréchet distributed:

$$G_{ik} = Pr[U \leq u] = F\left(\frac{ud_{ik}D_iQ_i^{1-\beta}}{w}\right), \quad (41)$$

$$G_{ik} = e^{-\Phi_{ik}e^{-\epsilon}}, \quad \Phi_{ik} = P_i(d_{ik}D_iQ_i^{1-\beta})^{-\epsilon}(w)^\epsilon$$

From all possible combination of place of residence and public service, the individuals choose the place that offers the highest utility. As the maximum of a sequence of Fréchet distributed random variables is also Fréchet distributed, the distribution of utility across all combination is:

$$1 - G(u) = 1 - \prod_{r=1}^S \prod_{s=1}^S e^{-\Phi_{rs}e^{-\epsilon}} \quad (42)$$

where the left hand side is the probability that an individual has an utility lower than u , and the right-hand side is one minus the probability that a individual has a utility level lower than u for all possible pairs of blocks or residence and public service. This leads to:

$$G(u) = e^{-\Phi u^{-\epsilon}}, \quad \Phi = \sum_{r=1}^S \sum_{s=1}^S \Phi_{rs} \quad (43)$$

Given that utility is here Fréchet distributed, we can derive the expected utility of moving to the metropolitan area:

$$\mathbb{E}[u] = \int_0^\infty \epsilon \Phi u^{-\epsilon} e^{-\Phi u^{-\epsilon}} du \quad (44)$$

Setting the following variable changes,

$$y = \Phi u^{-\epsilon}, \quad dy = -\epsilon \Phi u^{-(\epsilon+1)} du \quad (45)$$

we can then write the expected utility of moving to the city as:

$$\mathbb{E}[u] = \int_0^\infty \Phi^{1/\epsilon} y^{-1/\epsilon} e^{-y} dy. \quad (46)$$

This is equivalent to:

$$\mathbb{E}[u] = \gamma \Phi^{1/\epsilon}, \quad \gamma = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right) \quad (47)$$

where $\Gamma(\cdot)$ is the Gamma function; \mathbb{E} is the expectation operator and expectation is taken over the distribution of idiosyncratic utility. Population mobility then implies that the expected utility must equal the reservation utility in the wider economy.

$$\mathbb{E}[u] = \gamma \Phi^{1/\epsilon} = \gamma \left[\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_i w)^\epsilon \right]^{(1/\epsilon)} = \bar{U} \quad (48)$$

B Theory Appendix: Proof equilibrium for a given facility location strategy

Let us rewrite (17) using (16):

$$\pi_{Ri} = \left(\frac{\gamma}{\bar{U}}\right)^\epsilon \sum_{l=1}^K \frac{P_i (B_i w)^\epsilon}{(d_{il} D_i Q_i^{1-\beta})^\epsilon} \quad (49)$$

Noting that $H_{Ri} = \frac{\pi_{Ri}}{H}$, the land market clearing condition can be re-written as:

$$L_i = \frac{1}{H} \frac{(1 - \beta)w}{Q_i} \pi_{Ri} \quad (50)$$

Combining the above relationship with (49), land market clearing condition can be written as:

$$L_i = \frac{(1-\beta)w}{Q_i} \sum_{l=1}^K \frac{P_i(B_i w)^\epsilon}{(d_{il} D_i Q_i^{1-\beta})^\epsilon} \quad (51)$$

where we have chosen units to measure utility such that $\frac{1}{H} \times (\frac{\gamma}{U})^\epsilon = 1$, following Ahlfeldt et al. (2015). It follows that, for any given facility location strategy, we can obtain \mathbf{Q} . Given \mathbf{Q} , we can obtain $\boldsymbol{\pi}_R$.

We can then recover H from our measurement choice of utility: $\frac{1}{H} \times (\frac{\gamma}{U})^\epsilon = 1$. With population mobility (16), we have:

$$H = \left[\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j w)^\epsilon \right] \quad (52)$$

Q.E.D.

C Theory Appendix: Deriving the cost minimizing equilibrium: convergence and application

For a given number of facility locations K and the associated overall fixed cost, we can reformulate the State minimization problem (12) as:

$$\min_{c_1, \dots, c_K} \Pi = \sum_{k=1}^K \sum_{i=1}^N \overline{U_i^{-1}} H_{ik}^\omega d_{ik}, \quad \text{where } d_{ik} = e^{\nu \|x_i - c_k\|} \quad (53)$$

where $\|x_i - c_k\|$ is the Euclidean distance between x_i and c_k . Both d_{ik} and H_{ik} are a function of $\|x_i - c_k\|$. A natural iterative approach to solving (20) is to fix the probabilities π_{ik} , and optimize (20) with respect to the centers, then fixing the center, we can solve for the probabilities using (6), etc. until the optimum is reached.

Updating the probabilities The update of the probability of commuting between i and k is done using (6):

$$\pi_{ik} = \frac{P_i (d_{ik} D_i Q_i^{1-\beta})^{-\epsilon} (B_i)^\epsilon}{\sum_{j=1}^I \sum_{l=1}^K P_j (d_{jl} D_j Q_j^{1-\beta})^{-\epsilon} (B_j)^\epsilon}$$

Updating the centers The optimization of the center location can be done using a standard gradient optimization approach. Note that (20) is a separable function of the centers, which allows us to decompose the multi-facility problem into multiple single facilities problem as:

$$f(c_1, \dots, c_K) = \sum_{k=1}^K f_k(c_k), \quad \text{where} \quad f_k(c_k) = \sum_{i=1}^I \overline{U_i^{-1}} H_{ik}^\omega e^{\nu \|x_i - c_k\|}, \quad k \in K \quad (54)$$

Provided that $f_k(c_k)$ is convex (which we show below), $\nabla f_k(\mathbf{c}) = 0$ is necessary and sufficient for obtaining the minimum. Hence, we can optimize (20) for given commuting probabilities by setting $\nabla f_k(\mathbf{c}) = 0$, and solving for \mathbf{c} .

A necessary and sufficient condition for $f_k(c_k)$ to be convex is:

$$f_k''(\mathbf{c}) \geq 0.$$

When developing the second derivative of $f_k(\mathbf{c})$, we see that this condition is met for $\nu \geq 0$ which is given by definition:

$$f_k''(\mathbf{c}) = \sum_{i=1}^I \overline{U_{ik}^{-1}} H_{ik}^\omega \nu \frac{e^{\nu \|x_i - c_k\|}}{\|x_i - c_k\|} \left[\nu \frac{(x_i - c_k)^2}{\|x_i - c_k\|} + 1 + (x_i - c_k)^2 \right] \geq 0.$$

Q.E.D.

Using \mathbf{c} to denote the matrix of centers' coordinates, the gradient of $f_k(\mathbf{c})$ is given by:

$$\nabla f_k(\mathbf{c}) = - \sum_{i=1}^I \frac{\overline{U_{ik}^{-1}} H_{ik}^\omega \nu e^{\nu \|x_i - c_k\|}}{\|x_i - c_k\|} (x_i - c_k) \quad (55)$$

Setting $\nabla f_k(\mathbf{c})$ equal to 0, we get the optimal centers:

$$c_k^* = \sum_{i=1}^I \delta_k(x_i) x_i, \quad (56)$$

as convex combinations of the data points, with weights $\delta_k(x_i)$ given by:

$$\delta_k(x_i) = \frac{e^{\nu \|x_i - c_k\|} \overline{U_{ik}^{-1}} H_{ik}^\omega / \|x_i - c_k\|}{\sum_{j=1}^I e^{\nu \|x_i - c_k\|} \overline{U_{ik}^{-1}} H_{ik}^\omega / \|x_i - c_k\|} \quad (57)$$

From (56) and (57), we get K mappings $T_k : \mathbf{c} \rightarrow T_k(\mathbf{c})$:

$$T_k(\mathbf{c}) = \sum_{i=1}^I \left[\frac{e^{\nu\|x_i - c_k\|} \overline{U_{ik}^{-1}} H_{ik}^\omega / \|x_i - c_k\|}{\sum_{j=1}^I e^{\nu\|x_i - c_k\|} \overline{U_{ik}^{-1}} H_{ik}^\omega / \|x_i - c_k\|} \right] x_i, \quad \forall c_k \neq x_j. \quad (58)$$