A Macroeconomic Analysis of Corporate Tax Evasions [*]

Takeo Hori[†]   Noritaka Maebayashi[‡]   Keiichi Morimoto[§]

Abstract

We examine whether the world trend of decreasing corporate tax rate is desirable in view of tax evasions of firms by the theoretical and quantitative exercises. We explore the optimal corporate taxation under endogenous tax evasions by firms in an R&D based growth model. Using the model, we obtain the following implications. Decreasing corporate tax rate brings larger welfare cost than that in the usual discussion where endogenous tax evasions of firms are not considered. In terms of welfare as well as growth, corporate tax rates should not be set to low rates, for example, 20%.

Keywords: corporate tax, tax evasion, growth, welfare

1 Introduction

Is the world trend of the corporate tax cuts really desirable? In this paper, we tackle this question in view of tax evasions of firms by the theoretical and quantitative exercises. We explore the optimal corporate taxation under endogenous tax evasions by firms in an R&D based growth model. Using the model, we obtain the following implications. Decreasing corporate tax rate brings larger welfare cost than that in the usual discussion where endogenous tax evasions of firms are not considered. In terms of welfare as well as growth, corporate tax rates should not be set to low rates, for example, 20%.

In recent years, the governments of many countries have decreased the corporate income tax rates monotonically. Figure 1 shows the average rate of corporate income tax in OECD countries between 2000 and 2017. In this period, the average rate has greatly changed, from 32.49% to 24.18%.
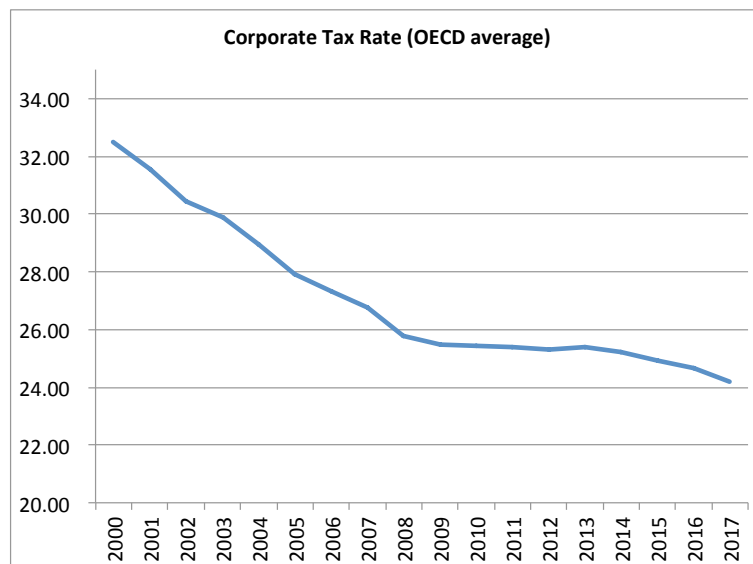

Figure 1: Average of corporate income tax rate in OECD countries between 2000 and 2017 (source: OECD database)

This monotonic behavior exhibits also in the country level: See Figure 2. Except from 2012 to 2015 in France, G7 countries almost decrease (or leave) the corporate income tax rates in this period. In particular, Canada, Germany, Japan and United Kingdom have been conducting the drastic tax cuts in the corporate sectors.

Why do the governments in OECD countries choose such a direction of tax policy in
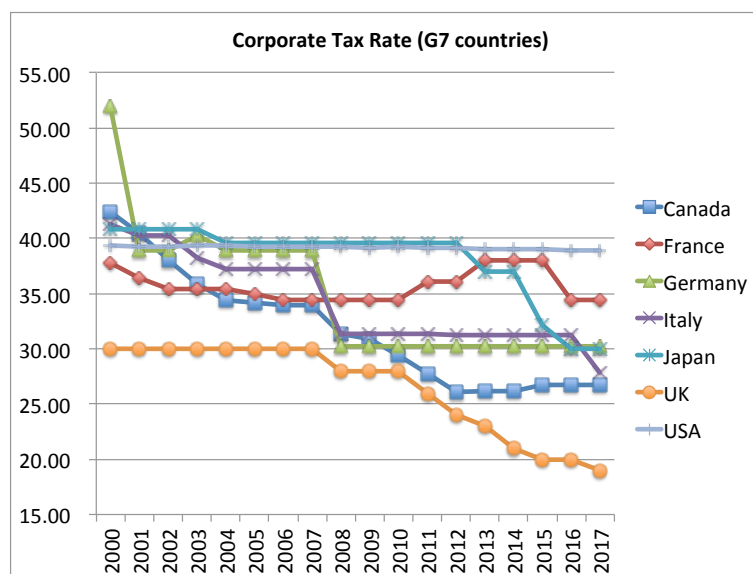
Figure 2: Corporate income tax rate in G7 countries between 2000 and 2017 (source: OECD database)

accordance? In general, there seems to be two aims for decreasing corporate income tax rates. First, a cut in corporate tax promotes private investment and leads to a rise in productivity in the macroeconomy. Second, the tax competition in the global economy requires the government to set a low tax rate to prepare efficient environment for the global business firms. The trend of cutting corporate income tax may last for the time being. According to the policy stance of President Trump, the government of the US decides to decrease the corporate tax rate to 21%. In Japan, the government plans to cut the corporate tax to about 20% under some condition. However, is such a policy line desirable? This is not necessarily obvious and not yet solved by formal economic arguments enough. Thus, we investigate it as the central question of the paper.

Corporate income tax is one of the important financial resources for public finance and macroeconomic policy in many countries. This partly covers the productive government expenditure on paying public services and forming infrastructure, which enhance long-run growth (see Footnote 1 below). In addition, corporate taxation affects the firms' behavior including pricing and production planning, and brings distortions of resource allocations. Thus, we explore the growth and welfare effect of corporate income tax in this paper.

Actually, corporate income tax occupies the considerable fraction of the annual revenue.

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For examples, the shares of corporate income tax to the total tax revenues are about 10% in the US in 2016 and 19% in Japan in 2015. However, the enforcements of corporate taxation are not enough in many countries: Only 1.1% of the taxed returns is examined in the US in 2016 and only 3.2% of all the corporations is audited in Japan in 2015. Accordingly, there seem to be tax evasions of huge scales in actual macroeconomies. For example, IRS (2016) reports about 44 billion dollars as the estimated tax gap of corporate income tax in annual average from 2008 to 2010 in the US. Some difficulties are emerging in practice of taxation. While the number of corporations and taxed affairs has been rapidly increasing in the developed countries, the budget for managing the tax agency is limited relative to the increase in corporations because of the tight fiscal conditions. Besides, it is also emphasized by the expansion of international business and the development of economic trading with information technology. Therefore, it is unrealistic to inspect most of the taxed corporation by increasing the expenditures on managing the tax agencies and we should study adjusting corporate income tax rate to realize the efficient resource allocation. This is because we focus on tax evasions by firms to discuss the optimal corporate taxation structure.

In this paper, we construct an R&D based growth model with productive public services, incorporating endogenous tax evasions by intermediate goods firms. To investigate the efficiency of the tax cuts, we analyze the growth- and welfare-maximizing corporate tax rates. For analytical tractability, we first use a simple model and show that the growth-maximizing tax rate is higher than the output-elasticity of public services, which is the optimal tax rate in the model without endogenous tax evasions of firms as the existing studies show. Besides, we show that the welfare-maximizing tax rate is higher than the growth-maximizing tax rate. It suggests that in terms of not only growth but welfare, the government should not lower the corporate tax rate so much. Next, we extend the model to obtain quantitative implications. Our quantitative analyses show that the optimal tax rate is higher than the rate which is (or will be) adopted in OECD countries for a sufficiently wide parameter region. This result is consistent with the empirical estimation by Aghion et al. (2016) mentioned below. It implies that the corporate tax rates adopted in recent years in many developed countries may be higher than the optimal ones.

Related Literature

Largely, our study lies in the literature starting from Barro (1990), which explores the optimal tax (or the optimal size of government spending) in an endogenous growth model with productive public service (capital) (e.g., Futagami et al, 1993; Glomm and Ravikumar, 1994; Turnovsky 1997). [1] These studies emphasize that the growth maximizing-income tax rate equals to the output elasticity of public capital and it coincides with the welfare maximizing one in the balanced growth path. This is so called Barro rule. This study investigates the optimal taxation theoretically and quantitatively in view of endogenous evasions of corporate tax in an R&D growth model with imperfectly competitive product market. [2]

Among the existing studies of tax evasions and growth, we should refer to Chen (2003) and Kafkalas et al. (2014). They extend Barro (1990)'s type model incorporating tax evasions and cause controversy over the rate of optimal income tax. Chen (2003) indicates that the optimal tax rate is larger than the output elasticity of productive public capital. On the other hand, Kafkalas et al. (2014) show that even with tax evasions, the optimal tax is equals to the output elasticity of productive public capital in line with Barro's rule. Although the findings from Chen (2003) and Kafkalas et al. (2014) are interesting, there are some reservations. First, they consider the tax evasions by household firm, that is, the firm is same as households. Therefore, there is no difference between corporate income tax and household's income tax, which lies under unrealistic assumption. Second, they assume exogenous tax evasions which occur necessarily at the constant rate regardless of economic environment such as fiscal policies. Third, they assume competitive goods market and therefore, tax evasions neither affect market power by firms nor cause distortionary effect on the tax structures. Our model differs from Chen (2003) and Kafkalas et al. (2014) in that we consider the corporate income tax, the firms' decision-making on tax evasions, and its distortionary effect on the optimal tax structures. [3]

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[1] Some empirical studies suggest the importance of productive public expenditure for economic growth. Abiad et al. (2016) show that increases in public investment in infrastructure raise output both in the short and long run.

[2] Agénor and Neanidis (2017) show empirically that public capital affects growth not only through productivity and innovation (R&D) capacity.

[3] There are some theoretical studies that investigate the role of public capital on growth under imperfectly competitive product market. Pereto (2007) explores this issue in an R&D based growth model where the market structure (entry and exit of firms) is endogenously determined. Pereto (2007) shows

This paper is comparable to Aghion et al. (2016). They treat the relationship among corporate taxation, growth, and welfare. Aghion et al. (2016)'s empirical analysis suggests that the relationship between corporate income tax and growth is an inverted-U shape and estimate welfare-maximizing corporate income tax rate of 42%. Based on this empirical finding, Aghion et al. (2016) construct an R&D based growth model in which public capital raises the expected returns to entrepreneurial efforts on R&D. The calibrated value of optimal corporate income tax rate in their model is 37% and is close to the estimated value of 42%. However, Aghion et al. (2016) do not consider tax evasions by firms but focus on the corruption between government and households. In this paper, we incorporate the tax evasions in an R&D based growth model with corporate taxation and suggest that the optimal corporate income tax rate is as high as Aghion et al. (2016)'s estimate when the firms choose to evade the corporate tax.

To sum up, Chen (2003) and Kafkalas et al. (2014) consider the tax evasions but do not treat corporate tax. Although Aghion et al. (2016) analyze the optimal corporate taxation, they do not incorporate tax evasions by firms. Therefore, to our best knowledge, this paper is a first one to tackle a macroeconomic analysis of corporate tax evasions.

Our study also relates to the literature on whether the Barro rule holds. Among the studies, there are no consensus. Futagami et al (1993) shows that welfare-maximizing tax rate is lower than the growth-maximizing one (the output elasticity of public capital), if the government evaluates the policy effects during the transition path caused by the accumulation of public capital. Ghosh and Roy (2004) and Agénor (2008) also find that the welfare-maximizing-tax rate is lower than the output elasticity of public infrastructure. The former considers the composite output externality between stock of public capital and flow of public services. The latter consider the public health capital in addition to the general infrastructure. On the other hand, Kalaitzidakis and Kalyvitis (2004) show that the growth- maximizing tax rate is larger than the output elasticity of public capital when the government faces the trade -off of public expenditure between new investment and maintenance of existing public capital. Chang and Chang (2015) compare

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that productive public capital is neutral to growth and therefore no optimal tax exists. On the other hand, Chang and Chang (2015) advocate that infrastructure affect growth and optimal tax rates exist in an endogenous growth model which includes monopolistic competition in the goods market and the unionization in the labor market. The recent empirical study of Agénor and Neanidis (2017) may supports the latter result.

growth-maximizing factor income taxes with welfare-maximizing ones in an endogenous growth model with market imperfection both in goods and labor markets. They show that welfare-maximizing productive government spending can be larger (lower) than the growth maximizing one if the government spending is financed by capital (labor) income taxes under reasonable parameter values. Therefore, while the existing studies on Barro rule treat neither corporate taxation nor tax evasions, we do the both in this paper.

2 Model

The economy is inhabitated by four kinds of agents, producers of final output, producers of intermediate goods, a representative household and a government. A representative household has an infinite planning horizon and perfect foresight and is endowed with L unit of labor. Labor moves freely across final goods and intermediate goods sectors.

2.1 The producers of final output

Final output is produced by perfectly competitive producers of final output in accordance with the following technology:

$$Y_t = AL_{Y,t}^{1-\alpha} \int_0^{N_t} (G_t x_{i,t})^\alpha di, \tag{1}$$

where $0 < \alpha < 1$, $A(> 0)$ is the scale parameter, Y_t is output, $L_{Y,t}$ is labor input in the final goods sector, $x_{i,t}$ is the ith type of specialized intermediate good, and N_t is the number of varieties of intermediates. Following Barro (1990), public services, G_t, increases the productivity of output. We take the final output is numeraire. Letting $p_{i,t}$ and w_t denote the price of intermediate good i and wage rate, respectively, we obtain the conditions for profit maximization as follows:

$$w_t = (1-\alpha)L_{Y,t}^{-\alpha} \int_0^{N_t} (G_t x_{i,t})^\alpha di = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{2}$$

$$p_{i,t} = \alpha AL_{Y,t}^{1-\alpha} G_t^\alpha x_{i,t}^{\alpha-1}. \tag{3}$$

Solving (2) and (3) with respect to $x_{i,t}$, we obtain the demand function for product of firm i:

$$x(p_{i,t}) = \frac{(1-\alpha)Y_t}{w_t}\left(\frac{\alpha AG_t^\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}}.$$

From (1) and (3), we obtain

$$\int_0^{N_t} p_{i,t}x_{i,t}di = \alpha Y_t. \tag{4}$$

2.2 Producers of intermediate goods

2.2.1 Entry into the intermediate goods market

In this economy, intermediate goods are produced (or invented) by monopolistically competitive firms. We assume that the economy starts with initial variety of intermediate goods $N_0 > 0$. Any firm needs to invest η unit of the final goods at time $t-1$ to invent an intermediate good and finances η through asset market. The firm that discovers a blueprint of intermediate goods and enters into the market can gain one period's monopoly profits. Such firms operate one period and exit from the market at the end of each period.

Furthermore, potential entrants/firms in the end of period $t-1$ draw their productivity b from distribution $F(b)$. We set the following assumptions in terms of b. First, b is *iid* over time as well as across firms. Second, b is private information and then other agents cannot know the value without any costs. Here, let $\pi_{i,t}^e$ denotes the expected after-tax operating profit of firm i at the beginning of period t, and it is a stochastic variable at that time because it can evade taxation stochastically when it tries. Potential investors/firms must choose whether or not to enter into the market in each period, taking account of these entry costs, η, and one period's expected operating profits, $\pi_{i,t}^e$.

In sum, the entry problem for each firm can be reduced to one of selecting, in period $t-1$, a period t product and price so as to maximize net discounted monopolistically competitive profits:

$$\Pi_{i,t-1} = \frac{1}{R_{t-1}}\int \pi_{i,t}^e dF(b) - \eta, \tag{5}$$

where R_{t-1} is the gross interest rate between periods $t-1$ and t. Free entry into the

intermediate goods market implies

$$\int \pi_{i,t}^e dF(b) = R_{t-1}\eta. \tag{6}$$

2.2.2 Maximization of operating profits

Consider next the maximization problem of operating profits in each firm. A firm that draws its productivity b needs $1/b$ unit of labor to produce one unit of intermediate good. The price of each intermediate good $p_{i,t}$ is assumed to be public information. Let $\pi_{i,t}$ and $\tilde{\pi}_{i,t}$ denote the true operating profit of the firm i and the operating profit that the firm i declares, respectively. The government impose corporate income tax on $\tilde{\pi}_{i,t}$. Under incomplete information, the government cannot know whether the declared profit of each firm, $\tilde{\pi}_{i,t}$, equals to their true profit, $\pi_{i,t}$, without any costs. This is because the productivity of each firm, b, is the private information and the government is unable to know the true profit of each firm.

Before proceeding to the case of incomplete information, let us consider the profit maximizing problem under perfect information. In the case of complete information, each firm's productivity is public information and the operating profit before tax is $\left(p_{i,t} - \frac{w_t}{b_i}\right)x(p_{i,t})$. Here, note that (3) indicates that production level of intermediate good, x_i, depends on $p_{i,t}$, which is public information, and therefore, so is the operating profit before tax. Accordingly, in the complete information economy, any firm declare true profit (i.e., $\tilde{\pi}_{i,t} = \pi_{i,t}$), otherwise the firm is punished by the government that knows true information regarding the firm's profit. The profit maximizing problem of firm i under complete information is to decide the price level, $p_{i,t}$, so as to maximize its after-tax operating profit, $(1-\tau)\left(p_{i,t} - \frac{w_t}{b_i}\right)x(p_{i,t})$, where, $\tau \in (0,1)$ is the announced corporate income tax rate. This yields $p_{i,t} = \frac{1}{\alpha b_i}w_t$ and the before-tax operating profit, $(1-\alpha)p_{i,t}x(p_{i,t})$.

Next, we move onto the profit maximizing problem under incomplete information. In the case of incomplete information, since b is private information, the government need inspection costs to know the true profit of the firm. We assume that the initial number of firm is normalized to be unity, $N_0 = 1$ and a large number of firm is distributed continuously between 0 and 1. This assumption indicates that it takes too much cost for the government to inspect all firms. Such the environment allows firms to report its profit

below the actual one ($\tilde{\pi}_{i,t} < \pi_{i,t}$) and evade corporate income tax payment. After-tax profit of the tax evading firm is represented as $\pi_{i,t} - \tau\tilde{\pi}_{i,t}$ because corporate tax is imposed on declared profit rather than on true profit. Let q represent the probability of audit and be constant over time. In the audited state, any taxable profit understatement, $\pi_{i,t} - \tilde{\pi}_{i,t}$, is subject to a penalty and is imposed an additional tax at the fixed rate $s(> 0)$. The amount of additional taxes of each firm is represented as $(1 + s)\tau[\pi_{i,t} - \tilde{\pi}_{i,t}]$. Therefore, after-tax profit of the detected firm is reduced to $\pi_{i,t} - \tau\tilde{\pi}_{i,t} - (1 + s)\tau[\pi_{i,t} - \tilde{\pi}_{i,t}]$. In the unaudited state, which takes place at the rate, $1 - p$, tax evading firms can avoid detection and after-tax profit of these firm is reduced to $\pi_{i,t} - \tau\tilde{\pi}_{i,t}$. Thus, the expected operating profit of firm i is given by

$$
\begin{aligned}
\pi_{i,t}^{e} &= (1 - q)[\pi_{i,t} - \tau\tilde{\pi}_{i,t}] + q\left\{\pi_{i,t} - \tau\tilde{\pi}_{i,t} - (1 + s)\tau[\pi_{i,t} - \tilde{\pi}_{i,t}]\right\} \\
&= \pi_{i,t} - \tau\tilde{\pi}_{i,t} - q(1 + s)\tau[\pi_{i,t} - \tilde{\pi}_{i,t}] \\
&= (1 - \tilde{\tau})\pi_{i,t},
\end{aligned}
\tag{7}
$$

where $\tilde{\tau}$ is the effective corporate tax rate and is defined as

$$
\tilde{\tau} \equiv \left\{[1 - q(1 + s)]\frac{\tilde{\pi}_{i,t}}{\pi_{i,t}} + q(1 + s)\right\}\tau,
\tag{8}
$$

taking the announced corporate tax rate, τ, as given.

Regarding the above profit maximizing problem under incomplete information, we obtain the following lemma.

Lemma. *Let $\tilde{p}_{i,t}$ denote the price level set by firm i at time t. Then, any firm declares the operating profit as*

$$
\tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}).
\tag{9}
$$

Proof: Because a firm declare the true profit (i.e., $\tilde{\pi}_{i,t} = \pi_{i,t}$), a firm maximizes the expected after-tax operating profit, $\pi_{i,t}^{e} = (1 - \tau)\pi_{i,t}$, which is derived by applying $\tilde{\pi}_{i,t} = \pi_{i,t}$ into (7) and (8). The price level set by the firm maximizing this expected profit is $\tilde{p}_{i,t} = \frac{1}{\alpha b_i}w_t$ and the operating profit declared is $\tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}) = (1 - \alpha)\frac{1}{\alpha b_i}w_t x\left(\frac{1}{\alpha b_i}w_t\right)$. Here, note the following two respects. First, both price level, $\tilde{p}_{i,t}$, and the operating profit declared, $\tilde{\pi}_{i,t}$, depend on the productivity of each firm that

the firm drew from F. Because productivity of each firm is private information, even though the price level, $\tilde{p}_{i,t}$, is public information and observable, only through $\tilde{p}_{i,t}$, the government cannot understand whether the declared profit, $\tilde{\pi}_{i,t} = (1-\alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t})$, is true one or not. In addition to this the government cannot know whether the price level of each firm is in accordance with the maximization of true profit. Second, contraposition of Proposition 1 is true, that is, if the firm that set its price level as $\tilde{p}_{i,t}$ does not declare the profit as $\tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t})$, the firm is found to be a liar. This means that if the firm that set its price level as $\tilde{p}_{i,t}$ attempts to tell a lie, the firm declare its profit as $\tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t})$ otherwise the government can know the firm as a liar immediately without costs. Thus, when the firm sets the price level as $\tilde{p}_{i,t}$ declares its profit as $\tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t})$, the two cases: *1* and *2* stated in Proposition 1 hold. $\square$

In contrast to the declared profit (9), the true profit of the firm i when its price level is set at $\tilde{p}_{i,t}$ is

$$\pi_{i,t} = \left(\tilde{p}_{i,t} - \frac{w_t}{b_i} \right) x(\tilde{p}_{i,t}). \tag{10}$$

From (9) and (10), we obtain

$$\pi_{i,t} \geq \tilde{\pi}_{i,t} \Leftrightarrow \tilde{p}_{i,t} \geq \frac{1}{\alpha b_i}w_t \tag{11}$$

(11) indicates that in the situation that the firm can set the price level as (at) $\tilde{p}_{i,t} > \frac{1}{\alpha b_i}w_t$, the firm has incentive to declare its profit, $\tilde{\pi}_{i,t}$, below true one, $\pi_{i,t}$. On the other hand, when $\tilde{p}_{i,t} = \frac{1}{\alpha b_i}w_t$, the firm cannot understate its profit and $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds. We proceed to under which condition $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds (does not bind). Let $\pi_{i,t}^{e,true}$ denote the expected after-tax operating profit when the firm declare the truth ($\pi_{i,t} = \tilde{\pi}_{i,t}$). Applying $\pi_{i,t} = \tilde{\pi}_{i,t}$ into (7) yields $\pi_{i,t}^{e,true} = (1 - \tau)\pi_{i,t}$, and therefore we obtain

$$\pi_{i,t}^e - \pi_{i,t}^{e,true} = \tau[1 - q(1 + s)](\pi_{i,t} - \tilde{\pi}_{i,t}) \tag{12}$$

Accordingly, $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds (does not bind) if $1 - q(1 + s) \leq (>)0$.

Each firm sets the price level, $\tilde{p}_{i,t}$ so as to maximize the expected operating profit (7)

subject to $\pi_{i,t} \geq \tilde{\pi}_{i,t}$. From (7), (9), (10) and (11), the first order condition is

$$
\frac{\partial \pi_{i,t}^e}{\partial \tilde{p}_{i,t}} = L_{Y,t}(\alpha A)^{\frac{1}{1-\alpha}} G_t^{\frac{\alpha}{1-\alpha}} \tilde{p}_{i,t}^{-\frac{1}{1-\alpha}}
$$
$$
\times \left\{ \frac{1 - q(1+s)\tau}{(1-\alpha)b_i} \left(-\alpha b_i + \frac{w_t}{\tilde{p}_{i,t}} \right) + [1 - q(1+s)]\tau\alpha \right\} = (<)0 \quad \text{for} \quad \tilde{p}_{i,t} > (=)\frac{1}{\alpha b_i}w_t
$$
(13)

Here, we set the following assumption.

Assumption. $\quad 1 - q(1+s)\tau > \max\{0, (1-\alpha)[1-q(1+s)]\tau\}$

Under Assumption, by (13), $\pi_{i,t}^e$ is a non-monotonic concave function of $\tilde{p}_{i,t}$ and the following unique $\tilde{p}_{i,t}(> 0)$ exists and satisfies the second order condition of maximizing (7).

$$
\tilde{p}_{i,t} = \frac{1}{\alpha b_i}\Gamma(\tau)w_t,
$$
(14)

$$
\Gamma(\tau) \equiv \frac{1 - q(1+s)\tau}{1 - q(1+s)\tau - (1-\alpha)[1-q(1+s)]\tau}.
$$

(13) and (14) results in the following two cases. First, $\Gamma(\tau) \leq 1$ holds if and only if $1 - q(1+s) \leq 0$ under which $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds and each firm declare its true operating profit. In this case, substituting $\pi_{i,t} = \tilde{\pi}_{i,t}$ and (10) both into (7) and (8), we obtain $\pi_{i,t}^e = (1 - \tau)\left(\tilde{p}_{i,t} - \frac{w_t}{b_i}\right)x(\tilde{p}_{i,t})$ and $\tilde{\tau} = \tau$, respectively. Thus, the price level of each firm is $\tilde{p}_t = \frac{1}{\alpha b}w_t$ and the effective tax rate $\tilde{\tau}$ coincides with the announced tax rate τ. Inserting $\tilde{p}_t = \frac{1}{\alpha b}w_t$ into (10) and comparing it with (9), we can confirm that the declared profit equals to the true profit:

$$
\pi_{i,t} = \tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}).
$$
(15)

Moreover, combining (15) with (7) and (8), we obtain the expected operating profit as follows:

$$
\pi_{i,t}^e = (1 - \tilde{\tau})(1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}),
$$
(16)

where, $\tilde{\tau} = \tau$.

Second, $\Gamma(\tau) > 1$ holds if and only if $1 - q(1+s) > 0$ under which each firm declare its operating profit below its true one, $\pi_{i,t} > \tilde{\pi}_{i,t}$. The price of each firm is determined by

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(14).[4] Substituting (14) into (10), we obtain the actual profit of each firm as follows:

$$\pi_t = \left(1 - \alpha\Gamma(\tau)^{-1}\right)\tilde{p}_{i,t}x(\tilde{p}_{i,t}) \tag{17}$$

Combining (17) with (7), we obtain

$$\pi_{i,t}^e = (1 - \tilde{\tau})\left(1 - \alpha\Gamma(\tau)^{-1}\right)\tilde{p}_{i,t}x(\tilde{p}_{i,t}), \tag{18}$$

where $\tilde{\tau}$ is given by (8).

We summarize the above results as the following proposition.

Proposition 1. *Maximization of the expected operating profit by each firm subject to (11) leads to the following cases:*

1. *if $1 - q(1+s) \leq 0$, $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds. Each firm set its price at $\tilde{p}_t = \frac{1}{\alpha b}w_t$ and declare its true operating profit. The declared operating profit and the expected operating profit are given by (15) and (16), respectively. In additon, the effective tax rate coinsides with the announced tax rate, $\tilde{\tau} = \tau$.*

2. *if $1 - q(1+s) > 0$, $\pi_{i,t} > \tilde{\pi}_{i,t}$ holds. Each firm sets its price in accordance with (14) and declare its operating profit below its true one. The declared operating profit is (9) whereas the true operating profit is (17). Therefore, the expected operating profit is given by (18). The effective tax rate in this case is given by (8) and the Appendix A proves that $\tau > \tilde{\tau} \in (0, \frac{1+\alpha q(1+s)}{1+\alpha})$, $d\tilde{\tau}/d\tau > 0$, $d(\tau/\tilde{\tau})/d\tau > 0$ and $d(\tau/\tilde{\tau})/d(q(1+s)) < 0$ hold.*

To understand the intuition behind proposition 1, we rewrite the first derivative of the expected operating profit with respect to $\tilde{p}_{i,t}$ as

$$\frac{\partial \pi_{i,t}^e}{\partial \tilde{p}_{i,t}} = [1 - q(1+s)\tau]\frac{\partial \pi_{i,t}}{\partial \tilde{p}_{i,t}} - [1 - q(1+s)]\tau\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}}, \tag{19}$$

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[4]When $1-q(1+s) > 0$, Assumption 1 is satisfied automatically because of $1-q(1+s)\tau > 1-q(1+s) > 0$ and $1 - q(1+s)\tau - (1-\alpha)[1 - q(1+s)]\tau = 1 - \{1 - \alpha[1 - q(1+s)]\}\tau > 0$.

where

$$\frac{\partial \pi_{i,t}}{\partial \tilde{p}_{i,t}} = L_{Y,t}(\alpha A)^{\frac{1}{1-\alpha}} G_t^{\frac{\alpha}{1-\alpha}} \tilde{p}_{i,t}^{-\frac{1}{1-\alpha}} [(1-\alpha)b_i]^{-1} \left(-\alpha b_i + \frac{w_t}{\tilde{p}_{i,t}}\right),$$

$$\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}} = -\alpha L_{Y,t}(\alpha A)^{\frac{1}{1-\alpha}} G_t^{\frac{\alpha}{1-\alpha}} \tilde{p}_{i,t}^{-\frac{1}{1-\alpha}}.$$

$\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}} < 0$ indicates that firm can understate its operating profit by raising price level $\tilde{p}_{i,t}$. When $1 - q(1+s) \leq 0$, the cost of understating the profit, $q(1+s)$, is large enough that understating the profit negatively affects the expected operating profit $\pi_{i,t}^e$ because $-[1 - q(1+s)]\tau\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}} > 0$ holds. Therefore, firm does not understate its operating profit but declare its true profit.

On the other hand, when $1 - q(1+s) > 0$, the cost of understating the profit, $q(1+s)$, is low enough that understating the profit positively affects the expected operating profit $\pi_{i,t}^e$ because $-[1 - q(1+s)]\tau\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}} > 0$ holds. Therefore, firm set the price higher ($\Gamma(\tau) > 1$ in (14)) to understate its profit and evade corporate tax payment. The second term of (19) indicates that the higher the announced corporate tax level, τ, is, the larger the incentive of understating the profit, $\tilde{\pi}_{i,t}$, by raising the price level, $\tilde{p}_{i,t}$. This reflect the fact that $\Gamma(\tau)$ is increasing in τ:

$$\Gamma'(\tau) = \frac{2(1-\alpha)[1 - q(1+s)]q(1+s)\tau}{\{1 - q(1+s)\tau - (1-\alpha)[1 - q(1+s)]\tau\}^2} > 0. \tag{20}$$

In addition, from the second term of (19), as the cost of understating the profit $q(1+s)$ is smaller, the incentive of understatement and setting higher price level become stronger. This reflects the fact that $\Gamma(\tau)$ is decreasing in $q(1+s)$:

$$\frac{\partial \Gamma(\tau)}{\partial q(1+s)} = -\frac{(1-\alpha)(1-\tau)\tau}{\{1 - q(1+s)\tau - (1-\alpha)[1 - q(1+s)]\tau\}^2} < 0.$$

2.3 Government

We assume that the revenue of the government relies only on the tax revenue from firms and the government keeps balanced budget in each period. From (7) and (8), the expected tax revenue of the government is $\int_0^{N_t} \int \{\tau\tilde{\pi}_{i,t} + q(1+s)\tau[\pi_{i,t} - \tilde{\pi}_{i,t}]\}dF(b)di = \tilde{\tau}\int_0^{N_t}\int \pi_{i,t}dF(b)di$. This revenue is allocated into productive government spending, G_t and inspection expen-

diture to detect tax evasion, M_t. Thus, the budget constraint of the government is given by

$$\tilde{\tau} \int_0^{N_t} \int \pi_{i,t} dF(b) di = G_t + M_t \tag{21}$$

Assume that spending a constant fraction, q, of government revenue on M_t, leads to succeed a detection of tax evasion by a firm at the probability q. The inspection expenditure is therefore given by

$$M_t = q\tilde{\tau} \int_0^{N_t} \int \pi_{i,t} dF(b) di. \tag{22}$$

(21) and (22) yield

$$G_t = (1-q)\tilde{\tau} \int_0^{N_t} \int \pi_{i,t} dF(b) di. \tag{23}$$

2.4 Household

The utility function of a representative household is specified as

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(C_t), \quad u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \tag{24}$$

$u(C_t) = \ln C_t$ when $\sigma = 1$. Here, C_t, $\rho(> 0)$ and $1/\sigma$ denote consumption in period t, the subjective discount rate and intertemporal elasticity of substitution, respectively. A representative household supply L unit of labor inelastically. The household's budget constraint is given by

$$W_t = R_{t-1}W_{t-1} + w_t L - C_t, \tag{25}$$

where, W_{t-1} is assets at the end of period $t-1$. Household's utility maximization yields the usual Euler equation

$$\frac{C_{t+1}}{C_t} = \left(\frac{R_t}{1+\rho} \right)^{1/\sigma}, \tag{26}$$

and the transversality condition.

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2.5 Equilibrium

Let L denote the total supply of labor. Labor market clears as

$$L = L_{Y,t} + \int_0^{N_t} \int \frac{x_{i,t}}{b} dF(b) di. \tag{27}$$

Aggregating free entry condition (6) over i and b, we obtain

$$\int_0^{N_t} \int \pi_{i,t}^e dF(b) di = R_{t-1} \eta N_t, \tag{28}$$

where aggregate entry cost/investment of intermediate goods producers/firms, ηN_t is financed by a representative household's asset at the end of period $t-1$ through asset market. Therefore, asset market clearing condition is given by

$$W_{t-1} = \eta N_t. \tag{29}$$

In the following, we describe the equilibriums under which (i) $\pi_{i,t} = \tilde{\pi}_{i,t}$ binds (i.e., $1 - q(1+s) \leq 0$) and (ii) $\pi_{i,t} > \tilde{\pi}_{i,t}$ (i.e., $1 - q(1+s) > 0$), respectively.

2.5.1 Equilibrium in the case of (i) $1 - q(1+s) \leq 0$

In this case, price level of firm i and j whose productivity is represented as b_i and b_j is $\tilde{p}_{i,t} = \frac{1}{\alpha b_i} w_t$ and $\tilde{p}_{j,t} = \frac{1}{\alpha b_j} w_t$. Combining these with (3) yields

$$\frac{p_{i,t}}{p_{j,t}} = \left(\frac{x_{i,t}}{x_{j,t}}\right)^{\alpha-1} = \frac{b_j}{b_i} \quad \Rightarrow \quad x_{i,t} = \left(\frac{b_j}{b_i}\right)^{\frac{1}{\alpha-1}} x_j \tag{30}$$

(30) together with (1),(2) and (3) rewrite $\tilde{p}_{i,t} = \frac{1}{\alpha b_i} w_t$ into

$$x_{i,t} = \frac{\alpha^2 L_{Y,t}}{(1-\alpha) b_i^{\frac{1}{\alpha-1}} \int b^{\frac{\alpha}{1-\alpha}} dF(b) N_t}. \tag{31}$$

From (27) and (31), we obtain

$$L_{Y,t} = \frac{1-\alpha}{1-\alpha+\alpha^2} L \tag{32}$$

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Substituting (32) into (31) leads to

$$x_{i,t} = \left(\frac{\alpha^2}{1-\alpha+\alpha^2}\right)\frac{b_i^{\frac{1}{1-\alpha}}}{\int b^{\frac{\alpha}{1-\alpha}}dF(b)}\frac{L}{N_t} \tag{33}$$

From (4) and (16), we obtain

$$\int_0^{N_t}\int \pi_{i,t}^e dF(b)di = (1-\tilde{\tau})(1-\alpha)\alpha Y_t, \tag{34}$$

and combining (34) with (28), we obtain

$$R_{t-1} = \frac{(1-\tilde{\tau})(1-\alpha)\alpha Y_t}{\eta N_t} \tag{35}$$

Substituting (2), (21), (29), (32) and (35) into (25) yields the resource constraint of this economy:

$$Y_t = C_t + \eta N_{t+1} + G_t + M_t, \tag{36}$$

where note that $(Y_t - \int_0^{N_t} \tilde{p}_{i,t}x_{i,t}di) + (\int_0^{N_t} \tilde{p}_{i,t}x_{i,t}di) = Y_t$ holds because no final goods are used in the production process of intermediate goods. (36) indicates that Y_t is allocated into consumption, C_t, investment in new intermediate goods producers, ηN_t, productive government spending, G_t and government inspection expenditure, M_t. Dividing both side of (36) by N_t and using (21) and (34) yield

$$\frac{N_{t+1}}{N_t} = \frac{1}{\eta}\left\{[1-\tilde{\tau}(1-\alpha)\alpha]\frac{Y_t}{N_t} - \frac{C_t}{N_t}\right\}, \tag{37}$$

and substituting (35) into (26), we obtain

$$\frac{C_{t+1}}{C_t} = \left[\frac{(1-\tilde{\tau})(1-\alpha)\alpha}{(1+\rho)\eta}\frac{Y_{t+1}}{N_{t+1}}\right]^{1/\sigma}. \tag{38}$$

Substituting (15) into (23) reduces to $G_t = (1-q)\tilde{\tau}(1-\alpha)\alpha Y_t$. Combining it with (1), (31) and (32), we obtain

$$\frac{Y_t}{N_t} = A^{\frac{1}{1-\alpha}}\{(1-q)\tilde{\tau}(1-\alpha)\alpha\}^{\frac{\alpha}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\left(\frac{L}{1-\alpha+\alpha^2}\right)^{\frac{1}{1-\alpha}}\left[\int b^{\frac{\alpha}{1-\alpha}}dF(b)\right]. \tag{39}$$

Next, let us define C_t/N_t and Y_t/N_t as $z_t \equiv C_t/N_t$ and $\Omega(\tilde{\tau}) \equiv Y_t/N_t$, respectively. From (37) and (38), we obtain

$$z_{t+1} = \frac{\eta^{1-\frac{1}{\sigma}}\{(1-\tilde{\tau})(1-\alpha)\alpha^2\Omega(\tilde{\tau})\}^{1/\sigma}z_t}{[1-\tilde{\tau}(1-\alpha)\alpha]\Omega(\tilde{\tau})-z_t} \tag{40}$$

The left-hand side (LHS) of (40) represents 45° line while the right-hand side (RHS) is monotonically convex function of z_t and takes zero when $z_t = 0$, and therefore a unique steady state exists. Note that z_t jumps to its steady state value z^* initially without transition because the steady state is unstable. The steady state value of z^* is calculated as

$$z^* = [1-\tilde{\tau}(1-\alpha)\alpha]\Omega(\tilde{\tau}) - \eta^{1-\frac{1}{\sigma}}\{(1-\tilde{\tau})(1-\alpha)\alpha^2\Omega(\tilde{\tau})\}^{1/\sigma}. \tag{41}$$

2.5.2 Equilibrium in the case of (ii) $1 - q(1+s) > 0$

In the case of (ii) $\pi_{i,t} > \tilde{\pi}_{i,t}$ (i.e., $1 - q(1+s) > 0$), price is determined as $\tilde{p}_t = \frac{1}{\alpha b}\Gamma(\tau)w_t$ according to (14). In the same manner of deriving (31), we can rewrite $\tilde{p}_t = \frac{1}{\alpha b}\Gamma(\tau)w_t$ into

$$x_{i,t} = \frac{\alpha^2 L_{Y,t}}{(1-\alpha)\Gamma(\tau)b_i^{\frac{1}{\alpha-1}}\int b^{\frac{\alpha}{1-\alpha}}dF(b)N_t}. \tag{42}$$

From (42) and (27), we obtain

$$L_{Y,t} = \frac{(1-\alpha)\Gamma(\tau)}{(1-\alpha)\Gamma(\tau)+\alpha^2}L, \tag{43}$$

$$x(\tilde{p}_{i,t}) = \left[\frac{\alpha^2}{(1-\alpha)\Gamma(\tau)+\alpha^2}\right]\frac{b_i^{\frac{1}{1-\alpha}}}{\int b^{\frac{\alpha}{1-\alpha}}dF(b)}\frac{L}{N_t}. \tag{44}$$

Substituting (18) into (4) yields

$$\int_0^{N_t}\int \pi_{i,t}^e dF(b)di = \left(1-\alpha\Gamma(\tau)^{-1}\right)\alpha Y_t. \tag{45}$$

Combining (45) with (28), we obtain

$$R_{t-1} = \frac{\left(1-\alpha\Gamma(\tau)^{-1}\right)\alpha Y_t}{\eta N_t}. \tag{46}$$

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Substituting (2), (21), (29), (32) and (46) into (25) yields resource constraint (36). Dividing both side of (36) by N_t and using (21) and (45), we obtain

$$\frac{N_{t+1}}{N_t} = \frac{1}{\eta} \left\{ \left[1 - \tilde{\tau} \left(1 - \alpha \Gamma(\tau)^{-1} \right) \alpha \right] \frac{Y_t}{N_t} - \frac{C_t}{N_t} \right\}. \tag{47}$$

Substituting (46) into (26), we obtain

$$\frac{C_{t+1}}{C_t} = \left[\frac{(1 - \tilde{\tau}) \left(1 - \alpha \Gamma(\tau)^{-1} \right) \alpha}{(1 + \rho)\eta} \frac{Y_{t+1}}{N_{t+1}} \right]^{1/\sigma}. \tag{48}$$

From (47) and (48), we find that a unique steady state exists and that there is no transition dynamics as in the case of (i).

Substituting (17) into (23) and using (4) yields $G_t = (1 - q)\tilde{\tau} \left(1 - \alpha\Gamma(\tau)^{-1} \right) \alpha Y_t$. Inserting it into (1) and using (43) (44), we obtain

$$\frac{Y_t}{N_t} = A^{\frac{1}{1-\alpha}} \left\{ (1 - q)\tilde{\tau} \left(1 - \alpha\Gamma(\tau)^{-1} \right) \alpha \right\}^{\frac{\alpha}{1-\alpha}} (1 - \alpha)\Gamma(\tau)\alpha^{\frac{2\alpha}{1-\alpha}} \left[\frac{L}{(1 - \alpha)\Gamma(\tau) + \alpha^2} \right]^{\frac{1}{1-\alpha}}$$

$$\times \left[\int b^{\frac{\alpha}{1-\alpha}} dF(b) \right]. \tag{49}$$

3 Optimal Tax Rates

In this section, we analyze optimal corporate tax rates in terms of growth and welfare.

3.1 Growth-maximizing Tax Rate

We obtain the following proposition.[5]

Proposition 2. *The growth-maximizing tax rates in case (i) and (ii) satisfy the following 1. and 2. respectively.*

1. *When each firm declares its true operating profit, the growth-maximizing effective corporate tax rate $(\tilde{\tau}^{GM})$ is consistent with the growth-maximizing announced tax rate (τ^{GM}), both of which is given by $\tilde{\tau}^{GM} = \tau^{GM} = \alpha$.*

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[5]We can show the first part immediately from (39) and (38). We provide a proof of the second part in Appendix B.

2. *When each firm understates its operating profit, both growth-maximizing effective corporate tax rate, $\tilde{\tau}^{GM}$, and growth-maximizing announced corporate tax rate, τ^{GM}, are larger than the output elasticity of public services, α. In addition, τ^{GM} becomes larger than $\tilde{\tau}^{GM}$. Thus, $\tau^{GM} > \tilde{\tau}^{GM} > \alpha$ holds.*

The first part of Proposition 2 is in line with the Barro (1990)'s growth-maximizing rule, that is, the tax rate that maximizes the long-run growth equals to the output elasticity of public services, α. This result is also consistent with that in Kafkalas et al. (2014), indicating that even with government spending for detection, the growth-maximizing tax rate coincides with Barro's one.

However, we show later that the result of $\tilde{\tau}^{GM} = \alpha$ breaks in this study when declared profit of each firm is below its true level. This is because of the following noticeable different respects between this study and Kafkalas et al. (2014). This study considers the corporate tax and endogenous tax evasion by firm under imperfect information and imperfect competitive market and whereas Kafkalas et al. (2014) consider household firm and tax base is income from final output and they assume exogenous tax evasion under the environment of perfect competition.

The result of $\tilde{\tau}^{GM} = \alpha$ is attributed to the trade-off between negative distortional effect of $\tilde{\tau}$ on the rate of return $(R_{t-1}\eta N_t)$ from investment in intermediate goods producers (ηN_t) and positive external effect of G_t on the rate of return $(R_{t-1}\eta N_t)$ from investment (ηN_t). Furthermore, from (39), we understand that N_t is the engine of endogenous growth, and therefore the structure of this model is similar to AK model of Barro (1990) type. Thus, the mechanism of deriving growth-maximizing public policy is in line with Barro (1990).

Next, we explain the intuition behind the second part of Proposition 2. To understate operating profit and evade tax, firms set the price higher than that without understatement of its profit $(\Gamma(\tau) > 1)$. This increase in pricing power by firms leads to the following effects. First, it increases the expected operating profit, $\pi_{i,t}^e$ (see (45)) and raises the return from investment, R_{t-1} (see (46)), directly. Second, an increase in pricing power by firms affects Y/N and generates some indirect effects on R_{t-1} (see (46)).

There are following effects of a rise in pricing power on Y/N. First, the higher price of intermediate goods decreases the amount of intermediate goods installed into final production (see (44)), which has negative effect on Y/N. Second, due to this decrease in

intermediate goods, labor supply moves from intermediate goods sector into final goods sector (see (43)), and therefore Y/N increases. Finally, a higher expected operating profit, $\pi_{i,t}^e(b)$ leads to raise tax revenue of the government and increase productive government expenditure, G_t (see (23)), which increases Y/N through its positive externality.

Because of $\Gamma'(\tau) > 0$ (see (20)) and $d\tau/d\tilde{\tau} > 0$ (see Proposition 1-2), the higher the effective tax rate is, the higher the price level of intermediate goods. The results from Proposition 2 indicates that positive effects on the return from investment R_{t-1} dominate the negative ones.

3.2 Welfare-maximizing Tax Rate

Next, we analyze the tax rate which maximizes social welfare. Remember that the economy jumps onto the balanced growth path at the initial period as explained in section 2. Letting the gross growth rate be $\hat{g}$, the equilibrium path of consumption is given by $C_t = \hat{g}^t C_0$. Substituting it into the life time utility function of the representative household, (24), we obtain

$$U = \frac{C_0^{1-\sigma}}{(1 - \sigma)\left[1 - (1 + \rho)^{-1}\hat{g}^{1-\sigma}\right]}, \tag{50}$$

where $1 > (1 + \rho)^{-1}\hat{g}^{1-\sigma}$ holds by the transversality condition. The social welfare is determined by the initial level of consumption and the long-run growth rate, which depend on corporate income tax rate, τ.

We obtain the following proposition. [6]

Proposition 3. *The welfare-maximizing tax rates in case (i) and (ii) satisfy the following 1. and 2. respectively.*

1. *When each firm declares its true operating profit, the welfare-maximizing announced tax rate (τ^{WM}) is higher than the growth-maximizing announced tax rate, τ^{GM}. Consequently, $\tau^{WM} > \tau^{GM} > \alpha$ holds.*

2. *Suppose that $q = 0$. Then, each firm understates its operating profit and a marginal increase in the announced tax rate at the growth-maximizing tax rate improves social welfare.*

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[6]Appendix C provides a proof.

Proposition 3 suggests that the government should keep a high rate on corporate income tax in terms of welfare. For intuitive interpretation, we focus on the marginal effect of raising tax rate at the growth-maximizing rate, τ^{GM}. At $\tau = \tau^{GM}$, because the welfare effect on the trend by a marginal increase of tax rate disappears, it affects only the initial level of consumption: $\mathrm{sign}\left\{\frac{dU}{d\tau}|_{\tau=\tau^{GM}}\right\} = \mathrm{sign}\left\{\frac{\partial C_0}{\partial \tau}|_{\tau=\tau^{GM}}\right\}$. Consider the case (i), where every firm is truth-telling: $q(1+s) > 1$. By equilibrium dynamics given by (40), assuming $N_0 = 1$ without loss of generality, we obtain

$$C_0 = \underbrace{[1 - \alpha(1-\alpha)\tau]\,\Omega(\tau)}_{=\,\text{disposable income}} - \eta \underbrace{\left[\eta^{-1}(1+\rho)^{-1}\alpha(1-\alpha)(1-\tau)\Omega(\tau)\right]^{\frac{1}{\sigma}}}_{=\,R/(1+\rho)},$$

where the first and second term will capture the income and intertemporal substitution effect respectively.[7] Because $\hat{g} = [\eta^{-1}(1+\rho)^{-1}\alpha(1-\alpha)(1-\tau)\Omega(\tau)]^{\frac{1}{\sigma}}$ by the Euler equation, (38), we obtain

$$C_0 = \frac{1 - \alpha(1-\alpha)\tau}{\eta^{-1}(1+\rho)^{-1}\alpha(1-\alpha)(1-\tau)}\hat{g}^{\sigma} - \eta\hat{g}. \tag{51}$$

By $\frac{\partial \hat{g}}{\partial \tau}|_{\tau=\tau^{GM}} = 0$, we can show $\frac{\partial U}{\partial \tau}|_{\tau=\tau^{GM}} > 0$. This indicates that a marginal increase in corporate tax rate exhibits only income effect and it is always positive at the growth-maximizing tax rate. [8] By (34), we find that a feature of corporate income tax leads to this result. The tax base of corporate income tax is the aggregate profit of the intermediate goods sector, which is smaller than aggregate income by the distributional ratio for it, $\alpha(1-\alpha)$. Raising tax rate amplifies aggregate income through productivity growth and also reduce the disposable income of household. Such a negative effect is relatively weak under corporate income taxation as mentioned above and the net marginal effect is positive, while positive and negative effects are canceled under household income taxation. One can confirm it by (51). When the tax system is household income tax, we find that the numerator of the first term of (51) is replaced by $1 - \tau$ according to the derivation. Thus, the initial level of consumption depends on only growth rate, and hence, welfare-maximization is equivalent to growth-maximization. This is the same result as Kafkalas et al. (2014) and Chen (2003). In other words, Proposition 3 is a new insight for corporate

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[7]See (34), (35), (37) and (38).

[8]Rigorously speaking, this is not enough to ensure the result of 1. of Proposition 3. Appendix C gives a formal proof.

income taxation in a general equilibrium framework.

As 2. of Proposition 3 suggests, the basic attribute dose not change when firms understate their profit. [9]

By similar algebra with (47), (48) and (49), we obtain the counterpart of (51):

$$C_0 = \frac{1 - \alpha \left[1 - \alpha\gamma(\tau)\right]\tilde{\tau}}{\eta^{-1}(1+\rho)^{-1}(1-\tilde{\tau})\left[1 - \alpha\gamma(\tau)\right]\alpha}\hat{g}^\sigma - \eta\hat{g}, \tag{52}$$

where $\gamma(\tau) = 1/\Gamma(\tau)$. Note that tax evasion by firms affects equilibrium consumption through $\gamma(\tau) \neq 1$ and $\tilde{\tau} \neq \tau$ in this case.

$$\left.\frac{\partial C_0}{\partial \tau}\right|_{\tau=\tau^{GM}} = \hat{g}^\sigma \frac{(1-\alpha\gamma(\tau))\left[1-\alpha(1-\alpha\gamma(\tau))\right]\frac{d\tilde{\tau}}{d\tau} + \alpha(1-\tilde{\tau})\gamma'(\tau)}{\eta^{-1}(1+\rho)^{-1}\alpha\left[(1-\alpha\gamma(\tau))(1-\tilde{\tau})\right]^2} \tag{53}$$

The second term of the numerator of the lefthand-side, $\alpha(1 - \tilde{\tau})\gamma'(\tau)$, is the additional effect by tax evasion of firms. Since $\gamma'(\tau) < 0$, the positive effect of raising tax rate is weakened under endogenous tax evasion. This is because each firm avoids a large reduction of profit by adjusting their price when they face a higher tax rate, which alleviates serious reduction of the (after-tax) capital income of household. Thus, the benefit from raising tax rate to increase public service is relatively small in the case of tax evasion. Nevertheless, the qualitative aspect of the result does not change because output itself is raised by the promotion of R&D investment. Note that the second part of Proposition 3 holds in the case of $q = 0$. Since the firms can not be accused in this case, the incentive to evade the tax is very strong. Hence, the strength of the effect mentioned above is at maximum for $q = 0$. The second assertion of Proposition 3 indicates that the similar result to the first assertion will also be true under the tax evasion even when it is most difficult to hold. We confirm this in the next section by a numerical analysis of an extended model.

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[9]Here, we should refrain from stating that a perfectly same result as the first assertion holds because we can not show it in mathematically rigorous ways in the case of tax evasion. However, we will illustrate $\tau^{WM} > \tau^{GM}$ in such a case by a numerical example in Section 1. Besides, we could not find any counterexample for a sufficiently wide parameter region. So that we conjecture that the same result as 1. may be true.

4 Quantitative Analysis

In this section, we explore some quantitative implications of the theoretical model.

4.1 An Extended Model

We start with a small revision of the model because the problematic restriction on the parameter lies in the previous form of production technology. It is that the output-elasticity of public services α must be set to be consistent with the elasticity of substitution between intermediate inputs $1/(1-\alpha)$. To resolve this, we change the production technology (1) into

$$Y_t = AL_{Y,t}^{1-\alpha} \int_0^{N_t} (a(G_t, N_t)x_{i,t})^\alpha di, \tag{54}$$

where

$$a(G_t, N_t) = G_t^\epsilon N_t^{1-\epsilon}, \qquad 0 < \epsilon < 1. \tag{55}$$

The composite externality (55) represents a combination of the role of knowledge spillover as in Benassy (1998), together with productive public services as in Barro (1990). This representation of the composite externality can be justified in following two reasons. First, as will become evident below and as stated by Chatterjee and Turnovsky (2012), it helps provide a plausible calibration of the aggregate economy, something that is generically problematic in the conventional one-sector endogenous growth model.[10] Under (54) and (55), the output-elasticitiy of public services is $\beta \equiv \alpha\epsilon(< \alpha)$, which differentiates α from the output-elasticity of public services. Second, all theoretical results obtained above are unchanged by this transformation of the production function:

Remark. *The qualitative results do not change in the extended model.*

1. *By replacing α with β, Proposition 2 holds:*

   - *When $1 - q(1 + s) \le 0$, $\tau^{GM} = \tilde{\tau}^{GM} = \beta$.*

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[10]Chatterjee and Turnovsky (2012) consider the composite externality from physical capital as in Romer (1986) and productive public spending as in Barro (1990) and Futagami, Morita and shibata (1993) and conduct numerical analyses including growth and welfare effects. Our application of (55) is in line with Chatterjee and Turnovsky (2012) because in our model, the stock of N_t replaces the role of physical capital and is both the engine of growth and the source of spillover and positive social returns to variety as discussed in Romer (1986) and applied in R&D-based growth models as in Agion and Howit (1998), Benassy (1998), Reretto (2007) and others.

	benchmark	source
ψ	1.059	Axtell (2001)
α	.8689	markup rate = .2
ϵ	.1150	$\partial \ln Y / \partial \ln G = .1$
τ	.3436	average of OECD countries
q	.089	Fullerton and Karayannis (1994)
s	.5	Fullerton and Karayannis (1994)
σ	1.5	Jones et al. (1993)
ρ	.0204	Jones et al. (1993)
L	1	normalization
$A^{\frac{1}{1-\alpha\epsilon}}/\eta$	34.1870	growth rate = .02

Table 1: Baseline Parameter Value

- *When $1 - q(1+s) > 0$, $\tau^{GM} > \tilde{\tau}^{GM} > \beta$.*

2. The same results as Proposition 3 hold.

Therefore, even though we take an additional externality (the spillover of knowledge) into account, the basic property of the benchmark model is maintained.

4.2 Calibration and Results

To conduct numerical exercises, we set the baseline parameter value as in Table 1. Appendix provides the detail of our calibration.

The distribution of firms' productivity is determined to make the curvature of the distribution function of firm sizes in the model equal that of the Pareto distribution estimated with US data by Axtell (2001). This requires $\psi = 1.059$. We choose $\alpha = 0.8689$ for the markup rate of firms to be 20%, which is a standard value of markup rate of firms with market power (eg. Rotemberg and Woodford (1999)). We set the parameter to measure the knowledge spillover, ϵ, to 0.1150 in such a way that the elasticity of output to input to public service equals 0.1. Although the estimates of the elasticity vary in some range in the empirical studies, 0.1 is one of the reasonable values. [11] Because the quantitative result is not sensitive to the elasticity, we fix it to the baseline value. We have to set a

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[11]From an empirical point of view, the output elasticity of infrastructure (or productive public services) has estimated and examined using data from many countries. Recent empirical studies (e.g., Röller and Waverman, 2001; Shioji, 2001; Esfahani and Ramírez, 2003; Kamps, 2006; Bom and Ligthart, 2014) indicate that the output elasticity of infrastructure (or productive public services) lies in the range 0.1–0.2, on average. More recent studies by Bom and Ligthart, 2014 and Caldeón et al. (2015) commonly indicates that the output elasticity of infrastructure is around 0.1.

baseline value of announced tax rate, τ. Since this model economy grows endogenously, the balanced growth rate depends on τ. We put $\tau = 0.3436$, which is the average tax rate of OECD countries adopted by Chen (2003). Of course, the announced tax rates in OECD countries rapidly fall in recent years as Figure 1 shows, but we use the above old value by the following reasons. First, we take the baseline value of detective rate and penalty tax rate, (q, s) from Fullerton and Karayannis (1994) as in Chen (2003). Second, the value of τ does not affect strongly the levels of growth- and welfare-maximizing tax rates. So that, the choice of the baseline value of τ makes little difference in the quantitative implications of our numerical exercises. We set $\sigma = 1.5$ and $\rho = 0.0204$ according to Jones et al. (1993). These are the standard values used in quantifying growth models. We set $L = 1$ for normalization. Finally, we choose the scale parameter, A, and the cost of developing one intermediate good, η such that the balanced growth rate equals 2%.
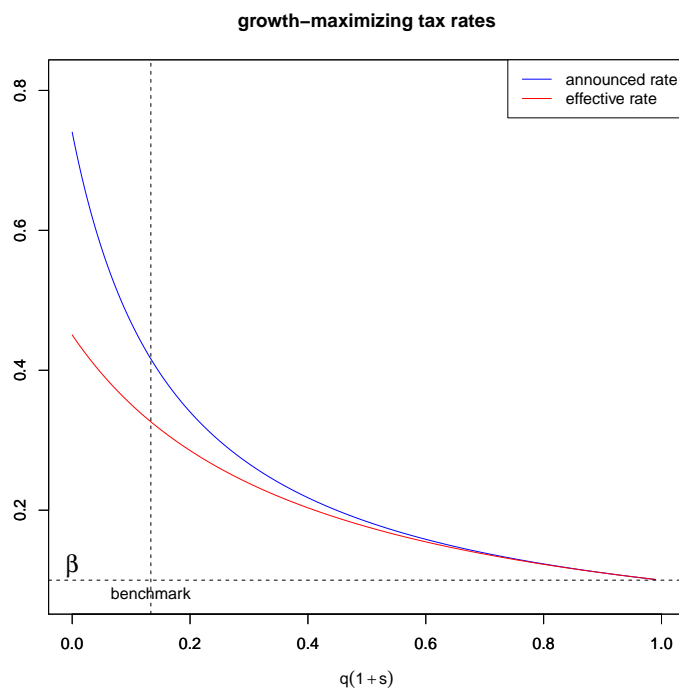

Figure 3: The relationship between the expected penalty rate and growth-maximizing tax rates

We compute the growth-maximizing announced and effective tax rates, τ^{GM} and $\tilde{\tau}^{GM}$. Figure 3 illustrates the result. Because the growth-maximizing tax rates depend on (not q and s separately but) the expected penalty rate to tax evasion, $q(1 + s)$, we show the

relationship between $q(1 + s)$ and τ^{GM} (and $\tilde{\tau}^{GM}$): See the proof of Proposition 2. The growth-maximizing tax rates are decreasing in the expected penalty rate. This is because the opportunity for tax evasions becomes smaller as the expected penalty rate become higher. In such a case, the positive effect of raising tax rate, which is explained in Section 3.1, is very limited. Thus, the difference between the growth-maximizing announced and effective tax rates is small. The growth-maximizing tax rates converge to the value of the output-elasticity of public service as $q(1 + s)$ goes to 1. This means the continuity of equilibrium allocation at the threshold between the cases of truth-telling and understatements, $q(1 + s) = 1$.
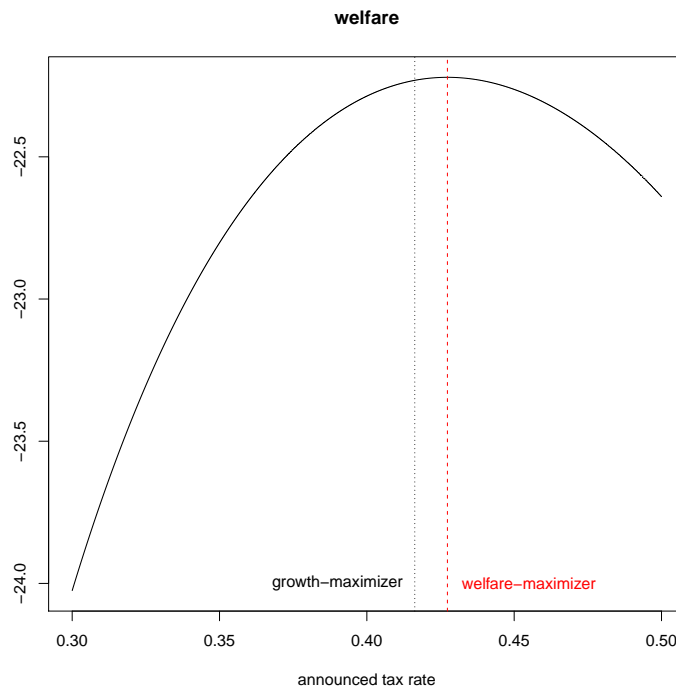

Figure 4: The welfare-maximizing tax rate

Next, we analyze the welfare-maximizing announced tax rate. Figure 4 shows the relationship between announced tax rate and social welfare for the benchmark parameter value. Social welfare is a single-peaked function of tax rate in our model as usual economic models. As Remark suggests, the welfare-maximizing tax rate is higher than the growth-maximizing tax rate. The difference between them is 0.0107 in this benchmark case, and we find it does not change so much according to parameter values. It is not surprising

markup rate	.1	.2	.3	.4	.5	.75
τ^{GM}	.5419	.4162	.3437	.2961	.2625	.2105
growth gain	.0074	.0014	.00003	.0010	.0034	.0127
τ^{WM}	.5608	.4273	.3518	.3031	.2689	.2165
$\tau^{WM} - \tau^{GM}$	.0188	.0107	.0082	.0070	.0064	.0060

Table 2: Summary of Quants (benchmark: markup rate = .2)

that the difference is small for various parameter values. The growth-maximizing tax rate optimizes the trend of the path of consumption. When the government set the tax rate far from the growth-maximizing rate, then the trend is strongly affected and it decreases equilibrium welfare to a large extent even though the initial level of consumption is raised: See Table 2 below.

Finally, we consider the quantitative implications. The markup rate is the key parameter which can make a large difference in the quantitative aspect of in this model. Remember that the theoretical key insights of our analysis come from the firms' tax evasions together with their pricing power. The policy effects primarily depends on the markup rate. Therefore, we conduct the numerical exercises for the various markup rates. We focus on the growth- and welfare maximizing tax rate, the difference between them, and the growth gain by maximizing growth rate.[12] Table 2 provides a summary of quants.

The growth gain by optimizing corporate tax is 0.14% for benchmark parameter value. Since the growth rate in our model is the long-run growth rate, this gain seems not small. This indicates that adjusting corporate income tax rate matters quantitatively.

For the benchmark parameter value, the growth- and welfare-maximizing tax rates are 41.62% and 42.73% respectively. These values are much higher than the actual rates in recent years in OECD countries but consistent with the estimate of Aghion et al. (2016). To generate a welfare-maximizing tax rate as low as the average level of OECD countries, markup rate is required to be higher than 50% according to Table 2, which is an unrealistic level. So that, by this calibrated model, we obtain a quantitative implication

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[12]We define the growth gain as the difference between the maximized growth rate and benchmark growth rate, 2%.

that we should reflect the monotonically decreasing trend of corporate income tax rate in the OECD countries.

5   Conclusion

This study investigates the optimal corporate taxation in an endogenous growth model with endogenous tax evasion strategies by firms that are engaged in R&D activity and have pricing power of intermediate goods. In our model, productive government services (or infrastructure) increase output and marginal returns of R&D firms but its revenue is under the influence of tax evasion strategies by R&D firms because it is financed through corporate income tax. We show first that the growth-maximizing corporate tax rate exists, which is larger than the output elasticity of productive public services. Second, the welfare-maximizing corporate tax rate are larger than the growth-maximizing one. Thus, the optimal point of view, corporate tax should be larger than the output elasticity of infrastructure. Our numerical exercises with plausible parameter values indicate that the optimal level of corporate income tax rate is far larger than the level which is (or will be) adopted in OECD countries. This result is also in line with the estimated value of around 40% by Agion et al. (2016).

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Appendix

A   Proof of (a) $\tau > \tilde{\tau} \in (0, \frac{1+\alpha q(1+s)}{1+\alpha})$, (b) $d(\tau/\tilde{\tau})/d\tau > 0$, (c) $d(\tau/\tilde{\tau})/d(q(1+s)) < 0$ and (d) $d\tilde{\tau}/d\tau > 0$.

Let us define

$$\gamma(\tau) \equiv \Gamma(\tau)^{-1} = 1 - \frac{(1-\alpha)[1-q(1+s)]\tau}{1-q(1+s)\tau}(<1). \tag{A.1}$$

Substituting (9) and (17) into (8), we have

$$\frac{\tilde{\tau}}{\tau} = [1 - q(1+s)]\frac{1-\alpha}{1-\alpha\gamma(\tau)} + q(1+s) \tag{A.2}$$

First, (a) $\frac{\tilde{\tau}}{\tau} < 1$ is obvious because of

$$[1 - q(1+s)]\frac{1-\alpha}{1-\alpha\gamma(\tau)} + q(1+s) - 1 = [1 - q(1+s)]\left(\frac{1-\alpha}{1-\alpha\gamma(\tau)} - 1\right) < 0$$

where, $\gamma(\tau) < 1$ and $1 - q(1+s) > 0$.

From the definition of $\gamma(\tau)$ and (A.2), $\lim_{\tau \to 0} \tilde{\tau} = 0$ and $\lim_{\tau \to 1} \tilde{\tau} = \frac{1+\alpha q(1+s)}{1+\alpha}$, and therefore we have $\tilde{\tau} \in (0, \frac{1+\alpha q(1+s)}{1+\alpha})$.

Second, (b) $d(\tau/\tilde{\tau})/d\tau > 0$ is obvious from the following reason. From the definition of $\gamma(\tau)$, we obtain

$$\gamma'(\tau) = -\frac{(1-\alpha)[1-q(1+s)]}{[1-q(1+s)\tau]^2} < 0. \tag{A.3}$$

(A.2) and (A.3) lead to $d(\tau/\tilde{\tau})/d\tau > 0$.

Third, we prove (d) $d(\tau/\tilde{\tau})/d(q(1+s)) < 0$. From (A.2),

$$\frac{d(\tilde{\tau}/\tau)}{d(q(1+s))} = 1 - \frac{1-\alpha}{1-\alpha\gamma(\tau)} + [1 - q(1+s)]\frac{d\left(\frac{1-\alpha}{1-\alpha\gamma(\tau)}\right)}{d(q(1+s))}, \tag{A.4}$$

where $1 - \frac{1-\alpha}{1-\alpha\gamma(\tau)} > 0$ and $\frac{d\left(\frac{1-\alpha}{1-\alpha\gamma(\tau)}\right)}{d(q(1+s))} > 0$ because $\gamma(\tau) < 1$ and $d\gamma(\tau)/dq(1+s) > 0$. Therefore, $d(\tau/\tilde{\tau})/d(q(1+s)) < 0$.

Finally, we prove (d) $d\tilde{\tau}/d\tau > 0$. From (A.2),

$$\frac{d\tilde{\tau}}{d\tau} = \frac{[1-q(1+s)](1-\alpha)}{[1-\alpha\gamma(\tau)]^2}[1 - \alpha\gamma(\tau) + \alpha\tau\gamma'(\tau)] + q(1+s) \tag{A.5}$$

In the following, we show that $\frac{d\tilde{\tau}}{d\tau} > 0$ for any $\tau \in (0,1)$. From (A.5), $\frac{d\tilde{\tau}}{d\tau} > 0$ if and only if

$$1 - \alpha\gamma(\tau) + \alpha\tau\gamma'(\tau) > -\frac{[1 - \alpha\gamma(\tau)]^2 q(1+s)}{[1 - q(1+s)](1-\alpha)} \qquad (A.6)$$

From $\gamma'(\tau) < 0$ and $\gamma''(\tau) < 0$,

$$\frac{d}{d\tau}[1 - \alpha\gamma(\tau) + \alpha\tau\gamma'(\tau)] = \alpha\tau\gamma''(\tau) < 0 \qquad (A.7)$$

This indicates that the LHS of (A.6) is decreasing in τ. Furthermore, it is obvious that the RHS of (A.6) is decreasing in τ. Thus, $\frac{d\tilde{\tau}}{d\tau} > 0$ for any $\tau \in (0,1)$ if the minimum value of the LHS of (A.6), $1 - \alpha\gamma(1) + \alpha\tau\gamma'(1)$, is larger than the maximum value of the RHS, $-\frac{[1-\alpha\gamma(0)]^2 q(1+s)}{[1-q(1+s)](1-\alpha)}$. By using $\gamma(0) = 1$, $\gamma(1) = \alpha$ and $\gamma'(1) = -\frac{1-\alpha}{1-q(1+s)}$, we obtain

$$1 - \alpha\gamma(1) + \alpha\tau\gamma'(1) - \left(-\frac{[1 - \alpha\gamma(0)]^2 q(1+s)}{[1 - q(1+s)](1-\alpha)} \right) = \frac{(1-\alpha)[1 - \alpha q(1+s)]}{1 - q(1+s)} > 0, \quad (A.8)$$

and therefore $\frac{d\tilde{\tau}}{d\tau} > 0$ for any $\tau \in (0,1)$.

B Proof of 2. of Proposition 2

From (49) and (48), growth maximization with respect to $\tilde{\tau}$ is equivalent to

$$\max_{\tilde{\tau}} \ f(\tilde{\tau}) = \ln(1 - \tilde{\tau})\tilde{\tau}^{\frac{\alpha}{1-\alpha}}(1 - \alpha\gamma(\tau))^{\frac{1}{1-\alpha}}\gamma(\tau) \left(\frac{\gamma(\tau)}{1 - \alpha + \alpha^2\gamma(\tau)} \right)^{\frac{1}{1-\alpha}}, \qquad (B.1)$$

subject to (A.2): $\frac{\tilde{\tau}}{\tau} = [1 - q(1+s)]\frac{1-\alpha}{1-\alpha\gamma(\tau)} + q(1+s)$. The first derivative of $f(\tilde{\tau})$ is

$$f'(\tilde{\tau}) = \left[\underbrace{-\frac{1}{1-\tilde{\tau}} + \frac{\alpha}{1-\alpha}\frac{1}{\tilde{\tau}}}_{\equiv \Psi_1(\tilde{\tau})} \right] + \frac{\alpha}{1-\alpha}\frac{d\tau}{d\tilde{\tau}}\gamma'(\tau) \left[\underbrace{\frac{1}{\gamma(\tau)} - \frac{1}{1-\alpha\gamma(\tau)} - \frac{\alpha}{1-\alpha+\alpha^2\gamma(\tau)}}_{\equiv \Psi_2(\tau)} \right].$$

$$(B.2)$$

It is obvious that $\Psi_1(\tilde\tau) = -\frac{1}{1-\tilde\tau} + \frac{\alpha}{1-\alpha}\frac{1}{\tau} = \frac{\alpha-\tilde\tau}{(1-\tilde\tau)(1-\alpha)\tilde\tau} \geq 0$ for $\tilde\tau \leq \alpha$. We next show that the sign of $\Psi_2(\tau)$ is negative for $\tilde\tau \leq \alpha$.

$$\mathrm{sign}\Psi_2(\tau) = [1-\alpha\gamma(\tau)][1-\alpha+\alpha^2\gamma(\tau)] - \gamma(\tau)[1-\alpha+\alpha^2\gamma(\tau)] - \alpha\gamma(\tau)[1-\alpha\gamma(\tau)]$$
$$= -\alpha^3\gamma(\tau)^2 - (1-\alpha)[(1+2\alpha)\gamma(\tau)-1] \tag{B.3}$$

Here, $\mathrm{sign}\Psi_2(\tau)$ satisfies the following properties: (I) and (II). (I) $\mathrm{sign}\Psi_2(\tau)$ is quadratic function with respect to $\gamma(\tau)$ and $\mathrm{sign}\Psi_2(\tau) = 0$ has a unique solution $\gamma(\tau)^*$ for $\gamma(\tau) > 0$ that satisfies $\Psi_2'(\gamma(\tau)^*) < 0$. (II) $\mathrm{sign}\Psi_2(\tau) < 0$ for $\gamma(\tau) > \frac{1}{1+2\alpha}$.

Furthermore, (A.1) and (A.2) indicate that $\gamma(\tau)$ is increasing in $q(1+s)$ and when $q(1+s) = 0$, $\gamma(\tau) = 1-(1-\alpha)\tau$, $\tau = \frac{\tilde\tau}{1-\alpha\tilde\tau}$ and $\gamma(\frac{\tilde\tau}{1-\alpha\tilde\tau}) = \frac{1-\tilde\tau}{1-\alpha\tilde\tau}$ hold. From $\frac{1-\tilde\tau}{1-\alpha\tilde\tau} - \frac{1}{1+2\alpha} = \frac{\alpha-\tilde\tau+\alpha(1-\tilde\tau)}{(1-\alpha\tilde\tau)(1+2\alpha)} > 0$, we obtain $\gamma(\frac{\tilde\tau}{1-\alpha\tilde\tau}) = \frac{1-\tilde\tau}{1-\alpha\tilde\tau} > \frac{1}{1+2\alpha}$ for $\tilde\tau \leq \alpha$. Thus, $\mathrm{sign}\Psi_2(\tau) < 0$ for $\tilde\tau \leq \alpha$. Combining $\Psi_1(\tilde\tau) \leq 0$ and $\Psi_2(\tilde\tau) < 0$ for $\tilde\tau \leq \alpha$ with $\gamma'(\tau) < 0$ ((A.2)) and $\frac{d\tilde\tau}{d\tau} > 0$ (Proposition 1-2), we obtain $f'(\tilde\tau) > 0$ for $\tilde\tau^{GM} > \alpha$. From the discussion so far, we find that if the growth-maximizing effective tax rate, $\tilde\tau^{GM}$, exists, $\tilde\tau^{GM} > \alpha$ holds.

Next, we derive the condition under which the existence of τ^{GM} is ensured. From the definitions of $\Psi_1(\tau)$ and $\Psi_2(\tau)$ as well as $\gamma(1) = \alpha$, $\gamma'(1) = -\frac{1-\alpha}{1-q(1+s)}$ and (A.5), we obtain

$$\Psi_1\left(\frac{1+\alpha q(1+s)}{1+\alpha}\right) = \frac{(1+\alpha)[\alpha-1+\alpha\{\alpha-q(1+s)\}]}{\alpha(1-\alpha)[1-q(1+s)][1+\alpha q(1+s)]} \tag{B.4}$$

$$\Psi_2(1) = \frac{(1-\alpha^2)(1-\alpha+\alpha^3)-\alpha}{\alpha(1-\alpha^2)(1-\alpha+\alpha^3)} \tag{B.5}$$

$$\lim_{\tau\to 1}\frac{d\tilde\tau}{d\tau} = \lim_{\tau\to 1}\frac{1}{d\tau/d\tilde\tau} = \frac{(1+\alpha)^2}{1+\alpha(1+\alpha)q(1+s)}, \tag{B.6}$$

From (B.2), (B.4), (B.5), (B.6) and $\gamma'(1) = -\frac{1-\alpha}{1-q(1+s)}$,

$$f'\left(\frac{1+\alpha q(1+s)}{1+\alpha}\right) < 0$$

if and only if

$$\frac{\alpha-1+\alpha\{\alpha-q(1+s)\}}{1+\alpha q(1+s)} - \frac{\alpha\{(1-\alpha^2)(1-\alpha+\alpha^3)-\alpha\}}{(1-\alpha+\alpha^3)[1+\alpha(1+\alpha)q(1+s)]} < 0 \tag{B.7}$$

Therefore, if (B.7) is satisfied, at least a growth maximizing effective tax rate, $\tilde{\tau}^{GM}$, in $\tilde{\tau} \in (\alpha, \frac{1+\alpha q(1+s)}{1+\alpha})$ exists.

C Proof of Proposition 3

Proof of 1.

The maximization condition of social welfare is $\frac{\partial U}{\partial \tau} = 0$. By (50), this is equivalent to

$$\left[1 - (1+\rho)^{-1}\hat{g}^{1-\sigma}\right]\frac{\partial C_0}{\partial \tau} + (1+\rho)^{-1}C_0\hat{g}^{\sigma}\frac{\partial \hat{g}}{\partial \tau} = 0. \tag{C.1}$$

Besides, by (51),

$$\frac{\partial C_0}{\partial \tau} = \frac{\eta(1+\rho)}{\alpha(1-\alpha)}\left[\frac{1-\alpha(1-\alpha)}{(1-\tau)^2}\hat{g}^{\sigma} + \frac{1-\alpha(1-\alpha)\tau}{1-\tau}\sigma\hat{g}^{\sigma-1}\frac{\partial \hat{g}}{\partial \tau}\right] - \eta\frac{\partial \hat{g}}{\partial \tau}. \tag{C.2}$$

Substituting (C.2) into (C.1) and rearranging it, we have

$$\frac{\partial \hat{g}}{\partial \tau} = -\frac{\eta\left[1 - (1+\rho)^{-1}\hat{g}^{1-\sigma}\right]\frac{(1+\rho)[1-\alpha(1-\alpha)]}{\alpha(1-\alpha)(1-\tau)^2}}{K}, \tag{C.3}$$

where

$$K = \eta\left[1 - (1+\rho)^{-1}\hat{g}^{1-\sigma}\right]\left[\frac{(1+\rho)[1-\alpha(1-\alpha)\tau]}{\alpha(1-\alpha)(1-\tau)}\sigma\hat{g}^{\sigma-1} - 1\right] + (1+\rho)^{-1}C_0\hat{g}^{-\sigma}.$$

Substituting (51) into the above equation, we obtain

$$K = \eta\left[\left[1 - (1+\rho)^{-1}\hat{g}^{1-\sigma}\right]\frac{(1+\rho)[1-\alpha(1-\alpha)\tau]}{\alpha(1-\alpha)(1-\tau)}\sigma\hat{g}^{\sigma-1} + \underbrace{\frac{1-\alpha(1-\alpha)\tau}{\alpha(1-\alpha)(1-\tau)}}_{>1} - 1\right] > 0.$$

Therefore, by (C.3), we find that

$$\left.\frac{\partial \hat{g}}{\partial \tau}\right|_{\tau=\tau^{WM}} < 0.$$

It implies $\tau^{GM} < \tau^{WM}$ because $\hat{g}$ is a single-peaked function of τ (see (38) and (39)).

Proof of 2.

Since sign$\{\frac{dU}{d\tau}|_{\tau=\tau^{GM}}\}$ = sign$\{\frac{\partial C_0}{\partial \tau}|_{\tau=\tau^{GM}}\}$, we show $\frac{\partial C_0}{\partial \tau}|_{\tau=\tau^{GM}} > 0$ for $q = 0$. By (52), it holds when $J_0 > 0$, which is defined by

$$J_0 = (1 - \alpha\gamma(\tau))[1 - \alpha(1 - \alpha\gamma(\tau))]\frac{d\tilde{\tau}}{d\tau} + \alpha(1 - \tilde{\tau})\gamma'(\tau). \tag{C.4}$$

Through simple algebra, we have $\gamma(\tau) = 1 - (1 - \alpha)\tau$ and $\tilde{\tau} = \left[\frac{1-\alpha}{1-\alpha\gamma(\tau)}\right]\tau$. Hence, utilizing these, (C.4) can be rearranged into $J_0 = \bar{J}_0 \cdot J(\tau)$, where

$$\bar{J}_0 = \frac{[1 - \alpha(1 - \alpha\gamma)](1 - \alpha)}{(1 - \alpha\gamma(\tau))[1 - \alpha(1 - \alpha)(1 - \alpha\tau)]} > 0,$$

$$J(\tau) = (1 - \alpha)^2[1 - \alpha + \alpha(1 + \alpha)\tau]. \tag{C.5}$$

Equation (C.5) ensures that $J(\tau) > 0$ for any $\tau \in [0, 1]$. This completes the proof.

D   Proof of Remark

Under the production technology of the final goods: (54) and (55), the wage rate: (2) and the price of intermediate goods: (3) are rewritten into $w_t = (1-\alpha)L_{Y,t}^{-\alpha}\int_0^{N_t}(a(G_t, N_t)x_{i,t})^\alpha di = (1 - \alpha)\frac{Y_t}{L_{Y,t}}$ and $p_{i,t} = \alpha A L_{Y,t}^{1-\alpha}a(G_t, N_t)^\alpha x_{i,t}^{\alpha-1}$, respectively. Other equations remain unchanged with the exceptions as follows. $\frac{\partial \pi_{i,t}}{\partial \tilde{p}_{i,t}}$ and $\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}}$ in (19) are changed into

$$\frac{\partial \pi_{i,t}}{\partial \tilde{p}_{i,t}} = L_{Y,t}(\alpha A)^{\frac{1}{1-\alpha}}a(G_t, N_t)^{\frac{\alpha}{1-\alpha}}\tilde{p}_{i,t}^{-\frac{1}{1-\alpha}}[(1 - \alpha)b]^{-1}\left(-\alpha b + \frac{w_t}{\tilde{p}_{i,t}}\right),$$

$$\frac{\partial \tilde{\pi}_{i,t}}{\partial \tilde{p}_{i,t}} = -\alpha L_{Y,t}(\alpha A)^{\frac{1}{1-\alpha}}a(G_t, N_t)^{\frac{\alpha}{1-\alpha}}\tilde{p}_{i,t}^{-\frac{1}{1-\alpha}}.$$

In the case of (i) $1 - q(1 + s) \leq 0$, (39) changes into

$$\frac{Y_t}{N_t} = A^{\frac{1}{1-\beta}}\{(1 - q)\tilde{\tau}(1 - \alpha)\alpha\}^{\frac{\beta}{1-\beta}}(1 - \alpha)^{\frac{1-\alpha}{1-\beta}}\alpha^{\frac{2\alpha}{1-\beta}}\left(\frac{L}{1 - \alpha + \alpha^2}\right)^{\frac{1}{1-\beta}}\left[\int_b b^{\frac{\alpha}{1-\alpha}}dF(b)\right]^{\frac{1-\alpha}{1-\beta}} \tag{D.1}$$

This results in transforming (40) and (41) into $z_{t+1} = \frac{\eta^{1-\frac{1}{\sigma}}\{\beta(1-\tilde{\tau})(1-\alpha)\alpha\Omega(\tilde{\tau})\}^{1/\sigma}z_t}{[1-\tilde{\tau}(1-\alpha)\alpha]\Omega(\tilde{\tau})-z_t}$ and $z^* = [1 - \tilde{\tau}(1 - \alpha)\alpha]\Omega(\tilde{\tau}) - \eta^{1-\frac{1}{\sigma}}\{\beta(1 - \tilde{\tau})(1 - \alpha)\alpha\Omega(\tilde{\tau})\}^{1/\sigma}$, respectively.

In the case of (ii) $1 - q(1 + s) > 0$, (49) is changed into

$$\frac{Y_t}{N_t} = A^{\frac{1}{1-\beta}} \left\{ (1-q)\tilde{\tau} \left(1 - \alpha\Gamma(\tau)^{-1}\right) \alpha \right\}^{\frac{\beta}{1-\beta}} \left\{ (1-\alpha)\Gamma(\tau) \right\}^{\frac{1-\alpha}{1-\beta}} \alpha^{\frac{2\alpha}{1-\beta}} \left(\frac{L}{(1-\alpha)\Gamma(\tau) + \alpha^2} \right)^{\frac{1}{1-\beta}}$$

$$\times \left[\int_b b^{\frac{\alpha}{1-\alpha}} dF(b) \right]^{\frac{1-\alpha}{1-\beta}} \tag{D.2}$$

This results in that $\tilde{\tau}^{GM} = argmax(1 - \tilde{\tau})\tilde{\tau}^{\frac{\beta}{1-\beta}}(1 - \alpha\gamma(\tau))^{\frac{1}{1-\beta}}\gamma(\tau)^{-\frac{1-\alpha}{1-\beta}} \left(\frac{\gamma(\tau)}{1-\alpha+\alpha^2\gamma(\tau)} \right)^{\frac{1}{1-\beta}}$.
(B.7) in Apendix B is changed into

$$\frac{\beta - 1 + \alpha\{\beta - q(1+s)\}}{1 + \alpha q(1+s)} - \frac{\alpha\{(1-\alpha^2)(1-\alpha+\alpha^3) - \alpha\}}{(1-\alpha+\alpha^3)[1+\alpha(1+\alpha)q(1+s)]}, \tag{D.3}$$

because (B.2) changes as

$$f'(\tilde{\tau}) = \left[\underbrace{-\frac{1}{1-\tilde{\tau}} + \frac{\beta}{1-\beta}\frac{1}{\tilde{\tau}}}_{\equiv\tilde{\Psi}_1(\tilde{\tau})} \right] + \frac{\alpha}{1-\alpha}\frac{d\tau}{d\tilde{\tau}}\gamma'(\tau) \left[\underbrace{\frac{1}{\gamma(\tau)} - \frac{1}{1-\alpha\gamma(\tau)} - \frac{\alpha}{1-\alpha+\alpha^2\gamma(\tau)}}_{\equiv\Psi_2(\tau)} \right],$$
(D.4)

where $\tilde{\Psi}_1 = \frac{\beta - \tilde{\tau}}{(1-\tilde{\tau})(1-\alpha)\tilde{\tau}} \geq 0$ for $\tilde{\tau} \leq \beta$ and $sign\Psi_2(\tau) < 0$ for $\tilde{\tau} \leq \beta$ because $\gamma(\frac{\tilde{\tau}}{1-\alpha\tilde{\tau}}) = \frac{1-\tilde{\tau}}{1-\alpha\tilde{\tau}} > \frac{1}{1+2\alpha}$ for $\tilde{\tau} \leq \beta$. and $\tilde{\Psi}_1 \left(\frac{1+\alpha q(1+s)}{1+\alpha} \right) = \frac{(1+\alpha)[\beta - 1 + \alpha\{\beta - q(1+s)\}]}{\alpha(1-\beta)[1-q(1+s)][1+\alpha q(1+s)]}$.

E A detail of calibration

We seek to obtain quantitative implications for the case of tax evasions for OECD countries.

- The distribution of productivity is set in such a way that the distribution of firm size is set to the Pareto distribution which is estimated by Axtell (2001).

  By (42) and (43), letting $N_t = 1$, we have

  $$\frac{x_i}{b_i} = \underbrace{\left[\frac{\alpha^2}{(1-\alpha)\Gamma(\tau) + \alpha^2} \right] \frac{L}{\int b^{\frac{\alpha}{1-\alpha}} dF(b)}}_{\equiv B} b_i^{\frac{\alpha}{1-\alpha}},$$

  where $\Gamma(\tau) = \frac{1-q(1+s)\tau}{1-q(1+s)\tau-(1-\alpha)\left[1-q(1+s)\right]\tau}$. This is the size of intermediate good firms. To make its distribution be a Pareto distribution, we set the distribution of b to the

Pareto distribution with scale parameter $\phi > 0$ and shape parameter $\psi > 1$. Then, letting SCALE $= B\phi$, because $\int b^{\frac{\alpha}{1-\alpha}} dF(b) = \frac{\psi-1}{\psi}\phi$ in B, the distribution of firm size is the Pareto distribution with scale parameter

$$\text{SCALE} = \frac{\alpha^2}{(1-\alpha)\Gamma(\tau) + \alpha^2} \frac{\psi - 1}{\psi} L \tag{E.1}$$

and shape parameter ψ. Since the shape parameter is the most important factor of firm distribution, we set $\psi = 1.059$ according to the estimate of Axtell (2001) which uses US data.

- Next, we control the markup rate of intermediate good firms. Letting the markup rate be μ, by (14),

$$\frac{\Gamma(\tau)}{\alpha} = 1 + \mu \tag{E.2}$$

As the benchmark value of μ, we adopt 0.2, a usual value of macroeconomic model with imperfect competition. [13]

- We set τ, q, s and f exogenously since these are policy parameters. We make use of these as free parameters if necessary. For the source of them, see Table 1 and the text.

- For (E.1) and (E.2), there is four undetermined parameters, α, L, SCALE, and ϕ. Here, we put $L = \phi = 1$ and determine α and SCALE by (E.1) and (E.2). Note that productivity b follows the Pareto distribution with scale parameter $\phi^{\frac{\alpha}{1-\alpha}}$ and shape parameter $\frac{1-\alpha}{\alpha}\psi$ since we assume $b^{\frac{\alpha}{1-\alpha}}$ follows the Pareto distribution with scale parameter ϕ and shape parameter ψ. Because the scale of productivity may be arbitrarily fixed whenever the distribution of firm size is properly controlled, we set $\phi = 1$. We simply normalize $L = 1$. Since the number of the intermediate good firms is a continuum, the minimum of firm size among them does not have to correspond to the minimal number of employee in actual data (and only the shape of the distribution, the curvature of density function, matters). Thus, we do not care

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[13]For example, Rotemberg and Woodford (1999) adopt this value. However, because markup rate is a key parameter of our analysis, we conduct numerical exercises for alternative values. See Table 2.

about the magnitude of parameter SCALE, which is determined by (E.1) for given α and the other parameters. Parameter α is pinned down by the condition of markup rate, (E.2).

- Finally, we explain the determination of α. By long but straightforward algebra, (E.2) can be rearranged as the quadratic equation with respect to α: $\Phi_2 \alpha^2 + \Phi_1 \alpha + \Phi_0 = 0$, where $\Phi_2 = (1+\mu)\big[1 - q(1+s)\big]\tau$, $\Phi_1 = (1+\mu)(1-\tau)$ and $\Phi_0 = -\big[1 - q(1+s)\tau\big]$. By $\Phi_2 > 0$ and $\Phi_1 > 0$, a necessary and sufficient condition for a unique solution in $(0, 1)$ is $\Phi_0 < 0$ and $\Phi_2 + \Phi_1 + \Phi_0 > 0$, which holds for any parameter set. The solution is given by $\alpha = \frac{-\Phi_1 + \sqrt{\Phi_1^2 - 4\Phi_2\Phi_0}}{2\Phi_2}$.