

Contribution to a public good under subjective uncertainty*

Anwesha Banerjee[†] and Nicolas Gravel[‡]

February 1st 2018

Abstract

This paper examines how voluntary contributions to a public good are affected by the contributors' heterogeneity in beliefs about the uncertain impact of their contributions. It assumes that contributors have Savagian preferences that are represented by a two-state-dependent expected utility function and different beliefs about the benefit that will result from the sum of their contributions. Under some conditions imposed on preferences, we establish general comparative static results on the effect of specific changes in the distribution of beliefs on the (Nash) equilibrium provision of public good. We specifically shows that the equilibrium public good provision is increasing with respect to first-order and second order stochastic dominance changes in the distribution of beliefs.

Keywords: Voluntary provision, public good, uncertainty, beliefs, optimism, consensus.

JEL classification codes: C72, H41

"The debate's over. The people who dispute the international consensus on global warming are in the same category now with the people who think the moon landing was staged on a movie lot in Arizona." (Al Gore)

"Whether Global Warming or Climate change. The fact is: We didn't cause it. We cannot change it." (Donald Trump)

1 Introduction

There are many circumstances where agents contribute to a public good about which they are *uncertain*. The fight against global warming through carbon emission reduction is an example of such a situation. As shown on Figure 1, there is considerable scientific uncertainty about the impact of carbon accumulation on the Earth temperature at, say, 2050 horizon. Moreover, as is also apparent in the picture, the probability distributions over the increase in the

*We are indebted, to Yann Bramoullé for his extremely insightful comments. We have also benefited from the comments and discussions we had with Sebastian Bervoets and Arunava Sen. The usual disclaimer of course applies fully.

[†]Aix-Marseille Univ. (Aix-Marseille School of Economics), CNRS, EHESS & Centrale Marseille, 5, Boul. Maurice Bourdet, 13001 Marseille France Cedex.

[‡]Centre de Sciences Humaines & Aix-Marseille School of Economics, 2, Dr Abdul Kalam Road, 11 0001, Delhi, India. Nicolas.gravel@csh-delhi.com.

Earth temperature associated to a specific scenario of carbon accumulation (the absence of any effort of reduction under a "business as usual" benchmark on Figure 1) produced with the best available models differ markedly among scientific teams.¹ This *heterogeneity of beliefs* about the environmental impact of carbon emissions is also reflected in the variety of opinions on this matter found in the public debate, and illustrated by the polarized views of the two leading American political figures quoted above.² There is little doubt that a person's belief on the impact of carbon emission on global warming will affect this person's propensity to make costly efforts in preventing climate change (see for example Roser-Renouf, Maibach, Leiserowitz, and Zhao (2014)). After all, had he been the US president, Al Gore would have certainly not taken the same decision *vis-à-vis* the Paris agreement on global climate change than that taken last June by Donald J. Trump. Other examples of situations involving uncertainty on the impact of individuals' contributions to the public good are contributions to charities or institutions by agents who are uncertain about their reliability or effectiveness.

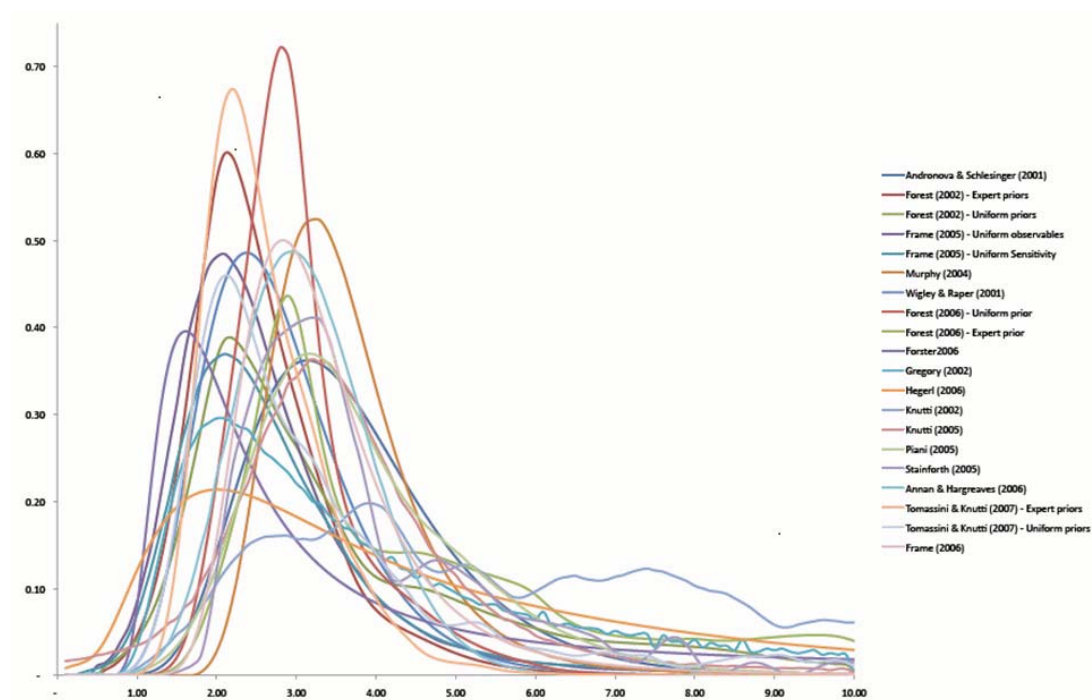


Figure 1: Estimated distributions of the increase in the Earth temperature in the next 50 years (source: Meinshausen, Meinshausen, Hare, Raper, Frieler, Knutti, Frame, and Allen (2009))

¹See also the latest IPCC report TF, D, G.K., M., and S.K. (2013), and the the survey provided by Doran and Zimmerman (2009).

²See also. Pelham (2009) for a survey on individuals opinion about global warming in 127 countries. An interesting analysis of beliefs polarization in the context of choice under ambiguity is provided by Dixit and Weibull (2007).

In this paper, we examine the impact of beliefs heterogeneity on the agents' contribution to a public good in a setting *à la* Bergstrom, Blume, and Varian (1986). We specifically ask two questions in such a setting:

1) Does the increase in some (or all) contributors' *optimism* about the impact of individuals' contributions to a public good increase the total amount of individual contributions ? That is, would the US make more global effort in reducing carbon emissions if some, or all, of the US citizens who currently share Donald Trump's beliefs about human contribution to global warming switch to Al Gore's view on this matter ?

2) Does an increase in the existing level of consensus in a community about the impact of individual contributions to a public good increase the overall level of provision ? That is, would Donald Trump and Al Gore contribute together more to the fight against global warming if they could get their different beliefs closer to each other ?

We address this question in a simple model in which the valuation of the benefit of any given sum of individual contributions is uncertain, and where individuals differ with respect to their perception of this uncertainty. Uncertainty concerns two possible states of the world: an optimistic one, in which a given sum of individuals' efforts yields a significant benefit in terms of public good provision, and a pessimistic one in which the benefit of this sum is subjectively perceived to be lower. Contributors differ by the probability that they attach to these two states. Optimistic people like Al Gore would attach probability close to 1 to the first state. At the other extreme, pessimistic (or skeptical) individuals like Donald Trump would attach almost zero probability to the state where individuals' efforts affect public good provision. Between these two extremes, possible contributors may attach various probabilities to either of these two states. We assume that every contributor evaluates the benefit of his/her contribution by the expectation - taken over his/her beliefs - of the same state-dependent expected utility that, just like in standard models of voluntary public good provision *à la* Bergstrom, Blume, and Varian (1986), is a function of two variables: individual effort and public good provision. In either the optimistic or the pessimistic state, the state-dependent utility function is decreasing with effort, increasing with the public good, and concave with respect to both goods. We also assume the utility function to exhibit a marginal disutility of effort that is not strictly decreasing with respect to the total amount of public good. In such a setting, and whatever is the distribution of beliefs among contributors, there will be a unique Nash equilibrium level of contribution. The main contribution of this paper is to identify the impact of specific changes in the distribution of beliefs on the Nash equilibrium aggregate level of contributions. We first establish easily that every individual's equilibrium level of contribution is increasing with respect to his/her own belief. This entails that individuals' Nash equilibrium levels of contributions will be ordered by their beliefs. We then show that an increase in optimism in the population in the sense of first-order dominance (see e.g. Hadar and Russell (1974)) leads to an increase in the overall level of contribution to the public good. The most important result of the paper concerns the impact of an *increase in the consensus* about the probability of being in the good state on the overall level of contribution. In the context of prevention of global warming, would a homogenization of individuals' beliefs about the impact of human activities on the Earth temperature increase the individuals' overall propensity to make carbon emission reduction efforts ? Answering this question

requires of course a definition of what it means for a distribution of beliefs to be “more homogenous” than another. Borrowing here again from the stochastic dominance literature, and exploiting the two-state feature of our framework, we define a distribution of beliefs to be more homogenous than another when the dominating distribution has the same average belief as the dominated one *and* has been obtained from the latter by a finite sequence of Pigou-Dalton transfers of probabilities attached by individuals to the optimistic state. We observe that the generalization of this plausible notion of homogenization to more than two-states is not immediate. We show that, under some additional condition on the utility function that concerns the behavior of the third derivative of the utility with respect to effort, the homogenization of beliefs in this sense always lead to an increase the overall amount of equilibrium aggregate contributions .

Our paper contributes to two strands of literature. Firstly, we add to the literature on uncertainty in public goods, with the analysis for heterogeneous agents. Secondly, we contribute to the literature on distributional comparative statics for aggregative games and games with strategic substitutes.

The literature on voluntary contributions to a public good, initiated largely by Bergstrom, Blume, and Varian (1986), is by now well-established (see Cornes and Sandler (1996) for a textbook survey). Yet there are relatively few contributions to this literature that have analyzed the impact of uncertainty on public good provision. Some of them, like Austen-Smith (1980) or Sandler, Sterbenz, and Posnett (1987) have considered uncertainty regarding the actions of others.³ Our paper does not have much to say on this matter. An early paper that examines the impact of uncertainty on the benefit of a public good is Gradstein, Nitzan, and Slutsky (1993), who analyzes the specific impact of price uncertainty on public good provision in a voluntary contribution setting à la Bergstrom, Blume, and Varian (1986). There is also a somewhat large literature, nicely summarized by Gradstein, Nitzan, and Slutsky (1992), which addresses the issue of uncertainty in general non-cooperative games without specific concern for games of voluntary provision of a public good. One finds also a sizable literature that address the issue of bargaining and negotiation on public good provision under uncertainty. For example, Kolstad (2007) studies self-enforcing international agreements under systematic or common uncertainty, while Bramoulle and Boucher (2010) extends the treaty formation model of Barrett (1994) to the case of uncertainty for both a public good and a public bad. However, these papers suppose risk neutrality from the part of the negotiators who are also often assumed to face the same uncertainty. Schumacher (2015) builds and empirically tests a model in which beliefs of individuals influence their willingness to contribute to expenditures toward a cause, such as preventing climate change. However, this paper assumes that the utility is linear with respect to the individual’s income, and that there are only two groups of individuals, which could be optimist or pessimists. These individuals are also assume to face a common shock which affect the percentage of their final income. Moreover, in their model, individual contributions affect the probability of the shock. Bramoulle and Treich (September 2009) have examined the impact of uncertainty regarding the benefit of collective action with risk-averse agents. Their results

³Classical papers on conjectural variations and "non-Nash conjectures" about the reaction of others such as Cornes and Sandler (1984), Cornes and Sandler (1985) and Sugden (1985) also belong to that stream of literature. See also Itaya and Shimomura (2001) for a dynamic version of conjectural variation model.

indicate that the introduction of uncertainty can, under some conditions, lower the amount of a public bad or increase the amount of a public good. However their paper assumes that all contributors face the same uncertainty and, therefore, does not address the issue of contributors' heterogeneity in their perception of uncertainty. The only paper that we are aware that allow for heterogeneity of beliefs about the public good provision resulting from contributors' efforts is Sakamoto (2014). Yet this paper is concerned with the effect of an increase in the ambiguity of the beliefs on equilibrium provision and the role played by the information acquisition on this ambiguity. It does not examine the impact of belief heterogeneity on aggregate public good provision.

As for the literature on distributional comparative statics in aggregative games, the literature that grows in the tradition of Topkis (1978) have established quite general results for games in which the actions of the players are strategic complements. A good summary of these results is provided by Milgrom and Shannon (1994). However, general results for games in which the players' strategies are strategic substitutes - like games of voluntary contributions to a public good - are much more sparse. Corchòn (1994) provides some powerful comparative static results in that setting for the case where players have strongly concave payoff functions. Corchòn (1994) results have generalized significantly by Acemoglu and Jensen (2013). However, these papers only consider the impact on the equilibrium of monotonic changes in the individual exogenous parameters of the models (for instance their beliefs) and do not explore the impact, on the equilibrium choice of strategies by the players, of changes in the distribution of those parameters among players. Jensen (June 2012.) provides some comparative static results on the effect of specific changes in the distribution of individual parameters in the context of Bayesian games.

The rest of the paper is organized as follows. The next section introduces our model of contribution to a public good with uncertainty and heterogeneous beliefs. The main comparative static results are provided and discussed in Section 3 and Section 4 concludes.

2 The Model

We consider a community made of a set $N = \{1, 2, \dots, n\}$ of n individuals (with $n \geq 2$). Any individual $i \in N$ must choose a level $e_i \in [0, \bar{e}]$ of effort (say in carbon emission reduction), where \bar{e} is some strictly positive number, interpreted to be the maximal amount of effort that any individual can provide. It does not matter for the analysis that this number be finite. However, it must be the same for all individuals. Any given profile $(e_1, \dots, e_n) \in [0, \bar{e}]^n$ of efforts made by the members of this community generates an aggregate public good $G = \sum_{i=1}^n e_i$ that they all value. There is, however, uncertainty regarding the subjective valuation made by an individual of any combination $(e, G) \in [0, \bar{e}] \times [0, n\bar{e}]$ of his/her effort and the aggregate public good produced by the sum of those efforts. We formulate this uncertainty as concerning two possible states of "optimism" (Al Gore) or skepticism (Donald Trump) about the effect of individuals efforts on public good provision. If the optimistic state o materializes, then a given combination (e, G) of effort and aggregate public good yields a utility of $U^o(e, G)$. On the other hand, if the pessimistic state p happens, then the utility provided by this very same combination is $U^p(e, G)$. Individuals differ

in terms of their *beliefs* about the likelihood of the optimistic state. Individual i believes the true state to be optimistic with probability $\pi_i \in [0, 1]$. Individuals are expected (state dependent) utility maximizers. An individual with belief π_i about the likelihood of the optimistic state thus evaluates the combination $(e, G) \in [0, \bar{e}] \times [0, n\bar{e}]$ of effort and aggregate public good by the expected state dependent utility $EU(\pi_i; e, G)$ defined by:

$$EU(\pi_i; e, G) = \pi_i U^o(e, G) + (1 - \pi_i) U^p(e, G) \quad (1)$$

We assume throughout that the functions U^o and U^p are at least thrice differentiable⁴ with respect to their two arguments and are both decreasing with respect to effort, increasing with respect to aggregate public good and strictly concave.⁵ We also assume that $U^p(e, G) \leq U^o(e, G)$ for any combination of effort and aggregate public good $(e, G) \in [0, \bar{e}] \times [0, n\bar{e}]$ (given effort and current aggregate public good, optimism is weakly preferable to pessimism). A degenerate case of this model happens of course when U^o and U^p are the same functions and when, as a result, there is no uncertainty about the benefit of contributing and, as a result, no heterogeneity among individuals. We avoid this degeneracy by assuming that the functions U^o and U^p are different. We actually make the stronger assumption that U^o and U^p satisfy, for any $(\tilde{e}, \tilde{G}) \in [0, \bar{e}] \times [0, n\bar{e}]$, the property that $U_j^o(\tilde{e}, \tilde{G}) \geq U_j^p(\tilde{e}, \tilde{G})$ for $j = e, G$, with at least one of the two inequalities being strict. That is, we assume that the additional benefit an individual gets from an additional aggregate public good quantity - given effort - is (weakly) stronger in the optimistic state than in the pessimistic one. This assumption also entails that the (subjective) marginal cost of effort - given public good provision - is lower in the optimistic state than in the pessimistic one. This assumption will be largely responsible for the comparative static result that increasing one's optimism will lead to an overall increase in the (Nash) equilibrium level of public good provision. Inverting the sign of these inequalities will naturally lead to inverting the direction of this comparative static effect. Of course the assumption that the ordering of the partial derivatives of the functions U^o and U^p is invariant to the choice of the particular combination of effort and public good at which the derivatives are evaluated is strong. The last assumption made on the functions U^j (for $s = o, p$) is to satisfy $U_{eG}^s(\tilde{e}, \tilde{G}) = U_{Ge}^s(\tilde{e}, \tilde{G}) \leq 0$ for any $(\tilde{e}, \tilde{G}) \in [0, \bar{e}] \times [0, n\bar{e}]$. This assumption rules out the possibility for the (subjective) marginal cost of effort - given public good provision and irrespective of the state - to be strictly increasing with respect to public good quantity. The weak formulation of this condition makes it compatible with the possibility that either (or both) the functions U^o and U^p be additively separable with respect to their two arguments. We finally assume that $U_e^s(0, 0) + U_G^s(0, 0) > 0 > U_e^s(\bar{e}, G) + U_G^s(\bar{e}, G)$ for any $G \in [0, n\bar{e}]$ and $s = o, p$. We denote by \mathcal{U} the class of all pairs of functions U^o and U^p that satisfy all those properties.

There are many problems of voluntary contribution to a public good under uncertainty that would fit in this framework. A simple one would consist in assuming, for $s = o, p$, that $U^s(e, G) = -C(e) + \Phi^s(G)$ for some state independent increasing and convex cost function C and some increasing and concave state

⁴The (partial) derivative of a function g with respect to its j th argument is denoted by g_j .

⁵That is, the function U^j (for $j = o, p$) satisfies $U^j(\lambda e + (1 - \lambda)e', \lambda G + (1 - \lambda)G') > \lambda U^j(e, G) + (1 - \lambda)U^j(e', G')$ for every $\lambda \in]0, 1[$ and every distinct combinations (e, G) and (e', G') of effort and aggregate public good.

dependent Φ^s . In the context of preventing global warming, such a specification would be quite natural. The cost of preventing global warming by devoting costly immediate effort in carbon emission could plausibly be independent from the subjective appraisal of the impact of aggregate carbon emissions on global warming. The state dependent function Φ^s would measure, on the other hand, the monetary benefit of global warming reduction in state s of belief about the impact of aggregate human efforts - as measured by G - on that reduction. It is quite natural to consider that this monetary benefit would be an increasing and concave function of the total effort in carbon emission reduction. As a matter of facts, several papers that have examined the negotiation process leading to international agreements on the prevention of global warming have considered specific state independent versions of this simple model. For example, Ulph (2004) has considered countries involved in such a negotiation process with linear utility function of the form $-be + cG$ for some strictly positive real numbers b and c . Kolstad (2005) has considered a quadratic version of the same model. The only additional assumption imposed by the general requirement that U^o and U^p belong to \mathcal{U} is the fact that $\Phi_G^o(\tilde{G}) > \Phi_G^p(\tilde{G})$ for any aggregate effort $\tilde{G} \in [0, n\bar{e}]$. But this seems to be somewhat plausible in this context. After all, the extra monetary benefit of a reduction in human carbon emission has all the chance to be larger in states where human contribution to global warming is believed to be high than in state where it is believed to be low.

But there are many other contexts where the set of assumptions on individual preferences associated to the requirement for the functions U^o and U^p to belong to \mathcal{U} .

Any distribution of beliefs $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ in the population generates the game in strategic (or normal) form $\mathcal{G}(\pi_1, \dots, \pi_n)$ in which N is the set of players, $[0, \bar{e}]$ is the strategy set of any such player, and $v^i(\pi_i, e_1, \dots, e_n) = EU(\pi_i; e_i, e_i + \sum_{j \neq i} e_j)$ is the payoff received by player i at the strategy profile $(e_1, \dots, e_n) \in [0, \bar{e}]^n$ when he/she holds belief π_i . It is easy to see that the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ is what has been called by ? an *aggregative game* (see also Dubey, Mas-Colell, and Shubik (1980) and Shubik (1984)). A (pure strategy) Nash equilibrium for the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ is an effort profile $(e_1^*, \dots, e_n^*) \in [0, \bar{e}]^n$ such that, for every individual $i \in N$ and every effort level $e_i \in [0, \bar{e}]$ for this individuals, one has:

$$EU(\pi_i; e_i^*, e_i^* + \sum_{j \neq i} e_j^*) \geq EU(\pi_i; e_i, e_i + \sum_{j \neq i} e_j^*) \quad (2)$$

We first establish, in the next proposition, that for any distribution of beliefs $(\pi_1, \dots, \pi_n) \in [0, 1]^n$, the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ admits a unique Nash equilibrium. Such a result is obviously an important preliminary step for identifying the effect of specific changes in the distribution of beliefs on the Nash equilibrium of the game. Such an endeavour can obviously not be achieved if Nash equilibria do not exist for some specification of the beliefs. Moreover, if there are many different Nash equilibria that can result from a particular distribution of beliefs, it is difficult to predict which of them would be achieved in such a case.

We start the formal analysis, whose methodology is very much inspired by ?, with the following technical lemma. This lemma establishes some monotonicity properties of function $T : [0, 1] \times [0, \bar{e}] \times [0, n\bar{e}] \rightarrow \mathbb{R}$ defined, for any $(\tilde{\pi}, \tilde{e}, \tilde{G}) \in$

$[0, 1] \times [0, \bar{e}] \times [0, n\bar{e}]$, by:

$$T(\tilde{\pi}, \tilde{e}, \tilde{G}) := \tilde{\pi}[U_e^o(\tilde{e}, \tilde{G}) + U_G^o(\tilde{e}, \tilde{G})] + (1 - \tilde{\pi})[U_e^p(\tilde{e}, \tilde{G}) + U_G^p(\tilde{e}, \tilde{G})] \quad (3)$$

This function T is the derivative of the expected state-dependent utility function of Expression (1) with respect to effort when considering the effect of effort on total public good provision given the efforts by others. This derivative, that is zero for any individual who contributes a positive amount at a Nash equilibrium of the game $G(\pi_1, \dots, \pi_n)$, plays for this reason a key role in the characterization of such Nash equilibria. The lemma that is (immediately) proved is the following.

Lemma 1 *Let $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ be a distribution of beliefs and let $\mathcal{G}(\pi_1, \dots, \pi_n)$ be the associated game in strategic form. Then, if any player i 's payoff of this game writes $v^i(\pi_i, e_1, \dots, e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1 - \pi_i) U^p(e_i, e_i + \sum_{j \neq i} e_j)$ for a pair of functions U^o and U^p in the set \mathcal{U} , the function T defined by (3) is strictly increasing with respect to π and strictly decreasing with respect to both e and G .*

Proof. *Since U^o and U^p are in \mathcal{U} , they are at least thrice differentiable. Hence, the function T defined by (3) is twice differentiable. Proving the result amounts therefore to verifying that:*

$$T_\pi(\tilde{\pi}, \tilde{e}, \tilde{G}) = U_e^o(\tilde{e}, \tilde{G}) - U_e^p(\tilde{e}, \tilde{G}) + U_{eG}^o(\tilde{e}, \tilde{G}) - U_{eG}^p(\tilde{e}, \tilde{G}) > 0 \quad (4)$$

$$\begin{aligned} T_e(\tilde{\pi}, \tilde{e}, \tilde{G}) &= \pi[U_{ee}^o(\tilde{e}, \tilde{G}) + 2U_{eG}^o(\tilde{e}, \tilde{G}) + U_{GG}^o(\tilde{e}, \tilde{G})] \\ &\quad + (1 - \pi)[U_{ee}^p(\tilde{e}, \tilde{G}) + 2U_{eG}^p(\tilde{e}, \tilde{G}) + U_{GG}^p(\tilde{e}, \tilde{G})] \\ &< 0 \end{aligned} \quad (5)$$

and,

$$\begin{aligned} T_G(\tilde{\pi}, \tilde{e}, \tilde{G}) &= \pi[U_{eG}^o(\tilde{e}, \tilde{G}) + U_{GG}^o(\tilde{e}, \tilde{G})] \\ &\quad + (1 - \pi)[U_{eG}^p(\tilde{e}, \tilde{G}) + U_{GG}^p(\tilde{e}, \tilde{G})] \\ &< 0 \end{aligned} \quad (6)$$

But Inequality (4) follows from the fact that functions U^o and U^p in \mathcal{U} satisfy $U_j^o(\tilde{e}, \tilde{G}) \geq U_j^p(\tilde{e}, \tilde{G})$ for $j = e, G$ with at least one of the two inequalities being strict. Inequalities (5) and (6) on the other hands are implied by the concavities of the functions U^o and U^p and the fact that they satisfy $U_{eG}^j(\tilde{e}, \tilde{G}) = U_{Ge}^j(\tilde{e}, \tilde{G}) \leq 0$ for $j = o, p$. ■

Equipped with this Lemma, we first establish the existence and uniqueness of a Nash equilibrium for the game $G(\pi_1, \dots, \pi_n)$ for any distribution (π_1, \dots, π_n) of beliefs.

Proposition 1 *Let $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ be a distribution of beliefs and let $\mathcal{G}(\pi_1, \dots, \pi_n)$ be the associated game in strategic form. Then, if any player i 's payoff of this game writes $v^i(\pi_i, e_1, \dots, e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1 - \pi_i) U^p(e_i, e_i + \sum_{j \neq i} e_j)$*

$\sum_{j \neq i} e_j$) for a pair of functions U^o and U^p in the set \mathcal{U} , then the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ admits a unique Nash equilibrium.

Proof. We first observe that by strict concavity of the functions U^o and U^p , the program:

$$\max_{e_i \in [0, \bar{e}]} \pi_i U^o(e_i, e_i + G_{-i}) + (1 - \pi_i) U^p(e_i, e_i + G_{-i}) \quad (7)$$

admits a unique solution for any π_i and any given real number $G \in [0, (n-1)\bar{e}]$. In effect, for any such π_i and G , the function $\Psi^{\pi_i G} : [0, \bar{e}] \rightarrow \mathbb{R}$ defined by

$$\Psi^{\pi_i G}(e) = \pi_i U^o(e, e + G) + (1 - \pi_i) U^p(e, e + G)$$

is continuous. By Weirstrass theorem, the maximization of a continuous function over a compact set (as is $[0, \bar{e}]$) admits a solution. The strict concavity of both U^o and U^p ensures the strict concavity of the function $\Psi^{\pi_i G}$ and, therefore, the uniqueness of the maximizer of this function for any π_i and G . Let $e^*(\pi_i, G)$ denote the value of this unique maximizer of $\Psi^{\pi_i G}$. It follows from Berge (1959) maximum theorem that e^* is a continuous function from $[0, 1] \times [0, (n-1)\bar{e}]$ to $[0, \bar{e}]$. It thus follows that, given the distribution of beliefs (π_1, \dots, π_n) , the function $\tilde{e}^* : [0, \bar{e}]^n \rightarrow [0, \bar{e}]^n$ defined, for any $(e_1, \dots, e_n) \in [0, \bar{e}]^n$, by $\tilde{e}^*(e_1, \dots, e_n) = (e^*(\pi_1, \sum_{j \neq 1} e_j), e^*(\pi_2, \sum_{j \neq 2} e_j), \dots, e^*(\pi_n, \sum_{j \neq n} e_j))$ is continuous. Since the domain of \tilde{e}^* is compact and convex, the function \tilde{e}^* admits a fixed point by Brouwer theorem. Any fixed point of \tilde{e}^* is clearly a Nash equilibrium. Hence a Nash equilibrium of the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ exists for any distribution of beliefs (π_1, \dots, π_n) . We now show that this equilibrium is unique. By contradiction, suppose $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ is a distribution of beliefs for which there are two distinct combinations of efforts (e_1^*, \dots, e_n^*) and $(\hat{e}_1, \dots, \hat{e}_n)$ that are Nash equilibria for the game $\mathcal{G}(\pi_1, \dots, \pi_n)$. Since (e_1^*, \dots, e_n^*) and $(\hat{e}_1, \dots, \hat{e}_n)$ are distinct, there exists some $i \in N$ for which $e_i^* \neq \hat{e}_i$. Without loss of generality (up to a change in the role of (e_1^*, \dots, e_n^*) and $(\hat{e}_1, \dots, \hat{e}_n)$ in the argument), we assume $0 \leq e_i^* < \hat{e}_i$. We consider two mutually exclusive cases:

- (i) $\sum_{j \in N} e_j^* \geq \sum_{j \in N} \hat{e}_j$ and,
- (ii) $\sum_{j \in N} e_j^* < \sum_{j \in N} \hat{e}_j$.

If case (i) holds, then, since $0 \leq e_i^* < \hat{e}_i$ for some individual i , there must be some individual h for which one has $0 \leq \hat{e}_h < e_h^*$. Since $0 > U_e^s(\bar{e}, G) + U_G^s(\bar{e}, G)$ for any $G \in [0, n\bar{e}]$ and $s = o, p$, one has $e_h^* < \bar{e}$. Hence e_h^* is in the interior of the interval $[0, \bar{e}]$ and, as a Nash equilibrium, must satisfy the first order condition of Program (7). Similarly, $\hat{e}_h \geq 0$ is by assumption a Nash equilibrium which may, or may not, be interior. One must thus have:

$$T(\pi_h, e_h^*, \sum_{j \in N} e_j^*) = 0 \geq T(\pi_h, \hat{e}_h, \sum_{j \in N} \hat{e}_j)$$

But since $\hat{e}_h < e_h^*$, this inequality is incompatible with the properties, established in Lemma 1, that T is strictly decreasing with respect to both e and G .

If case (ii) holds, then we have $0 \leq e_i^* < \hat{e}_i$ for some individual i and $\sum_{j \in N} e_j^* < \sum_{j \in N} \hat{e}_j$

$\sum_{j \in N} \hat{e}_j$. For the same reason than before, \hat{e}_i is interior to the interval $[0, \bar{e}]$ while e_i^* is either zero or in the interior of that same interval. Since by assumption both e_i^* and \hat{e}_i are part of a Nash equilibrium, they satisfy (using the first order conditions of Program (7)):

$$T(\pi_i, \hat{e}_i, \sum_{j \in N} \hat{e}_j) = 0 \geq T(\pi_i, e_i^*, \sum_{j \in N} e_j^*)$$

But again, since both $e_i^* < \hat{e}_i$ and $\sum_{j \in N} e_j^* < \sum_{j \in N} \hat{e}_j$, this inequality incompatible with the fact, established in Lemma 1, that T is strictly decreasing with respect to e and G . ■

Because of proposition 1, we denote, for every distribution of beliefs $(\pi_1, \dots, \pi_n) \in [0, 1]^n$, by $\mathbf{e}^*(\pi_1, \dots, \pi_n) = (e_1^*(\pi_1, \dots, \pi_n), \dots, e_n^*(\pi_1, \dots, \pi_n))$ the unique Nash equilibrium of the game $\mathcal{G}(\pi_1, \dots, \pi_n)$ associated to (π_1, \dots, π_n) . In the next proposition, we establish that for any such distribution of beliefs $(\pi_1, \dots, \pi_n) \in [0, 1]^n$, the individuals' levels of contributions at the (unique) Nash equilibrium of the associated game will be weakly ordered by their beliefs. We also establish that, for those individuals who contribute positively to the public good, their levels of contribution will be strictly increasing with respect to their belief. This result, which is of interest in its own right, will also play an important role in the two additional comparative static results of this paper. For one thing, it implies that any Nash equilibrium combination of efforts will be entirely determined by the individuals' beliefs in the following sense that up to a belief threshold, nobody will contribute while everyone with belief above the threshold will contribute a strictly positive amount. Moreover, those positive contributors, who will always exist thanks to the assumption that $U_e^s(0, 0) + U_G^s(0, 0) > 0$ for $s = o, p$, will be strictly ordered by their beliefs. The proved result is the following.

Proposition 2 *Let $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ be a distribution of beliefs and let $\mathbf{e}^*(\pi_1, \dots, \pi_n)$ be the unique Nash equilibrium of the associated game in normal form $\mathcal{G}(\pi_1, \dots, \pi_n)$. Then, if any player i 's payoff of this game writes $v^i(\pi_i, e_1, \dots, e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1 - \pi_i) U^p(e_i, e_i + \sum_{j \neq i} e_j)$ for a pair of functions U^o and U^p in the set \mathcal{U} , then one has $\pi_i \geq \pi_h \implies e_i^*(\pi_1, \dots, \pi_n) \geq e_h^*(\pi_1, \dots, \pi_n)$. Moreover, for any individuals h and i such that $e_i^*(\pi_1, \dots, \pi_n) > 0$ and $e_h^*(\pi_1, \dots, \pi_n) > 0$, one has $\pi_i > \pi_h \implies e_i^*(\pi_1, \dots, \pi_n) > e_h^*(\pi_1, \dots, \pi_n)$.*

Proof. *In order to prove the first statement of the Proposition, let $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ be a distribution of beliefs and $\mathbf{e}^*(\pi_1, \dots, \pi_n)$ be the unique Nash equilibrium of the associated game and assume by contradiction that there are two individuals h and i such that $\pi_i \geq \pi_h$ and $e_i^*(\pi_1, \dots, \pi_n) < e_h^*(\pi_1, \dots, \pi_n)$. This entails that $e_h^*(\pi_1, \dots, \pi_n) > 0$. Since $0 > U_e^s(\bar{e}, \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) + U_G^s(\bar{e}, \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n))$ for $s = o, p$, $e_h^* < \bar{e}$. Hence $e_h^*(\pi_1, \dots, \pi_n)$ is in the interior of the interval $[0, \bar{e}]$ and satisfies therefore the first order condition of Program (7):*

$$T(\pi_h, e_h^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) = 0$$

while $e_i^*(\pi_1, \dots, \pi_n)$ satisfies:

$$T(\pi_i, e_i^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) \leq 0$$

but, when combined with the assumption that $e_i^*(\pi_1, \dots, \pi_n) < e_h^*(\pi_1, \dots, \pi_n)$, these two inequalities are clearly incompatible with the increasingness of T with respect to π and the strict decreasingness of T with respect to e established in Lemma 1. As for the second statement of the lemma, let again $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ be a distribution of beliefs and $e^*(\pi_1, \dots, \pi_n)$ be the unique Nash equilibrium of the associated game. Assume that h and i are two individuals such that $e_i^*(\pi_1, \dots, \pi_n) > 0$, $e_h^*(\pi_1, \dots, \pi_n) > 0$ and $\pi_i > \pi_h$. By contradiction, assume that $e_i^*(\pi_1, \dots, \pi_n) \leq e_h^*(\pi_1, \dots, \pi_n)$. For the same reason as before, both levels of contributions $e_h^*(\pi_1, \dots, \pi_n)$ and $e_i^*(\pi_1, \dots, \pi_n)$ are in the interior of the interval $[0, \bar{e}]$ and satisfy therefore the first order condition of Program. (7):

$$T(\pi_h, e_h^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) = 0 = T(\pi_i, e_i^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n))$$

But, when combined with $e_i^*(\pi_1, \dots, \pi_n) \leq e_h^*(\pi_1, \dots, \pi_n)$ and $\pi_i > \pi_h$, this equality is incompatible with the strict increasingness of T with respect to π and the decreasingness of T with respect to e established in Lemma 1. ■

An obvious consequence of Proposition 2 is that individuals' contributions at a Nash equilibrium are (weakly increasing) function of their beliefs *only*. In particular, any permutation of a distribution of beliefs $(\pi_1, \dots, \pi_n) \in [0, 1]^n$ will have no effect on the total sum of efforts provided at equilibrium and will only lead to the very same permutation of the individuals' contributions. Because of this, we can restrict attention, in what follows, to distributions of beliefs such that $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$. For such ordered distribution of beliefs, we now establish our first comparative static result. Specifically, we show that the total amount of contribution to the public good at a Nash equilibrium will never diminish when there is an improvement in optimism in the sense of first order stochastic dominance. Recall that an (ordered) distribution of beliefs (π_1, \dots, π_n) first order stochastically dominates the (ordered) distribution (π'_1, \dots, π'_n) if and only if it is the case that $\pi_i \geq \pi'_i$ for every individual i . We then establish the following result.

Proposition 3 *Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distributions of beliefs in $[0, 1]^n$ satisfying $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$ and $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$. Then, if $\pi'_i \geq \pi_i$ for all i and the payoff functions of the associated games in strategic form both result from functions U^o and U^p in the set \mathcal{U} , one has $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) \geq$*

$\sum_i e_i^(\pi_1, \dots, \pi_n)$. Moreover, if the Nash equilibria of the two games are such that $e_i^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_i^*(\pi_1, \dots, \pi_n) > 0$ for all i and the two distributions of beliefs are distinct, then one has $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) > \sum_i e_i^*(\pi_1, \dots, \pi_n)$.*

Proof. *For the first statement of the Proposition, let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distributions of beliefs satisfying $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$*

and $\pi'_i \geq \pi_i$ for all i and assume by contradiction that $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) < \sum_i e_i^*(\pi_1, \dots, \pi_n)$. For this inequality to hold, there must be some individual h such that $0 \leq e_h^*(\pi'_1, \dots, \pi'_n) < e_h^*(\pi_1, \dots, \pi_n)$. Since $e_h^*(\pi_1, \dots, \pi_n)$ is interior of $[0, \bar{e}]$, it follows from the first order condition of Program (7) that:

$$T(\pi_h, e_h^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) = 0 \geq T(\pi'_h, e_h^*(\pi'_1, \dots, \pi'_n), \sum_{j \in N} e_j^*(\pi'_1, \dots, \pi'_n))$$

But this inequality is incompatible with the strict increasingness of T with respect to π and its strict decreasingness with respect to both e and G established in Lemma 1. For the second statement of the proposition, let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distinct distributions of beliefs satisfying $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$ and $\pi'_i \geq \pi_i$ for all i and assume that $e_i^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_i^*(\pi_1, \dots, \pi_n) > 0$ for all i . Suppose by contradiction that $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) \leq \sum_i e_i^*(\pi_1, \dots, \pi_n)$. Since (π_1, \dots, π_n) and (π'_1, \dots, π'_n) are distinct, there is an individual h such that $\pi'_h > \pi_h$. Since $e_h^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_h^*(\pi_1, \dots, \pi_n) > 0$, both $e_h^*(\pi'_1, \dots, \pi'_n)$ and $e_h^*(\pi_1, \dots, \pi_n)$ are interior to $[0, \bar{e}]$ and satisfy, for this reason, the first order conditions of Program (7):

$$T(\pi_h, e_h^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) = 0 = T(\pi'_h, e_h^*(\pi'_1, \dots, \pi'_n), \sum_{j \in N} e_j^*(\pi'_1, \dots, \pi'_n))$$

Since, by Lemma 1, T is strictly increasing in π and strictly decreasing with respect to both e and G , the only way to make this equality compatible with $\pi'_h > \pi_h$ and $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) \leq \sum_i e_i^*(\pi_1, \dots, \pi_n)$ is to have:

$$e_h^*(\pi_1, \dots, \pi_n) < e_h^*(\pi'_1, \dots, \pi'_n) \quad (8)$$

Moreover, since $e_i^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_i^*(\pi_1, \dots, \pi_n) > 0$ for all i , all such individual levels of contributions are interior and satisfy the first order conditions:

$$T(\pi_i, e_i^*(\pi_1, \dots, \pi_n), \sum_{j \in N} e_j^*(\pi_1, \dots, \pi_n)) = 0 = T(\pi'_i, e_i^*(\pi'_1, \dots, \pi'_n), \sum_{j \in N} e_j^*(\pi'_1, \dots, \pi'_n))$$

for $i = 1, \dots, n$. Since $\pi'_i \geq \pi_i$ for all i and T is strictly increasing in π and strictly decreasing with respect to both e and G by Lemma 1, this equality can only hold if

$$e_i^*(\pi_1, \dots, \pi_n) \leq e_i^*(\pi'_1, \dots, \pi'_n) \quad (9)$$

holds for all i . But combining the strict inequality (3) with the n weak inequalities (9) lead to the conclusion that:

$$\sum_i e_i^*(\pi_1, \dots, \pi_n) < \sum_i e_i^*(\pi'_1, \dots, \pi'_n)$$

a contradiction. ■

We now examine the impact, on the overall Nash contributive effort, of an increase in the *consensus* that may exist in the community as to the likelihood

that individual efforts can have significant impact on public good provision. In the example of global warming discussed earlier, former vice-president Al Gore was referring to the emergence of a consensus about the human origin of the currently observed carbon accumulation in the atmosphere. Debate and discussion among individuals are indeed likely to increase the existing consensus on that matter. Of course a consensus can *a priori* be reached around any "average" level of optimism. But suppose we take this average level of optimism as given. What is the effect - on the total contribution to the public good - of bringing everybody in the society closer to this average level of consensus? This is the question that we now address.

Answering this question requires of course a definition of what it means for a distribution of beliefs to be "more consensual" than another. To motivate our proposed definition of that notion, consider again the case of global warming and imagine that Donald Trump and Al Gore are forming a community. Assume that Donald Trump initially assigns a zero probability to the (optimistic) scenario in which human efforts in carbon emission reduction prevents global warming while Al Gore assigns the opposite polar probability of 1 to that same scenario. Observe that the average probability assigned to the optimistic scenario in this two-individuals community is $1/2$. We could now imagine D. Trump and A. Gore trying to convince each other of the validity of their respective beliefs. One could of course be more convincing than the other and therefore more successful in bringing the other closer to his view. But suppose that the two guys are of equal convincing power, and manage to get their belief closer. For example, at the end of the discussion, Donald Trump belief could be $1/4$, while Al Gore one could be $3/4$. The average probability assigned in the population to the optimistic scenario would still be $1/2$, but the two members of the community would be closer to each other (and to this average). In such a case, we would say that the consensus in the society has increased.

Specifically, the definition of "increase in consensus" is based on the notion of Lorenz domination of one distribution of beliefs over another or, equivalently (see for example Dasgupta, Sen, and Starrett (1973)) on the idea that one distribution of beliefs has been obtained from the other by a finite sequence of "Pigou-Dalton" transfer of beliefs. We specifically consider the following definition of one distribution of beliefs to be more consensual than another.

Definition Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distributions of beliefs in $[0, 1]^n$ such that $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$ and $\sum_{i \in N} \pi_i = \sum_{i \in N} \pi'_i$.

We say that (π_1, \dots, π_n) is more consensual than (π'_1, \dots, π'_n) if and only if, for any $k \in N$, it is the case that $\sum_{i=1}^k \pi_i \geq \sum_{i=1}^k \pi'_i$.

As is well-known from the inequality measurement literature, and in particular the Hardy-Littlewood-Polya theorem (see e.g. Berge (1959)), there is an equivalent definition of "more consensual than" that can be expressed in terms of bilateral Pigou-Dalton transfers. As it turns out, this equivalent definition will be easier to use for establishing the last comparative static result of this paper.

We start by the following definition of what it means for a distribution of beliefs to be obtained from another by a *bilateral Pigou-Dalton transfer*.

Definition Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distributions of beliefs in $[0, 1]^n$ such that $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$. We say that (π_1, \dots, π_n)

has been obtained from (π'_1, \dots, π'_n) by a bilateral Pigou-Dalton transfer if there exists two individuals g and h and a non-negative number δ such that $\pi_i = \pi'_i$ for all $i \notin \{h, g\}$ and $\pi_g = \pi'_g + \delta \leq \pi'_h - \delta = \pi_h$.

In words, a Pigou-Dalton transfer is the formal description of a balanced "debate" between optimistic individual h (Gore) and pessimistic individual g (Trump). At the end of this balanced debate, Trump has gained δ of optimism but this gain has been exactly counterbalanced by the loss of optimism by Gore by exactly that same δ .

The well-known Hardy-Littlewood-Polya theorem elegantly demonstrated by Berge (1959) establishes an equivalence between the fact for one distribution of beliefs to be more consensual than another as per Definition 2 and the possibility of going from the less to the more consensual distribution by a finite sequence of bilateral Pigou-Dalton transfer. For later use, we state formally this theorem as follows.

Theorem 1 (*Hardy-Littlewood-Polya*) *Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distributions of beliefs in $[0, 1]^n$ such that $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$. Then (π_1, \dots, π_n) is more consensual than (π'_1, \dots, π'_n) as per Definition 2 if and only if there exists a sequence of $t \in \mathbb{N}_+$ distributions of beliefs (with $t \geq 2$) $(\pi_1^k, \dots, \pi_n^k) \in [0, 1]^n$, for $k = 1, \dots, t$ such that*

- (i) $(\pi_1^1, \dots, \pi_n^1) = (\pi_1, \dots, \pi_n)$,
- (ii) $(\pi_1^t, \dots, \pi_n^t) = (\pi'_1, \dots, \pi'_n)$ and
- (iii) $(\pi_1^k, \dots, \pi_n^k)$ has been obtained from $(\pi_1^{k+1}, \dots, \pi_n^{k+1})$ by a bilateral Pigou-Dalton transfer as per Definition 2 for all $k = 1, \dots, t - 1$.

We now examine the impact that an increase in the consensus in a given community in the sense of Definition 2 can have on the aggregate contributive effort at the non-cooperative Nash equilibrium. As it turns out, the set of assumptions made thus far on the utility functions - namely that U^o and U^p belong to \mathcal{U} - does not suffice in obtaining clear cut conclusions on that matter. Intuitively, if an individual "transfers" part of his/her optimism to someone else, this has two conflicting effects. On the one hand, the "giver" of optimism will tend to reduce his/her contribution while the "receiver" of optimism will increase his/her. The two forces are clearly playing in opposite direction. Hence, some additional condition on the utility functions are required to predict the relative strength at which these two opposite forces will play. As it turns out, the following set of conditions on U^o and U^p are sufficient, when applied to functions that belong to \mathcal{U} , for establishing the result that an increase in consensus - in the sense of Definition 2 - will increase the aggregate amount of contributions.

Condition 1 *For any $G \in [0, n\bar{e}]$, and any e and e' such that $e \geq e'$, one has:*

- (i) $U_j^o(e, G) - U_j^p(e, G) \geq U_j^o(e', G) - U_j^p(e', G)$ for $j = e, G$ and
- (ii) $U_{kl}^s(e', G) \geq U_{kl}^s(e, G)$ for $s = o, p$, $k = e, G$ and $l = e, G$

In plain English the part (i) of this condition requires that the increase (decrease) of the marginal benefit (cost) of increasing (decreasing) global (individual) effort be non-decreasing with respect to individual effort. Hence, as individuals increase their individual effort for a given total quantity of effort, they feel more difference in the marginal benefit (or cost) of additional effort between the two states. The part (ii) of the condition requires the concavity of

the state dependent function U^s (for $s = o, p$) to be stronger, for any given aggregate quantity of efforts, when the individual perform a low effort than when he or she performs a large one. This condition is certainly demanding. It is actually not necessary for the comparative static result to come. But it may be useful to assess its strength in the somewhat specific but plausible context of the additively separable monetary evaluation of the benefit to global warming prevention discussed above. In this context, we had $U^s(e, G) = -C(e) + \Phi^s(G)$. In such a setting, Condition 1 (i) would hold trivially, and Condition 2 (ii) would be equivalent to requiring that the function C had a positive third derivative. In words, this amount to requiring that the increase in marginal cost be decreasing with effort (or that the marginal cost curves be increasing and concave). This is clearly a restrictive condition. But it does not strike us as being unreasonable.

We now turn to the last comparative static result of this paper. Contrary to what was obtained for the previous results (for example Proposition 8), the result we are about to state - namely that aggregate effort will increase with consensus - holds only when all community members are strict contributors. There is an obvious reason for this. Suppose in effect that, at some Nash equilibrium, someone with very optimistic belief is contributing while another person, endowed with more pessimistic belief, is not contributing at all. Imagine then that a small Pigou-Dalton transfer of belief take place between these two individuals, everything else remaining the same. Suppose that the increase in optimism of the non-contributor brought about by the transfer is not sufficient for making him/her a contributor. Then the transfer will only end up reducing the optimism of the set of active contributors. As shown in Proposition 8, this will lead to a reduction in the total amount of contributions by those contributors. Hence, in a case like this, a bilateral Pigou-Dalton transfer will actually lead to a reduction in the total contributive effort of the community.

However, if everybody contributes both before and after the increase in consensus, then the latter will cause the total contributive effort to increase. The formal statement of this result is as follows.

Proposition 4 *Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distinct distributions of beliefs in $[0, 1]^n$ satisfying $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$ and $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$. Suppose that (π_1, \dots, π_n) is more consensual than (π'_1, \dots, π'_n) as per Definition 2 and the payoff functions of the associated games in strategic form both result from functions U^o and U^p in the set \mathcal{U} that satisfy Condition 1. Suppose also that $e_i^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_i^*(\pi_1, \dots, \pi_n) > 0$ for all i . Then one has $\sum_i e_i^*(\pi_1, \dots, \pi_n) > \sum_i e_i^*(\pi'_1, \dots, \pi'_n)$.*

Proof. *By Theorem 1, (π_1, \dots, π_n) is more consensual than (π'_1, \dots, π'_n) if and only if (π_1, \dots, π_n) has been obtained from (π'_1, \dots, π'_n) by a finite sequence of bilateral Pigou-Dalton transfers. Hence, in order to prove each statement of the Proposition, it is clearly sufficient to prove that a bilateral Pigou-Dalton performed between a pair of individuals (say h and i) in a community of strict contributors lead to an increase in the sum of individuals' contributions. We actually provide the proof for a "small" Pigou-Dalton transfer of δ , for which the approximation provided by calculus is adequate (A MORE GENERAL PROOF WILL SOON BE AVAILABLE). Specifically, we consider (π_1, \dots, π_n) and (π'_1, \dots, π'_n) such that*

$\pi_j = \pi'_j$ for all $j \notin \{h, i\}$ and
 $\pi_i = \pi'_i - \delta \geq \pi'_h + \delta = \pi_h$

with δ suitably small. We can approximate the impact on (interior) equilibrium of such a small Pigou-Dalton transfer by differentiating totally the first order conditions of Program 7 above and evaluating the differential at the Nash equilibrium associated to the distribution of beliefs (π'_1, \dots, π'_n) . Denote by (e_1^*, \dots, e_n^*) and $G^* = \sum_{j=1}^n e_j^*$ the distribution of efforts and aggregate effort (respectively) associated to this Nash equilibrium. Doing the total differentiation yields, for individuals h and i

$$\begin{aligned} T_e(\pi'_h, e_h^*, G^*)de_h + T_G(\pi'_h, e_h^*, G^*)dG &\equiv -T_\pi(\pi'_h, e_h^*, G^*)d\pi_h \\ T_e(\pi'_i, e_i^*, G^*)de_i + T_G(\pi'_i, e_i^*, G^*)dG &\equiv -T_\pi(\pi'_i, e_i^*, G^*)d\pi_i \end{aligned}$$

while for the other individuals j (if any), whose beliefs have not changed, the total differentiation of their equilibrium first order condition is simply:

$$T_e(\pi'_j, e_j^*, G^*)de_j + T_G(\pi'_j, e_j^*, G^*)dG \equiv 0$$

Exploiting the fact that $d\pi_i = \pi_i - \pi'_i = (\pi'_i - \delta) - \pi'_i = -\delta = -d\pi_h = \pi'_h - \pi_h = \pi'_h - (\pi'_h + \delta)$ and the strict monotonicity of T established in Lemma 1, one can write these equations as:

$$\begin{aligned} de_h &\equiv -\delta \frac{T_\pi(\pi'_h, e_h^*, G^*)}{T_e(\pi'_h, e_h^*, G^*)} - \frac{T_G(\pi'_h, e_h^*, G^*)}{T_e(\pi'_h, e_h^*, G^*)}dG \\ de_i &\equiv \delta \frac{T_\pi(\pi'_i, e_i^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)} - \frac{T_G(\pi'_i, e_i^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)}dG \end{aligned}$$

and:

$$de_j \equiv -\frac{T_G(\pi'_j, e_j^*, G^*)}{T_e(\pi'_j, e_j^*, G^*)}dG$$

for all $j \notin \{h, i\}$. Summing these equations over all individuals and rearranging yields, after exploiting the fact that $\sum_{j=1}^n de_j = dG$:

$$dG = \frac{\delta \left[\frac{T_\pi(\pi'_i, e_i^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)} - \frac{T_\pi(\pi'_h, e_h^*, G^*)}{T_e(\pi'_h, e_h^*, G^*)} \right]}{1 + \sum_{j=1}^n \frac{T_G(\pi'_j, e_j^*, G^*)}{T_e(\pi'_j, e_j^*, G^*)}}$$

The denominator of this expression is strictly positive for all functions U^o and U^p in the set \mathcal{U} . Hence, the sign of dG - that is the fact that aggregate effort increases or decreases - depends upon the sign of the expression:

$$\frac{T_\pi(\pi'_i, e_i^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)} - \frac{T_\pi(\pi'_h, e_h^*, G^*)}{T_e(\pi'_h, e_h^*, G^*)}$$

This expression is weakly positive if and only if:

$$\frac{T_e(\pi'_h, e_h^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)} \geq \frac{T_\pi(\pi'_h, e_h^*, G^*)}{T_\pi(\pi'_i, e_i^*, G^*)} \quad (10)$$

Thanks to Proposition 2, one has $e_h^*(\pi'_1, \dots, \pi'_n) \leq e_i^*(\pi'_1, \dots, \pi'_n)$. Hence it follows from part (i) of Condition 1 that:

$$\begin{aligned} T_\pi(\pi'_h, e_h^*, G^*) &= U_e^o(e_h^*, G^*) - U_e^p(e_h^*, G^*) + U_G^o(e_h^*, G^*) - U_G^p(e_h^*, G^*) \\ &\leq U_e^o(e_i^*, G^*) - U_e^p(e_i^*, G^*) + U_G^o(e_i^*, G^*) - U_G^p(e_i^*, G^*) \\ &= T_\pi(\pi'_i, e_i^*, G^*) \end{aligned}$$

Moreover, it also follows from part (ii) of Condition 1 that:

$$\begin{aligned} T_e(\pi'_h, e_h^*, G^*) &= \pi'_h[U_{ee}^o(e_h^*, G^*) + 2U_{eG}^o(e_h^*, G^*) + U_{GG}^o(e_h^*, G^*)] \\ &\quad + (1 - \pi'_h)[U_{ee}^p(e_h^*, G^*) + 2U_{eG}^p(e_h^*, G^*) + U_{GG}^p(e_h^*, G^*)] \\ &\geq \pi'_i[U_{ee}^o(e_i^*, G^*) + 2U_{eG}^o(e_i^*, G^*) + U_{GG}^o(e_i^*, G^*)] \\ &\quad + (1 - \pi'_i)[U_{ee}^p(e_i^*, G^*) + 2U_{eG}^p(e_i^*, G^*) + U_{GG}^p(e_i^*, G^*)] \end{aligned}$$

Hence one has :

$$\frac{T_e(\pi'_h, e_h^*, G^*)}{T_e(\pi'_i, e_i^*, G^*)} \geq 1 \geq \frac{T_\pi(\pi'_h, e_h^*, G^*)}{T_\pi(\pi'_i, e_i^*, G^*)}$$

and Inequality (10) follows. ■

Proposition 3 shows that increasing optimism in the community in the sense of 1st order stochastic dominance increases the total effort that individuals are willing to devote to the production of a public good when they behave non-cooperatively. Proposition 4 shows, under the additional condition 1, that increasing the consensus among the community members for a given average level of optimism also increases the total effort that the community members are willing to make for providing the public good. An obvious corollary to these two propositions is the favorable impact, on global effort, of a combination of an increase in optimism - in the sense of first order dominance - and an increase in consensus, in the form of a sequence of Pigou-Dalton transfer. Specifically consider two ordered distributions of beliefs (π_1, \dots, π_n) and (π'_1, \dots, π'_n) such that, for any $k = 1, \dots, n$, one has

$$\sum_{j=1}^k \pi_j \geq \sum_{j=1}^k \pi'_j \quad (11)$$

Observe that, contrary to what was the case for the definition of an increase in consensus, we do not require the average optimism to be the same. It is, for instance, possible to have $\sum_{j=1}^n \pi_j > \sum_{j=1}^n \pi'_j$ so that the community with belief (π_1, \dots, π_n) is more optimistic in average (or in total if the population size is the same) than (π'_1, \dots, π'_n) . The requirement that Inequality (11) holds for all $k = 1, \dots, n$ between two distributions (π_1, \dots, π_n) and (π'_1, \dots, π'_n) is usually referred to as Generalized Lorenz dominance (see e.g. Shorrocks (1983)) It is then an immediate corollary of Propositions 3 and 4 that the following holds.

Corollary 1 *Let (π_1, \dots, π_n) and (π'_1, \dots, π'_n) be two distinct distributions of beliefs in $[0, 1]^n$ satisfying $\pi_1 \leq \pi_2 \leq \dots \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq \dots \leq \pi'_n$ and $\sum_{j=1}^k \pi_j \geq \sum_{j=1}^k \pi'_j$ for every $k = 1, \dots, n$. Suppose that the payoff functions of the associated games in strategic form both result result from functions U^o and U^p in the set \mathcal{U} that satisfy Condition 1. Suppose also that $e_i^*(\pi'_1, \dots, \pi'_n) > 0$ and $e_i^*(\pi_1, \dots, \pi_n) > 0$ for all i . Then one has $\sum_i e_i^*(\pi'_1, \dots, \pi'_n) > \sum_i e_i^*(\pi_1, \dots, \pi_n)$.*

3 An alternative interpretation

The model thus far holds for any general U^o and U^p , and is interpreted in terms of people contributing effort in order to derive utility from a public good. As mentioned earlier in Section 2, we can reformulate the same model to the case of abatement efforts to counter a public bad.

Consider the case of a group of agents, such as in Bramouille and Treich (September 2009), who emit pollution, and are trying to abate the pollution by investing effort. Each agent abates by investing an effort e . Effort is costly, and agents incur a cost $C(e)$ which does not vary across the states optimistic, or pessimistic. The cost function is increasing and strictly convex in e . Agents benefit from the sum of abatement, $G = \sum_i e_i$, and get benefit $\Phi^o(G)$ in the optimistic state, and $\Phi^p(G)$ in the pessimistic state. The benefit functions Φ^o and Φ^p are increasing and strictly concave. Each agent holds a subjective belief π_i about the occurrence of the optimistic state optimistic.

This is clearly an application of our model to the case where the utility functions are additively separable, and the cost of effort is invariant across the two states. The assumptions in section 2 will simplify to the requirements that $\Phi^o(G) > \Phi^p(G)$, and $\Phi_e^o(G) > \Phi_e^p(G)$. The expected utility function of an agent i therefore becomes:

$$EU(\pi_i, e, G) = \pi_i \Phi^o(G) + (1 - \pi_i) \Phi^p(G) - C(e)$$

Simple calculations following the proof for Proposition 4 show that condition 1 in this new model transforms to the requirement that $C_{eee} > 0$. That is, a necessary and sufficient condition for increasing consensus to increase the sum of abatement efforts is that the third derivative of the cost function must be strictly positive. The intuition for this is not immediately obvious, but it is clear that the convexity of the cost function in itself is not enough to guarantee an increase in the total pollution abatement in society when the consensus in beliefs increases as in Proposition 4: the cost of abatement must be increasingly convex.

4 Conclusion

This paper has examined the problem, for a community of individuals, of contributing to a public good when there is subjective uncertainty about the impact of the aggregate contribution on the considered public good. The problem of making individual efforts for reducing carbon emission with the aim of preventing global warming is a good example of this kind of situations. Other examples are provided by individual decisions to resort to vaccination to prevent the global risk of getting disease, or, in developing countries, by decisions to defecate in toilet rather than openly. When individuals are subjectively uncertain about the impact of the sum of their individual contributions on the benefit that they receive from the public good and differ in terms of this uncertainty, they may try to modify the beliefs held by others about this through debate or, perhaps, activism. This paper has examined to what extent an increase in the average belief that the sum of efforts impacts favorably the benefit of the public good lead a community of individuals to increase their effort. It has also examined to what extent an increase in the consensus about these beliefs - in the sense of

Lorenz dominance - leads to such an increase in aggregate effort. It has shown this to be the case under somewhat strong, but not unreasonable, conditions on contributors' subjective evaluation of the benefit of the public good and the cost of their private effort.

The analysis performed in this paper is yet incomplete in many respects. One of its limitation is the two-state setting in which it is framed. As modeled in this paper, an individual contributor is facing indeed only two states of the world: an "optimistic" one in which the sum of the community members is impacting significantly the benefit that she receives from the public good, and a "pessimistic" one where the impact of the community's aggregate contribution on the benefit of the public good is less favorable. It would obviously be interesting to generalize the analysis to more than two states. But doing so is not as straightforward as it may seem. For one thing, it leaves open the question of defining what it means for the consensus about the beliefs in the occurrence of the various state to increase in the society. When there are only two states, each assigned with some probability, it is natural to define an increase in consensus by Lorenz dominance over one of the two probabilities (that sum to one). If there are, say, three states, then each state comes with a (possibly) different probability, and making the probabilities assigned to a state by two different individuals closer in the sense of a Pigou-Dalton transfer does not mean making the probabilities assigned by these two individuals closer. There seems therefore to be the need of developing a theory of what it means for two probability distributions to be closer to each other.

References

- ACEMOGLU, D., AND M. K. JENSEN (2013): "Aggregate Comparative Statics," *Games and Economic Behavior*, 81, 27–49.
- AUSTEN-SMITH, D. (1980): "INDIVIDUAL CONTRIBUTION TO PUBLIC GOODS," *Economic Letters*, 5, 359–361.
- BARRETT, S. (1994): "Self-enforcing international environmental agreements," *Oxford Econ. Papers*, 46, 878–894.
- BERGE, C. (1959): *Espaces topologiques et fonctions multivoques*. Dunod, Paris.
- BERGSTROM, T., L. BLUME, AND H. VARIAN (1986): "On the Private Provision of Public Goods," *Journal of Public Economics*, 29, 25–49.
- BRAMOULLE, Y., AND V. BOUCHER (2010): "Providing global public goods under uncertainty," *Journal of Public Economics*, 94, 591–603.
- BRAMOULLE, Y., AND N. TREICH (September 2009): "CAN UNCERTAINTY ALLEVIATE THE COMMONS PROBLEM?," *Journal of the European Economic Association*, 7(5), 1042–1067.
- CORCHÓN, L. C. (1994): "Comparative statics for aggregative games. The strong concavity case.," *Mathematical Social Sciences*, 28, 151–165.
- CORNES, R., AND T. SANDLER (1984): "The Theory of Public Goods: Non-Nash Behaviour," *Journal of Public Economics*, 23, 367–379.

- (1985): “On the Consistency of Conjectures with Public Goods,” *Journal of Public Economics*, 27, 125–129.
- (1996): *The Theory of Externalities, Public Goods, and Club Goods*, Cambridge Books. Cambridge University Press.
- DASGUPTA, P., A. K. SEN, AND D. STARRETT (1973): “Notes on the Measurement of Inequality,” *Journal of Economic Theory*, 6, 180–187.
- DIXIT, A. K., AND J. W. WEIBULL (2007): “Political polarization,” *Proceedings of the National Academy of Sciences*, 104(18), 7351–7356.
- DORAN, P., AND M. ZIMMERMAN (2009): “Examining the scientific consensus on climate change,” *Eos Trans Am Geophys Union*, 90, 21–22.
- DUBEY, P., A. MAS-COLELL, AND M. SHUBIK (1980): “Efficiency Properties of Strategic Market Games,” *Journal of Economic Theory*, 22, 339–362.
- GRADSTEIN, M., S. NITZAN, AND S. SLUTSKY (1992): “The effect of uncertainty on interactive behaviour,” *Economics Journal*, 102(412), 554–561.
- GRADSTEIN, M., S. NITZAN, AND S. SLUTSKY (1993): “Private provision of public goods under price uncertainty,” *Social Choice and Welfare*, 10, 371–382.
- HADAR, J., AND W. RUSSELL (1974): “Stochastic Dominance in Choice under Uncertainty,” in *Essays on Economics Behavior under Uncertainty*, ed. by M. S. Balch, D. L. McFadden, and S. Wu. North Holland, Amsterdam, UK.
- ITAYA, J.-I., AND K. SHIMOMURA (2001): “A dynamic conjectural variations model in the private provision of public goods: a differential game approach,” *Journal of Public Economics*, 81, 153–172.
- JENSEN, M. K. (June 2012.): “Distributional comparative statics,” Department of Economics Discussion paper, University of Birmingham.
- KOLSTAD, C. D. (2005): “Piercing the Veil of Uncertainty in Transboundary Pollution Agreements,” *Environmental and Resource Economics*, 31, 21–34.
- (2007): “Systematic uncertainty in self-enforcing international environmental agreements,” *Journal of Environmental Economics and Management*, 53, 68–79.
- MEINSHAUSEN, M., N. MEINSHAUSEN, W. HARE, S. C. B. RAPER, K. FRIELER, R. KNUTTI, D. J. FRAME, AND M. R. ALLEN (2009): “Greenhouse-gas emission targets for limiting global warming to 2°C,” *Nature*, 458, 1158–1163.
- MILGROM, P., AND C. SHANNON (1994): “Monotone comparative statics,” *Econometrica*, 62, 157–180.
- PELHAM, B. W. (2009): “Awareness, opinions about global warming vary worldwide,” Gallup Poll.

- ROSER-RENOUF, C., E. W. MAIBACH, A. LEISEROWITZ, AND X. ZHAO (2014): “The genesis of climate change activism: from key beliefs to political action,” *Climatic Change*, 125(2), 163–178.
- SAKAMOTO, H. (2014): “Public Bads, Heterogeneous Beliefs, and the Value of Information,” Discussion papers e-13-009, Graduate School of Economics Project Center, Kyoto University.
- SANDLER, T., F. P. STERBENZ, AND J. POSNETT (1987): “FREE RIDING AND UNCERTAINTY,” *European Economic Review*, 31, 1605–1617.
- SCHUMACHER, I. (2015): “How Beliefs Influence The Willingness To Contribute To Prevention Expenditure,” *American Journal of Agricultural Economics*, 91(5), 1417–1432.
- SHORROCKS, A. F. (1983): “Ranking Income Distributions,” *Economica*, 50, 3–17.
- SHUBIK, M. (1984): *Game Theory in the Social Sciences: concepts and solutions*. MIT Press, Cambridge, MA.
- SUGDEN, R. (1985): “Consistent Conjectures and Voluntary Contributions to Public Goods: Why the Conventional Theory Doesn’t Work,” *Journal of Public Economics*, 27, 117–124.
- TF, S., Q. D, P. G.K., T. M., AND A. S.K. (eds.) (2013): *Intergovernmental Panel on Climate Change (2013) Climate Change 2013: The Physical Science Basis*. Cambridge University Press, Cambridge, United Kingdom and NY, USA.
- TOPKIS, D. (1978): “Minimizing a submodular function on a lattice.,” *Operational Research*, 26, 305–321.
- ULPH, A. (2004): “Stable International Environmental Agreements with a Stock Pollutant, Uncertainty and Learning,” *The Journal of Risk and Uncertainty*, 29, 53–73.