The value of information on deadlines

Marco Serena*†

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Abstract

In each period, the harder an agent works on a task, the more likely the final completion of the task is. The agent receives a reward if she completes the task, but pays the costs of working. At a certain period, an exogenous deadline will suddenly stop the agent from working. We compare two information regimes; one where the agent knows when she will be stopped (deadline awareness), and one where the agent is not informed (deadline unawareness). We find that the expected probability of completing the task is greater under deadline awareness (unawareness) when the reward is low (high). We extend the result to an agent with time-inconsistent preferences, and we find that when the agent is a (sufficient) procrastinator the scope for deadline unawareness vanishes regardless of the reward.

Keywords: Deadline; Information; Productivity; Procrastination.

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1 Introduction

> "*In ten hours a day you have time to fall twice as far behind your commitments as in five hours a day*".
>
> *The Asimov corollary to Parkinson's law*

There is abundance of situations where the timespan to work on a task is limited and the successful completion of the task depends on how hard an agent works. An employee works on a project until the deadline to submit the final report. A PhD candidate works on his/her thesis until the scholarship expires. In many such situations, there is some degree of control over the agent's awareness of when the deadline is. A firm might be more or less transparent with its employees on deadlines. An economic department might leave the PhD candidate uncertain when the funding

*Max Planck Institute for Tax Law and Public Finance. *Email*: marco.serena85@gmail.com

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will be cut. In this paper we study the impact of such (un)awareness on the agent's productivity on the task.

We address the question through the lens of a simple model where an agent faces either a tight (one period) or long (two periods) deadline to complete a task. In each period the agent either works or shirks. Working, as opposed to shirking, is more costly, but it also increases the probability of completing the task. In fact, the probability of completing the task depends on the overall (in the one or two periods) choice to work or shirk. The agent eventually obtains a reward if the task is completed. In this simple setting, we compare two information regimes, which we name "awareness" and "unawareness." Under awareness, the agent knows whether she will be given one or two periods, so she can plan in advance her working/shirking decisions with no uncertainty. Under unawareness, the agent is not informed of whether she will be given one or two periods; thus, the first-period decision to work or shirk is preemptively taken by the agent under uncertainty about the second period. In our basic model (Section 2) we study whether the expected probability of completing the task is greater under awareness or unawareness.

In Theorem 1 we find that for high (low) rewards the probability of completing the task is greater under unawareness (awareness). We consider two cases — namely, whether the probability of completing the task exhibits increasing or decreasing returns to working — and we show that Theorem 1 holds in both cases. Nevertheless, the intuition behind Theorem 1 is diametrically opposed in these two cases and we therefore keep them separate. We sketch here the intuition under decreasing returns to working, and in Section 2 we formalize it and complete it with the analogous case of increasing returns to working.[1] The intuition is in two steps:

1. *Ex-post trade-off between awareness and unawareness.* Consider a one-period deadline. Awareness of such a tight deadline incentivizes working because the agent wants to make the most of the initially higher returns to working (decreasing returns to working), while unawareness of such a tight deadline incentivizes shirking because the agent believes that there is a chance of having a second period to top-up her first-period action. Consider a two-period deadline. Under unawareness of such a long deadline the agent is more incentivized to work (than under awareness) out of fear that the deadline might actually be tight, and thus that she will suddenly be stopped after the first period. Thus, the ex-post trade-off favors awareness if there is one period and unawareness if there are two periods. Which of these two opposing forces ex-ante prevails when comparing awareness and unawareness?

2. *Ex-ante unraveling of the ex-post trade-off between awareness and unawareness.* For every deadline (tight or long) and information (awareness or unawareness), there is a sufficiently high reward to make the agent work. The key is the placement of these reward-thresholds. Awareness of a tight deadline is the most favorable information regime to incentivize working; thus, the reward-threshold is *low*. Awareness of a long deadline is the worst information

[1] Decreasing (increasing) returns to working means that the more times the agent works the less (more) working increases the probability of completing the task. See, Section 2 for a formal definition.

regime to incentivize working; thus, the reward-threshold is *high*. Unawareness is an intermediate information regime as opposed to awareness of a tight and awareness of a long deadline; thus, the reward-threshold is *medium*. It follows that if the reward is between low and medium, awareness dominates unawareness, and if the reward is between medium and high, unawareness dominates awareness. This is Theorem 1.

Deadlines are often seen as a useful commitment device to overcome procrastination; in the second half of the paper (Section 3), we ask whether the scope for deadline (un)awareness found in our basic model (Theorem 1) still holds when the agent is a procrastinator. Procrastinating behaviors are typically modeled as time-inconsistent preferences, pioneered by Phelps and Pollak (1968) and further developed, among others, by Akerlof (1991), Laibson (1997), O'Donoghue and Rabin (1999a,b). Time-inconsistent preferences may be an obstacle to sticking to a planned sequence of actions. Deadlines can overcome this obstacle. In fact, empirical evidence shows that people do self-impose binding deadlines (Wertenbroch, 1998; Trope and Fishbach, 2000) and that those self-imposed deadlines improve task performance (Ariely and Wertenbroch, 2002). In Section 3 we incorporate time-inconsistent preferences into our setting in perhaps the easiest way given our stylized model; namely, in the first period the agent underestimates the second-period cost of working. Thus, when the second period comes and the procrastinator realizes that the cost of working is higher than she believed it would be, she may regret her first-period action and revise the second-period action that she planned in the first period. Thus, we investigate the optimal information regime under procrastination.

A procrastinator tends to postpone work to the second period, but uncertainty over the existence of a second period makes the postponed work less likely to take place, thus decreasing the probability of the task being completed. This negative effect of unawareness is prevented by making the procrastinator aware of the exact number of periods.[2] We find that when the agent is a sufficiently severe procrastinator (i.e., her first-period underestimation of the second-period costs of working is sufficiently high), then regardless of the size of the reward there is no more scope for unawareness.

Literature review. In order to have a simple and meaningful comparison of the two information regimes, we model the deadline as *exogenous* and *stochastic*. Such deadlines have proven useful in other strands of literature. First, in the bargaining literature, Zwick, Rapoport and Howard (1992) analyze a model where players alternate offers to split a pie, and any time an offer is rejected there is an exogenous probability of the game being terminated.[3] Agreements tend to be reached near the deadline (as in Ma and Manove, 1993; Roth, Murnighan, and Schoumaker, 1988), and thus negotiators concede more the more looming the deadline is; deadline awareness speeds up concessions by the counterpart. In fact, Moore (2004) shows that negotiators can achieve better outcomes by revealing their final deadlines in

[2] If the probability of a second period is very low, the negative effect of unawareness is mitigated; in fact, a very high probability of having only one period wipes away the effect of a strong underestimation of the second period effort for a severe procrastinator. See Theorem 2.

[3] See also Fanning (2016) and Simsek and Yildiz (2016) for bargaining model with such deadlines.

negotiation. Fanning (2016) provides a rationale for the fact that agreements tend to be reached near the deadline via a bargaining model with uncertain deadline. Second, in the auction literature, the stochasticity of the deadline is useful to avoid having a substantial fraction of the bidding take place at the last minute before the deadline.[4] Candle auctions display exogenous and stochastic deadlines; the end of the auction occurs when a candle goes out (Füllbrunn and Abdolkarim, 2012). Electronic versions of the candle auctions are nowadays in use; for instance, in the Xetra intraday auction of the Frankfurt stock exchange a two-minute call period is followed by a "random end" phase which can last up to 30 seconds.

Our paper is different from the above papers in that we introduce a stochastic and exogenous deadline in a setting where an agent's productivity is to be incentivized. Likewise, Varas (2017) finds that under some conditions in a dynamic multi-tasking model the optimal contract to stimulate productivity entails a stochastic deadline.[5] Saez-Marti and Sjogren (2008) propose a discrete-time model where in each period an agent decides whether or not to work, given the opportunity cost of working of that period. They investigate the optimal deadline set by a principal to incentivize the agent to work, and they provide conditions under which a stochastic deadline is optimal. As opposed to Varas (2017) and Saez-Marti and Sjogren (2008), rather than investigating the optimality of a stochastic deadline taking the information regime as given, we do the opposite here.[6] In doing so, we are motivated by the abundance of real-life situations where an exogenous source may terminate the available time to complete the task; for instance, when a different and more important task pops up and the agent has to move her efforts to this new task, or in case of a random external distraction which the agent cannot prevent, but can statistically anticipate.

Like the present paper, Green and Taylor (2017) model an agent who chooses at each point in time whether to work (which increases the probability of success of the project) or shirk (which increases the agent's private benefit). As opposed to our model, completing the task requires two successful breakthroughs, the first being privately observed by the agent, while the second being publicly observed. They study the extraction of information *from* the agent; namely, what contract incentivizes a truthful report of the first unobservable and unverifiable breakthrough by the agent. The optimal contract involves a form of stochastic and exogenous deadline; randomly and secretly fix a date on which, if the agent has not yet reported a breakthrough, the agent gets fired. They show that this contract induces the agent not only to work, rather than shirk, but also to report truthfully whether and when

[4] This last-moment bidding is called "sniping." For evidence on sniping, see Bajari and Hortacsu (2003), Hayne et al. (2003), and Wilcox (2000). Experimentally, sniping has been confirmed by Ariely et al. (2005).

[5] Similarly, endogenous deadlines are studied, for instance, in Saez-Marti Sjogren (2008), Green and Taylor (2017), Bonatti and Horner (2011). In our setting, we take the deadline as exogenous and focus on the effects of different information regimes on the realization of the deadline itself.

[6] We assume an exogenous deadline in order to exclusively single out the effect of its awareness on productivity. In fact, the trade-off which is typical in the literature on endogenous deadlines is that longer deadlines increase the chance of completing the task but also increase shirking. Our model with exogenous and stochastic deadline is rather meant to capture the trade-off in the choice of the information regime; namely deadline awareness is beneficial in case of a tight deadline but backfires in case of a long deadline (or the opposite in case of increasing, rather than decreasing, returns to working).

the first breakthrough is achieved. In contrast, we study the provision of information *to* the agent, and ask whether more information concerning the occurrence of an exogenous deadline is beneficial or detrimental to the probability of completing the task.

The study of procrastination when the goal is to stimulate performance of the procrastinating agent is not novel. O'Donoghue and Rabin (1999b) introduced the seminal concept of time-inconsistent ($\beta - \delta$)-preferences in a model where an agent in each period decides whether to complete a task and incur a performance cost, or shirk for that period in the hope of a more favorable future period.[7] O'Donoghue and Rabin (2008) propose a model where an agent with time-inconsistent preferences (what we call a procrastinator) chooses in each period to work or shirk, and two periods of working are required for the completing the task. They show that procrastinators might pay the first-period cost of working to begin projects, but then never finish. Models investigating how to stimulate an agent with time-inconsistent preferences to act, like the above, typically assume that the agent is aware of the deadline and rather investigate the agent's awareness of being time-inconsistent. Here, we focus on the agent's awareness of the deadline itself.

2 Basic Model

A risk-neutral agent obtains a reward $V > 0$ if she successfully completes a task within a deadline. We model the deadline as a stochastic number of periods that the agent has so as to work on completing the task; with probability $q \in (0, 1)$ the agent has one period, with probability $1 - q$ the agent has two periods.[8] In every period, the agent either works (W) or shirks (S). The agent dislikes working; every time she chooses W her disutility is $c > 0$, whereas shirking entails no disutility.[9] However, working, as opposed to shirking, entails a greater probability p of completing the task, and thus to eventually obtain the reward V. The agent's utility is the difference between the expected benefit from completing the task pV, and the disutility of working, if any. The value of p depends on whether the agent works or shirks; in particular, throughout the paper we assume that

- $p_W > p_S$
- $p_{WW} > p_{WS} = p_{SW} > p_{SS}$

The former means that if there is only one period, task's completion is more likely if the agent works than if she shirks. The latter means that if there are two periods, task's completion is more likely in case the agent works in both periods than if she works in one period and shirks in the other, than if she shirks in both periods. Notice that there is no form of discounting across periods; working in the

[7] In Section 3 we adopt a stylized version of the ($\beta - \delta$)-preferences.

[8] Throughout the paper, we rule out $q \in \{0, 1\}$ because information, which is the target of our analysis, would play no role in affecting the task's completion. For distributions over longer timespans, the analysis quickly becomes highly complicated.

[9] That is, we normalize the disutility of S to 0.

first period and shirking in the second period is as costly and beneficial to the agent as shirking in the first period and working in the second period (costs are c and benefits are $p_{WS}V = p_{SW}V$). Thus, we simply write p_{WS}. The lack of discounting provides a useful benchmark to single out in Section 3 the intertemporal effects of procrastination only.

Throughout the paper, we distinguish two alternative cases:[10]

- **(Conc)** $p_W - p_S > p_{WS} - p_{SS} > p_{WW} - p_{WS}$
- **(Conv)** $p_W - p_S < p_{WS} - p_{SS} < p_{WW} - p_{WS}$

(Conc) is a *decreasing* returns to working assumption; the additional returns of working rather than shirking if there is a single period is greater than if there is another period where the agent shirked, which is in turn greater than if there is another period where the agent worked. In other words, the more *times* the agent works the less working as opposed to shirking increases the probability of completing the task. Conversely, **(Conv)** is an *increasing* returns to working assumption.[11] A special case of **(Conc)** and **(Conv)** is that; 1) W and S are scalars with $W > S$, 2) W and S are summable across periods, i.e., $p(a_1 + a_2)$ where $a_1, a_2 \in \{W, S\}$ are the first-period and second-period (if any) actions, and 3) $p' > 0$, and $p'' < 0$ under **(Conc)** or $p'' < 0$ under **(Conv)**.

The goal of the paper is to analyze two opposing information regimes. In information regime A the agent observes the realization of the deadline, which is one period in subgame $A1$ and two periods in subgame $A2$. In information regime U the agent does not observe the realization of the deadline, and thus when she decides whether to work or shirk in the first period, she faces the risk of being stopped at the end of the first period and she hopes that her first period working/shirking action results in a successful completion of the task; in other words, under U the agent learns the number of periods when it is too late to account for this information in her first-period action. The agent's strategy is $s^{A1} \in \{S, W\}$ in $A1$, $s^{A2} \in \{SS, WS, SW, WW\}$ in $A2$, and $s^U \in \{S\tilde{S}, W\tilde{S}, S\tilde{W}, W\tilde{W}\}$ in U, where the tilde on top of the second-period action denotes uncertainty over the existence of a second period. We denote by p^A and p^U the expected probability of completing the task respectively under A and U. In summary, in the probability of completing the task p, *subscripts* denote actions (e.g., p_{SW}, p_W), and *superscripts* denote information regimes (p^A and p^U). The following example clarifies the analysis of the game.

Example of analysis. *For instance, the agent's expected payoff of action W is $p_W V - c$, of action SS is $p_{SS}V$, and of action $W\tilde{S}$ is $q(p_W V - c) + (1-q)(p_{WS}V - c)$.*

[10] All inequalities are strict to avoid having to discuss trivial case distinctions.

[11] No ranking of single one period probabilities and single two periods probabilities is needed. That is, for instance, we do not have to take a stand on whether $p_W \lesseqgtr p_{SS}$. The reason is as follows. Under awareness, the agent knows the number of periods, and thus only compares p_W and p_S, or p_{WW}, p_{WS}, and p_{SS}. Under unawareness, the agent compares the profitability of each possible strategy by comparing differences between two (possibly) different one-period actions and two (possibly) different two-period actions, and thus we only need to sign the difference of differences of probabilities, as we do in **(Conc)** or **(Conv)**.

*If the parameter constellation — i.e., c, q, and all the p's — and one between **(Conc)** or **(Conv)** are such that the three actions — W, SS, and $W\tilde{S}$ — maximize the agent's expected payoff respectively in $A1$, $A2$, and U, then the expected probability of completing the task under A equals $p^A = qp_W + (1-q)p_{SS}$, and under U equals $p^U = qp_W + (1-q)p_{WS}$, so $p^A < p^U$ follows; in words, we say that A dominates U. We then conclude that U is the optimal information regime.*

Thus, in order to characterize the optimal information regime, we need to first characterize the optimal action of the agent for any given parameter constellation, for **(Conc)** and **(Conv)**, and in each subgame ($A1$, $A2$, and U). We relegate these characterizations to Appendix A (under **(Conc)** in Lemma 4 and under **(Conv)** in Lemma 5). The optimal information regime is in Theorem 1.[12]

Theorem 1 *Fix $\{c, q\}$. Assume either **(Conc)** or **(Conv)**. There exists a triple $\{V_1, V_2, V_3\}$ with $V_1 < V_2 < V_3$ such that;*

- $\forall V \in (0, V_1) \cup (V_3, \infty)$, $p^A = p^U$
- $\forall V \in (V_1, V_2)$, $p^A > p^U$
- $\forall V \in (V_2, V_3)$, $p^A < p^U$.

Proof. See Appendix A. ■

In the remainder of this Section we provide the intuition behind Theorem 1, through the visual help of tables 1 to 4, following the two steps spelled out in the Introduction; first the ex-post trade-off, and second the ex-ante unraveling of the ex-post trade-off.

Ex-post trade-off between awareness and unawareness.

We illustrate the ex-post trade-off through an example for **(Conv)** and **(Conc)**. Consider a researcher working on a paper for a special issue in either a top-tier journal or a minor journal, and the agent is either given a month (one period) or a year (two periods) to submit the paper. When writing for a top-tier journal, good probabilities of success require a massive workload, while a small workload is virtually pointless; in this sense, **(Conv)** exemplifies top-tier journals. On the other hand, in order to successfully write for a minor journal, relatively small workloads suffice for a sizeable probability of success and excessive workloads thus have a limited impact; in this sense, **(Conc)** exemplifies minor journals.

Consider a minor journal; the **(Conc)** line of Table 1. In case of a month to write for a minor journal, an unaware agent (U) partially relies on the possibility of having a whole year and is thus more prone to shirking than under the awareness (A) of having only a month. Therefore, in case of a month to write for a minor journal,

[12] All the results of the paper are defined over open intervals. At the thresholds, such as V_1 and V_2 in Theorem 1, the agent is indifferent between two actions, thus according to which one the agent picks, either $p^A > p^U$ or $p^A < p^U$. Since this is not interesting, we save on space and do not break indifferences. The same will hold for thresholds of q, and in Section 3 for thresholds of the parameter β.

A dominates U. In case of a whole year to write for a minor journal, awareness (A) of having a whole year to write for a minor journal makes the agent more prone to shirking, while an unaware agent (U) is "afraid" of being asked to submit the paper at the end of the first month, and is thus more prone to working. Therefore, in case of a whole year to write for a minor journal, U dominates A.

Consider a top-tier journal; the **(Conv)** line of Table 1. In case of a whole year to write for a top-tier, awareness (A) of such a long deadline gives the agent more hope of making it than unawareness (U), where the agent is more prone to shirk, having possibly only a month to write for a top-tier. Therefore, in case of a whole year to write for a top-tier, A dominates U. In case of a month to write for a top-tier, awareness of such a tight deadline deters the agent from having any hope of making it, while unawareness gives her some hope of perhaps having a whole year. Therefore, in case of a month to write for a top-tier, U dominates A.

	q	$1-q$
	One period; (*a month*)	Two periods; (*a year*)
(Conc); *minor journal*	A	U
(Conv); *top-tier journal*	U	A

Table 1. Ex-post trade-off.

The above discussion of the ex-post trade-off between A and U is a crucial building block for the intuition behind Theorem 1. In the remainder of this Section we unravel this ex-post trade-off from an ex-ante viewpoint. In tables 2 to 4 we visualize the optimal action of the agent for different values of V (horizontally) and for $A1$, $A2$, and U (vertically). Notice that in both **(Conc)** and **(Conv)** cases, for a sufficiently large V the agent works regardless of the information regime and the number of periods; thus, the information regime affects neither the agent's action nor the expected probability of completing the task — that is, $p^A = p^U = qp_W + (1-q)p_{WW}$. Similarly, for a sufficiently small V the agent unconditionally shirks and A and U are thus outcome equivalent. Intermediate values of V are more interesting and are discussed in the remainder of this Section, under **(Conc)** and **(Conv)**.

Ex-ante unraveling of the ex-post trade-off; (Conc).

Table 2 shows the optimal action under **(Conc)**, which we explain in what follows. First, SW and WS are outcome equivalent, thus we simply write WS.

Second, $S\tilde{W}$ is never chosen. The reason is that a necessary condition to choose $S\tilde{W}$ is that it is preferred to both $W\tilde{S}$ and $S\tilde{S}$, but to be preferred to $W\tilde{S}$ the reward V has to be sufficiently *high* so $W \succ S$ (in case of two periods, $S\tilde{W}$ and $W\tilde{S}$ are outcome equivalent) and to be preferred to $S\tilde{S}$ the reward V has to be sufficiently *low* so $SS \succ WS$ (in case of one period, $S\tilde{W}$ and $S\tilde{S}$ are outcome equivalent). The former threshold of V (s.t. $W \sim S$) is greater than the latter (s.t. $SS \sim WS$) because the cost differential is identical, but the benefit differential is greater in the former case by **(Conc)** (i.e., $p_W - p_S > p_{WS} - p_{SS}$). Thus, there is no reward V for which $S\tilde{W}$ is chosen. This immediately implies that the threshold of V below which the agent stops to unconditionally work is the same in U and $A2$ (see, Table 2).

Third, under **(Conc)** the less the agent works, the more productive it is to work, and thus certainty of having just one period of time ($A1$) incentivizes work more than the possibility (U) or certainty ($A2$) of having a second period, as discussed in the ex-post trade-off. For this reason, $A1$ is the *best* case to incentivize work, and thus in $A1$ the threshold of V above which the agent works is lower than that of $A2$ and U. On the other hand, certainty of having two periods of time ($A2$) is the *worst* case to incentivize work, and thus in $A2$ the threshold of V above which the agent works is greater than that of $A1$ and U. Not surprisingly, the power of information regime U to incentivize work is in-between that of $A1$ and $A2$, as is the threshold of V above which the agent works. The first three rows of Table 2 follow.

The optimal information regime immediately follows. For low V's A is optimal because if there are two periods the agent will shirk both in A and U, but if there is one period the agent will work in A and shirk in U. For high V's U is optimal because if there is one period the agent works both in A and U, but if there are two periods the agent works in U and not in A. For all the other more extreme values of V, A and U are outcome equivalent. Theorem 1 under **(Conc)** follows.

$A1$:	S	W			
$A2$:	SS			WS	WW
U :	$S\tilde{S}$		$W\tilde{S}$		$W\tilde{W}$
Optimal:		A	U		

Table 2: **(Conc)**, actions of the agent.

Ex-ante unraveling of the ex-post trade-off; (Conv).

Tables 3 and 4 show the optimal action under **(Conv)**, which we explain in what follows. First, SW and WS are never chosen. The reason is that under **(Conv)** high marginal returns to work are reached only for great overall workloads, and thus either the agent is sufficiently sure of having enough time to achieve such high overall workloads, or it is not worthy for her to work at all.[13]

Second, $W\tilde{S}$ is never chosen, because as opposed to $S\tilde{W}$ it yields less extreme and more moderate overall workloads, and this is not optimal since the returns to working are increasing. On the other hand, $S\tilde{W}$ is chosen if the probability q of one period is sufficiently high (compare tables 3 and 4) because if instead the agent is sufficiently confident of having two periods, she is better-off choosing $W\tilde{W}$ rather than $S\tilde{W}$.

Third, under **(Conv)** certainty of having just one period of time ($A1$) is the biggest deterrence of work because the agent is sure of not being able to make the most of the high marginal returns to working reached at high overall levels of workload, as discussed in the ex-post trade-off. For this reason, $A1$ is the *worst* information regime to incentivize work, and thus in $A1$ the threshold of V above which the agent works is greater than that of $A2$ and U. On the other hand, certainty

[13]Technically, **(Conv)** and linear costs of working directly imply that either SS or WW is preferred to mixtures of W and S.

of having two periods of time ($A2$) is *best* case to incentivize work, and thus in $A2$ the threshold of V above which the agent works is lower than that of $A1$ and U. Not surprisingly, the power of information regime U to incentivize work is in-between that of $A1$ and $A2$, and so is the threshold of V above which the agent works.

The optimal information regime immediately follows. For low V's A is optimal because if there is one period the agent will shirk both in A and U, but if there are two periods the agent will work in both periods in A and shirk in at least one period in U (depending on q). For high V's U is optimal because if there are two periods the agent will work in both A and U, but if there is one period the agent will work in U and shirk in A. For all the other more extreme values of V, A and U are outcome equivalent. Theorem 1 under **(Conv)** follows.

<table>
<tr><td>$A1$:</td><td colspan="3">S</td><td>W</td></tr>
<tr><td>$A2$:</td><td>SS</td><td colspan="3">WW</td></tr>
<tr><td>U :</td><td colspan="2">$S\tilde{S}$</td><td colspan="2">$W\tilde{W}$</td></tr>
<tr><td>Optimal:</td><td></td><td>A</td><td>U</td><td></td></tr>
</table>

Table 3: **(Conv)**, actions of the agent if $q < \bar{q} \equiv \frac{(p_{WW}-p_{WS})-(p_{WS}-p_{SS})}{(p_{WW}-p_{WS})-(p_W-p_S)}$.

<table>
<tr><td>$A1$:</td><td colspan="4">S</td><td>W</td></tr>
<tr><td>$A2$:</td><td>SS</td><td colspan="4">WW</td></tr>
<tr><td>U :</td><td colspan="2">$S\tilde{S}$</td><td>$S\tilde{W}$</td><td colspan="2">$W\tilde{W}$</td></tr>
<tr><td>Optimal:</td><td></td><td>A</td><td>A</td><td>U</td><td></td></tr>
</table>

Table 4: **(Conv)**, actions of the agent if $q > \bar{q}$.

3 Procrastinator

Procrastinating behaviors are widespread in real-life. The literature typically captures procrastination with time-inconsistent preferences.[14] In our highly stylized setting there is at most *one* future period, and thus a natural easy way to introduce procrastination in our model is to assume that in the first period the agent underestimates the second-period cost of working by a multiplicative factor $\beta \leq 1$.[15] We call the agent affected by such time-inconsistent preferences a procrastinator. In each of the (possibly) two periods, the agent is modeled as a separate self who chooses the

[14]Seminal contributions are, for instance, the quasi-hyperbolic discounting (Phelps and Pollak, 1968; Laibson, 1997), the (β, δ)-*preferences* (O'Donoghue and Rabin, 1999a), and the salience-based preference (Akerlof, 1991).

[15]If we were to strictly follow the (β, δ)-preferences we should discount the reward V by β too because the reward (if any) is realized at the end of the second period, but since V is realized at most once we can set βV to a new V, save on notation and obtain the same qualitative results. Recall that we have no discounting in the present paper, so as to keep the focus of Section 2 on the value of information over deadlines, and of Section 3 on the effects that arise from introducing procrastination in the agent's preferences.

optimal action given her current preferences and her perceptions of future behavior (perception-perfect strategies, see O'Donoghue and Rabin, 1999b). The preferences of the first-period self underestimate the second-period cost of working by β, while the second-period self is not affected by underestimation.

In Section 2 we analyzed a rational agent, in the sense that her preferences were time-consistent ($\beta = 1$).[16] A rational agent always sticks to her first-period plan of action when the second period comes. In contrast, a procrastinator chooses in the first period a plan of action (for both periods) under underestimation of the second-period cost of working, so when the second period comes, she "realizes" that the second-period cost of working is actually higher than she believed it would be in the first period, but it is too late to go back in time and change the first-period action. However, she is still on schedule should she wish to revise the action that she planned for the second period in the first period. Thus, in this Section we distinguish *planned* and *actual* actions for the second period; to denote the former we use brackets for the first-period plan of the second-period action — e.g., $S(W)$ — while the latter is denoted as in Section 2 — e.g., SW. For instance, a procrastinator planning $S(W)$ expects a payoff of $p_{WS}V - \beta c$, but after shirking in the first period she realizes that the cost of working is actually c rather than βc, but she has already irreversibly sank her first-period action S. Thus, if she sticks to the original plan SW she obtains $p_{WS}V - c$, whereas if she chooses SS she obtains $p_{SS}V$.

We ask whether and how analyzing a procrastinator rather than a rational agent changes the agent's actions, the probability of completing the task, and finally the optimal information regime. Lemma 7 and Lemma 8 in Online Appendix B characterize the agent's actions, whose analysis is more convoluted than in case of rationality because planned and actual actions are characterized for all β's. Despite this analytical hurdle, the optimal information regime turns out to be neatly characterizable; in Theorem 2 under **(Conc)** and in Theorem 3 under **(Conv).**

Theorem 2 *Fix $\{c, q\}$. Assume **(Conc)** and that the agent is a procrastinator.*

When $q < \bar{q}$ and $\beta < \frac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)(p_{WS}-p_{SS})}$, there exists a pair $\{V_1, V_2\}$ with $V_1 < V_2$ such that;

- $\forall V \in (0, V_1) \cup (V_2, \infty)$, $p^A = p^U$
- $\forall V \in (V_1, V_2)$, $p^A > p^U$

Otherwise, there exists a triple $\{V_1, V_2, V_3\}$ for which the result of Theorem 1 holds.

Proof. See Online Appendix B.1. ■

[16] Despite the analysis of this section under $\beta = 1$ yielding the same results as Section 2, we keep the two sections entirely separate for two reasons. First, when $\beta = 1$ in the second period the agent neither regrets the first-period action nor revises the second-period planned action, whereas when $\beta \to 1$ regrets and revisions are both possible and have to be analyzed. This adds a considerable extra layer of analysis. Second, our simple model of Section 2 is of self-contained interest, and extentions other than that of a procrastinator might be worth pursuing; thus, having easy access to the neat characterization of the $\beta = 1$ case might be useful for future research.

Intuition behind Theorem 2.

We will refer to *regret* when in the second period if the procrastinator could go back in time she would strictly prefer to change her first-period action (paying its cost). We will refer to an *innocuous adjustment of plans* when the planned and actual actions differ, but if the procrastinator could go back in time and change her first-period action she would at most obtain the same payoff ("it is not too late"). We redraw *Table 2* below for a procrastinator *(Table 5)*.

The actual actions under A *are not affected* by β. Under $A1$, β trivially plays no role, whereas under $A2$ we show that only innocuous adjustments of plans could occur. First, consider W as a first-period action. A procrastinator never plans $W(S)$ because $S(W)$ yields the same benefit, but it is cheaper in the eyes of her first-period self. Thus, if a procrastinator works in the first period she must have planned to work in the second period too — i.e., $W(W)$ — which implies that she must have preferred $W(W) \succ S(W)$ and, if so, her second-period self must also prefer $WW \succ SW$. The latter, together with $SW \sim WS$, implies that $WW \succ WS$; in words, a procrastinator never regrets having worked in the first period, and she sticks to the plan of working in the second period too, WW. Second, consider S as a first-period action. If a procrastinator planned $S(S)$, the actual action is also SS. If a procrastinator planned $S(W)$, once she realizes that working is more costly than she believed it would be, it is not too late to avoid such a cost and shirk in the second period too; SS. Thus, the only effect of β is that there is an extra region of parameters where a procrastinator plans $S(W)$, but ends up choosing SS; an innocuous adjustment of plans.[17]

The actual actions under U *are affected* by β. Consider action $S\tilde{W}$, and recall that it was never chosen by a rational agent (*Table 2*) due to uncertainty over the existence of the second period, so $W\tilde{S} \succ S\tilde{W}$. Now, the lower the β, the more tempting $S(\tilde{W})$; in fact, for a sufficiently low β, the plan is $S(\tilde{W})$. The resulting actual action is $S\tilde{S}$ for a low V, and $S\tilde{W}$ for a high V. Thus, a sufficiently low β enables $S\tilde{W}$ to be chosen (see *Table 5*); in fact, $S\tilde{W}$ is chosen between $S\tilde{S}$ and $W\tilde{S}$.[18] Furthermore, the V such that $S\tilde{W} \sim S\tilde{S}$ and such that $SW \sim SS$ is the same because the first period "cancels out." Thus, the main novelty of introducing a sufficiently low β is that there is a new region where $S\tilde{W}$ is chosen under U, while actions $\{W, SW\}$ are chosen under A, so A is optimal. For all the other more extreme values of V, A and U are outcome equivalent. Theorem 2 follows.

$A1$:	S	W			
$A2$:	SS		SW		WW
U :	$S\tilde{S}$		$S\tilde{W}$	$W\tilde{S}$	$W\tilde{W}$
Optimal:		A	A		

Table 5: **(Conc)**, actual actions of a procrastinator, if $q < \bar{q}$ and $\beta < \frac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)(p_{WS}-p_{SS})}$.

[17] Formally, this region is $V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WS}-p_{SS}}\right)$, as can be seen comparing s^{A2} and (s^{A2}) in Lemma 7 of the Online Appendix B.1.

[18] Note that together with a sufficiently low β, also $q < \bar{q}$ is required, otherwise the likelihood of the second period, which drives the choice of $S\tilde{W}$, is too low. In words, even if I think that the cost tomorrow is very low (low β), a very high probability of not having a tomorrow at all (high q) will make me want to work immediately today.

Theorem 3 *Fix $\{c, q\}$. Assume **(Conv)** and that the agent is a procrastinator.*

When $\beta < \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})}$, there exists a pair $\{V_1, V_2\}$ with $V_1 < V_2$ such that;

- $\forall V \in (0, V_1) \cup (V_2, \infty)$, $p^A = p^U$
- $\forall V \in (V_1, V_2)$, $p^A > p^U$

Otherwise, there exists a triple $\{V_1, V_2, V_3\}$ for which the result of Theorem 1 holds.

Proof. See Online Appendix B.2. ■

Intuition behind Theorem 3.

The actual actions under A *are not affected* by β for a reason very similar to the one described in the intuition behind Theorem 2. Thus, we only draw the final conclusion here. The only effect of β is that there is an extra region of parameters where a procrastinator plans $S(W)$, but ends up choosing SS; an innocuous adjustment of plans.[19]

The actual actions under U *are affected* by β. We do not draw another table because there are two minor changes in the agent's actions, as opposed to *Table 4*. In what follows we describe them in words. First, as under **(Conc)**, now action $W\tilde{S}$ can be chosen by a procrastinator. The optimality of $W\tilde{S}$ exhibits between $S\tilde{W}$ and $W\tilde{W}$, which was the cutoff of V where we moved from $p^A > p^U$ to $p^A < p^U$ (see, *Table 4*); thus, the new region where $W\tilde{S}$ is chosen does not affect the result that for low V's $p^A > p^U$ and for high V's $p^A < p^U$. Second, for a sufficiently low β, the V above which $W\tilde{W} \succ S\tilde{W}$ becomes greater than the V above which $W \succ S$, because the latter V is not affected by β while the former V decreases in β.[20] In fact, the first-period underestimation of the second-period cost of work increases the temptation for the procrastinator to plan to work in both periods so as to make the most of the high returns of high overall workloads due to **(Conv)**. The increase of the V above which $W\tilde{W}$ is chosen wipes away the scope for U; that is, Theorem 3.

4 Conclusions

We investigate whether the productivity of an agent on a certain task benefits from the agent's awareness of the deadline to complete the task. We find that the agent's awareness (unawareness) of the deadline maximizes the agent's productivity if the reward for task's completion is low (high). This finding suggests a scope for deadline concealment; a principal — who wants the agent to complete the task without

[19] Formally, this region is $V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right)$, and it exists only if $\beta < \beta_5$, as can be seen comparing s^{A2} and (s^{A2}) in Lemma 8 of the Online Appendix B.2.

[20] For a sufficiently low β, $W\tilde{S}$ is never chosen, as can be seen in s^U of Lemma 8 of the Online Appendix B.2. The reason is that a very low β yields a severe first-period underestimation of the second-period cost, and thus a severe temptation to choose W in the first period. Due to **(Conv)**, when the second period comes, the sunk W of the first period makes it profitable to keep working so as to make the most out of the high returns of high overall workloads due to **(Conv).**

internalizing the agent's cost of working — commits to concealing the deadlines of "important" tasks. Additionally, we challenge the optimality of deadline concealment by testing its robustness when the agent is characterized by procrastinating behavior. For sufficiently severe procrastinators, we find that the case for deadline concealment vanishes.

Being the first cut investigating the value of information on deadlines to stimulate productivity, our model is admittedly very stylized. Thus, it leaves several doors open for future research. Among them, we find the following appealing. We assumed that the success or failure of completing the task is only realized at the end of the game — that is, only when the agent submits the project which she handled for one or two periods she will observe whether the project is successful or not. An extension is to allow for interim realizations of success; that is, after working in the first period, the project might already yield a success, so no more effort by the agent is required. This extension sharply changes the analysis, and thus it is hard to foresee how the optimal information regime would change.

Appendix A: Basic Model - Lemmas and Proofs

Lemma 4 *Fix $\{c, q\}$. Under **(Conc)**, the agent's optimal action is*

$$s^{A1} = \begin{cases} S & \text{if } V < \frac{c}{p_W - p_S} \\ W & \text{if } V > \frac{c}{p_W - p_S} \end{cases}$$

$$s^{A2} = \begin{cases} SS & \text{if } V < \frac{c}{p_{WS} - p_{SS}} \\ WS & \text{if } V \in \left(\frac{c}{p_{WS} - p_{SS}}, \frac{c}{p_{WW} - p_{WS}}\right) \\ WW & \text{if } V > \frac{c}{p_{WW} - p_{WS}} \end{cases}$$

$$s^{U} = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{(1-q)(p_{WS} - p_{SS}) + q(p_W - p_S)} \\ W\tilde{S} & \text{if } V \in \left(\frac{c}{(1-q)(p_{WS} - p_{SS}) + q(p_W - p_S)}, \frac{c}{p_{WW} - p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW} - p_{WS}} \end{cases}$$

Proof of Lemma 4. Under $A1$ the utility of S and W is respectively $p_W V - c$ and $p_S V$. Thus, the optimal strategy under $A1$ is

$$s^{A1} = \begin{cases} S & \text{if } V < \frac{c}{p_W - p_S} \\ W & \text{if } V > \frac{c}{p_W - p_S} \end{cases}$$

W is chosen if $V > \frac{c}{p_W - p_S}$, and S is chosen if $V < \frac{c}{p_W - p_S}$.

Under $A2$ the utility of WW, WS and SS is respectively $p_{WW}V - 2c$, $p_{WS}V - c$ and $p_{SS}V$. Thus,

$$s^{A2} = \begin{cases} SS & \text{if } V < \frac{c}{p_{WS} - p_{SS}} \\ WS & \text{if } V \in \left(\frac{c}{p_{WS} - p_{SS}}, \frac{c}{p_{WW} - p_{WS}}\right) \\ WW & \text{if } V > \frac{c}{p_{WW} - p_{WS}} \end{cases}$$

Notice that this ranking of thresholds of V relies on **(Conc)**.

Under U, *first* $S\tilde{W}$ is never chosen since $\nexists V : S\tilde{W} \succ W\tilde{S} \vee S\tilde{W} \succ S\tilde{S}$. In fact,

$$\begin{aligned} S\tilde{W} &\succ W\tilde{S} \iff (1-q)(p_{WS}V - c) + qp_S V > (1-q)(p_{WS}V - c) + q(p_W V - c) \\ &\iff V < \frac{c}{p_W - p_S} \end{aligned}$$

and

$$\begin{aligned} S\tilde{W} &\succ S\tilde{S} \iff (1-q)(p_{WS}V - c) + qp_S V > (1-q)p_{SS}V + qp_S V \\ &\iff V > \frac{c}{p_{WS} - p_{SS}} \end{aligned}$$

and by **(Conc)** $\frac{c}{p_W - p_S} < \frac{c}{p_{WS} - p_{SS}}$.

Second, we rank the three remaining strategies; $S\tilde{S}$, $W\tilde{S}$, and $W\tilde{W}$.

$$\begin{aligned} S\tilde{S} &\succ W\tilde{S} \iff (1-q)p_{SS}V + qp_S V > (1-q)(p_{WS}V - c) + q(p_W V - c) \\ &\iff V < \frac{c}{(1-q)(p_{WS} - p_{SS}) + q(p_W - p_S)} \end{aligned}$$

and

$$W\tilde{S} \succ W\tilde{W} \iff (1-q)(p_{WS}V-c)+q(p_WV-c)>(1-q)(p_{WW}V-2c)+q(p_WV-c)$$
$$\iff V<\frac{c}{p_{WW}-p_{WS}}$$

By **(Conc)** we can rank the two above thresholds of V,

$$\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$$
$$\iff p_{WW}-p_{WS}<(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)$$

and since by **(Conc)** $p_{WW}-p_{WS}<p_{WS}-p_{SS}<p_W-p_S$, then the above ranking of thresholds of V holds. Thus, the optimal strategy under U follows. ■

Lemma 5 *Fix $\{c,q\}$. Under **(Conv)**, the agent's optimal action is*

$$s^{A1} = \begin{cases} S & \text{if } V<\frac{c}{p_W-p_S} \\ W & \text{if } V>\frac{c}{p_W-p_S} \end{cases}$$

$$s^{A2} = \begin{cases} SS & \text{if } V<\frac{2c}{p_{WW}-p_{SS}} \\ WW & \text{if } V>\frac{2c}{p_{WW}-p_{SS}} \end{cases}$$

$$\text{If } q<\bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V<\frac{(1-q)2c+qc}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \\ W\tilde{W} & \text{if } V>\frac{(1-q)2c+qc}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \end{cases}$$

$$\text{If } q>\bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V<\frac{c}{p_{WS}-p_{SS}} \\ S\tilde{W} & \text{if } V\in\left(\frac{c}{p_{WS}-p_{SS}},\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right) \\ W\tilde{W} & \text{if } V>\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \end{cases}$$

where $\bar{q}\equiv\frac{(p_{WW}-p_{WS})-(p_{WS}-p_{SS})}{(p_{WW}-p_{WS})-(p_W-p_S)}$.

Proof of Lemma 5. Under $A1$, s^{A1} coincides with the one under **(Conc)**, whereas under $A2$ **(Conv)** implies that $\frac{c}{p_{WW}-p_{WS}}<\frac{c}{p_{WS}-p_{SS}}$ and thus the region where WS is optimal disappears. Also, $WW\succ SS \iff p_{WW}V-2c>p_{SS}V$, and thus

$$s^{A2} = \begin{cases} SS & \text{if } V<\frac{2c}{p_{WW}-p_{SS}} \\ WW & \text{if } V>\frac{2c}{p_{WW}-p_{SS}} \end{cases}$$

Under U, *first* strategy $W\tilde{S}$ is never chosen because $\nexists V: W\tilde{S}\succ S\tilde{S}\vee W\tilde{S}\succ W\tilde{W}$. In fact,

$$W\tilde{S} \succ S\tilde{S} \iff (1-q)(p_{WS}V-c)+q(p_WV-c)>(1-q)p_{SS}V+qp_SV$$
$$\iff V>\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}$$

and

$$W\tilde{S} \succ W\tilde{W} \iff (1-q)(p_{WS}V-c)+q(p_WV-c)>(1-q)(p_{WW}V-2c)+q(p_WV-c)$$
$$\iff V<\frac{c}{p_{WW}-p_{WS}}$$

and by **(Conv)** $\frac{c}{p_{WW}-p_{WS}} < \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}$.

Second, whether $S\tilde{W}$ is chosen depends on q. In fact, for $S\tilde{W}$ to be chosen $S\tilde{W} \succ S\tilde{S} \vee S\tilde{W} \succ W\tilde{W}$. That is,

$$S\tilde{W} \succ S\tilde{S} \iff V > \frac{c}{p_{WS}-p_{SS}}$$

and

$$\begin{aligned} S\tilde{W} \quad &\succ \quad W\tilde{W} \iff (1-q)(p_{WS}V-c)+qp_SV > (1-q)(p_{WW}V-2c)+q(p_WV-c) \\ &\iff \quad V < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \end{aligned}$$

Thus, $S\tilde{W}$ is chosen if and only if

$$\begin{aligned} \frac{c}{p_{WS}-p_{SS}} \quad &< \quad \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \\ &\iff \quad (1-q)(p_{WW}-p_{WS})+q(p_W-p_S) < p_{WS}-p_{SS} \\ &\iff \quad q > \bar{q} \equiv \frac{(p_{WW}-p_{WS})-(p_{WS}-p_{SS})}{(p_{WW}-p_{WS})-(p_W-p_S)} \end{aligned}$$

where $\bar{q} \in (0,1)$ by **(Conv)**.

Case I. If $q < \bar{q}$, then only $S\tilde{S}$ and $W\tilde{W}$ can be chosen, and in particular,

$$\begin{aligned} S\tilde{S} \quad &\succ \quad W\tilde{W} \iff (1-q)p_{SS}V+qp_SV > (1-q)(p_{WW}V-2c)+q(p_WV-c) \\ &\iff \quad V < \frac{(1-q)2c+qc}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \end{aligned}$$

The optimal strategy s^U under $q < \bar{q}$ follows.

Case II. If $q > \bar{q}$, then $S\tilde{S}, S\tilde{W}$ and $W\tilde{W}$ can be chosen. From the above, we know the three thresholds of V relevant for pairwise rankings of the three possible actions; $S\tilde{S}, S\tilde{W}$ and $W\tilde{W}$. These three thresholds of V are ranked as follows,

$$\frac{c}{p_{WS}-p_{SS}} < \frac{(1-q)2c+qc}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}$$

In fact, simple algebra shows that both the first and the second inequality are equivalent to $q > \bar{q}$. The optimal strategy s^U under $q > \bar{q}$ follows. ■

Proof of Theorem 1. **(Conc)** implies that the relevant thresholds of V of $\{s^{A1}, s^{A2}, s^U\}$ are ranked as follows,

$$\underbrace{\frac{c}{p_W-p_S}}_{V_1} < \underbrace{\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}}_{V_2} < \underbrace{\frac{c}{p_{WS}-p_{SS}}}_{V_3} < \underbrace{\frac{c}{p_{WW}-p_{WS}}}_{V_4}$$

(V_1, V_2, V_3) will turn out to be the ones of Theorem 1.

First of all, when the reward is sufficiently low ($V < V_1$), S is always the dominant action, and thus regardless of the information regime, the agent's expected action is to unconditionally shirk, and thus $p^A = p^U = qp_S + (1-q)p_{SS}$. Similarly, when $V > V_4$, $p^A = p^U$.

When $V \in (V_3, V_4)$, the optimal strategies are W and WS under A, and $W\tilde{S}$ under U. Thus, once again, the information regime does not affect the agent's expected action, and $p^A = p^U$.

The remaining two regions are more interesting. When $V \in (V_1, V_2)$, the optimal strategies are W and SS under A, and $S\tilde{S}$ under U. Since $qp_W + (1-q)p_{SS} > qp_S + (1-q)p_{SS}$, $p^A > p^U$. Conversely, when $V \in (V_2, V_3)$, the optimal strategies are W and SS under A, and $W\tilde{S}$ under U. Since $qp_W + (1-q)p_{SS} < qp_W + (1-q)p_{WS}$, $p^A < p^U$.

(Conv), Case I. If $q < \bar{q}$, **(Conv)** implies that the relevant thresholds of V of $\{s^{A1}, s^{A2}, s^U\}$ are ranked as follows,

$$\underbrace{\frac{2c}{p_{WW} - p_{SS}}}_{V_1} < \underbrace{\frac{(1-q)2c + qc}{(1-q)(p_{WW} - p_{SS}) + q(p_W - p_S)}}_{V_2} < \underbrace{\frac{c}{p_W - p_S}}_{V_3}$$

where the first inequality is proved in what follows.

$$\begin{aligned} \frac{2c}{p_{WW} - p_{SS}} &< \frac{(1-q)2c + qc}{(1-q)(p_{WW} - p_{SS}) + q(p_W - p_S)} \\ &\iff \frac{(1-q)(p_{WW} - p_{SS}) + q(p_W - p_S)}{p_{WW} - p_{SS}} < 1 - q + \frac{q}{2} \\ &\iff \frac{p_W - p_S}{p_{WW} - p_{SS}} < \frac{1}{2} \\ &\iff p_W - p_S < \frac{(p_{WW} - p_{WS}) + (p_{WS} - p_{SS})}{2} \end{aligned}$$

The lowest and highest regions (i.e., $V < V_1$ or $V > V_3$) yields S or W to be unconditionally chosen, and thus $p^A = p^U$. When $V \in (V_1, V_2)$, $p^A > p^U$ because the optimal strategies are S and WW under A, and $S\tilde{S}$ under U. When $V \in (V_2, V_3)$, $p^A < p^U$ because the optimal strategies are S and WW under A, and $W\tilde{W}$ under U.

(Conv), Case II. If $q > \bar{q}$, **(Conv)** implies that the relevant thresholds of V of $\{s^{A1}, s^{A2}, s^U\}$ are ranked as follows,

$$\underbrace{\frac{2c}{p_{WW} - p_{SS}}}_{V_1} < \frac{c}{p_{WS} - p_{SS}} < \underbrace{\frac{c}{(1-q)(p_{WW} - p_{WS}) + q(p_W - p_S)}}_{V_2} < \underbrace{\frac{c}{p_W - p_S}}_{V_3}$$

Notice that now there is an extra threshold as opposed to Case I because $S\tilde{W}$ could be optimal.

The lowest and highest regions (i.e., $V < V_1$ or $V > V_3$) yields S or W to be unconditionally chosen, and thus $p^A = p^U$. When $V \in (V_1, V_2)$, $p^A > p^U$ because the optimal strategies are S and WW under A, and $S\tilde{S}$ ($S\tilde{W}$) below (above) $\frac{c}{p_{WS} - p_{SS}}$ under U. When $V \in (V_2, V_3)$, $p^A < p^U$ because the optimal strategies are S and WW under A, and $W\tilde{W}$ under U. ■

Appendix B: Section 3 - Lemmas and Proofs

Notation. By definition, a procrastinator might *plan* a certain sequence of action in the first period, *implement* the first-period action, and when the second period comes, *change* the plan by realizing that the costs of working are actually greater than she believed it would be. We denote the first period plan (under the curse of underestimation) by (s^{A2}) and (s^U), which for instance take values $S(W)$ or $S(\tilde{W})$. The brackets indicate a planned action (i.e., the first-period plan for the second-period action) which might eventually not be implemented. Thus, if the agent does not change her initial plan, *planned* and *actual* actions coincide; that is, $(s^{A2}) = s^{A2}$ and $(s^U) = s^U$.

We start with preliminary results on ranking of some crucial thresholds of β's which will play an important role throughout the proofs of this Appendix.

Lemma 6 *Define*

$$\beta_1 \equiv \frac{p_{WW} - p_{WS}}{q(p_W - p_S) + (1-q)(p_{WW} - p_{WS})} \qquad \beta_2 \equiv \frac{p_{WS} - p_{SS}}{q(p_W - p_S) + (1-q)(p_{WS} - p_{SS})}$$

$$\beta_3 \equiv \frac{p_{WS} - p_{SS}}{q(p_W - p_S) + (1-q)(p_{WW} - p_{WS})} \qquad \beta_4 \equiv \frac{p_{WW} - p_{WS}}{q(p_W - p_S) + (1-q)(p_{WS} - p_{SS})}$$

Then,

$$\beta_3 > 1 \iff q > \bar{q} = \frac{(p_{WW} - p_{WS}) - (p_{WS} - p_{SS})}{(p_{WW} - p_{WS}) - (p_W - p_S)}$$

Under ***(Conc)***,

$$\beta_3 > \beta_2 > \beta_1 > \beta_4$$

Under ***(Conv)***,

$$\beta_3 < \beta_2 < \beta_1 < \beta_4$$

Proof. Under **(Conc)**, $\beta_3 > \beta_2$ and $\beta_1 > \beta_4$. But also, $\beta_2 > \beta_1$ since $\beta_2 > \beta_1 \iff [q(p_W - p_S) + (1-q)(p_{WW} - p_{WS})](p_{WS} - p_{SS}) > [q(p_W - p_S) + (1-q)(p_{WS} - p_{SS})](p_{WW} - p_{WS}) \iff p_{WS} - p_{SS} > p_{WW} - p_{WS}$. Under **(Conv)**, $\beta_1 < \beta_4$ and $\beta_3 < \beta_2$. But also, $\beta_2 < \beta_1$ since, as above, $\beta_2 < \beta_1 \iff p_{WS} - p_{SS} < p_{WW} - p_{WS}$. ■

Appendix B.1: The (Conc) case

Lemma 7 *Fix $\{c,q\}$. Assume **(Conc)** and that the agent is a procrastinator.*

*The agent's optimal **planned** action in the first period is (s^{A1}) as s^{A1} in Lemma 4, and*

$$(s^{A2}) = \begin{cases} S(S) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(W) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W(W) & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta > \beta_2,\ (s^U) = \begin{cases} S(\tilde{S}) & \text{if } V < \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} \\ W(\tilde{S}) & \text{if } V \in \left(\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}, \frac{\beta c}{p_{WW}-p_{WS}}\right) \\ W(\tilde{W}) & \text{if } V > \frac{\beta c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta \in (\beta_1,\beta_2),\ (s^U) = \begin{cases} S(\tilde{S}) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(\tilde{W}) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right) \\ W(\tilde{S}) & \text{if } V \in \left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{\beta c}{p_{WW}-p_{WS}}\right) \\ W(\tilde{W}) & \text{if } V > \frac{\beta c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta < \beta_1,\ (s^U) = \begin{cases} S(\tilde{S}) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(\tilde{W}) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right) \\ W(\tilde{W}) & \text{if } V > \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \end{cases}$$

*The agent's **actual** action is $s^{A1} = (s^{A1})$, s^{A2} as in Lemma 4, and*

$$\text{If } \beta > \beta_2,\ s^U = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} \\ W\tilde{S} & \text{if } V \in \left(\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta \in (\beta_1,\beta_2) \begin{cases} \text{If } \beta < \beta_7 \textbf{ and } q < \bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{p_{WS}-p_{SS}} \\ S\tilde{W} & \text{if } V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right) \\ W\tilde{S} & \text{if } V \in \left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases} \\ \text{If } \beta > \beta_7 \textbf{ or } q > \bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V < \frac{c[1-\beta(1-q)]}{q(p_W-p_S)} \\ W\tilde{S} & \text{if } V \in \left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases} \end{cases}$$

$$\text{If } \beta < \beta_1 \begin{cases} \text{If } q > \bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \\ W\tilde{S} & \text{if } V \in \left(\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases} \\ \text{If } q < \bar{q},\ s^U = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{p_{WS}-p_{SS}} \\ S\tilde{W} & \text{if } V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right) \\ W\tilde{S} & \text{if } V \in \left(\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases} \end{cases}$$

where $\beta_7 = \frac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)(p_{WS}-p_{SS})}$.

Proof. Under $A1$ β plays no role and thus s^{A1} is the same as in Lemma 4.

Under $A2$ the utility of the four possible actions $W(W)$, $W(S)$, $S(W)$ and $S(S)$ is respectively $p_{WW}V - c - \beta c$, $p_{WS}V - c$, $p_{WS}V - \beta c$ and $p_{SS}V$, thus action WS is never chosen as first-period planned action since SW is preferred. Thus, the optimal planned action of the first period under $A2$ is[21]

$$(s^{A2}) = \begin{cases} S(S) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(W) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W(W) & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

However, when the second period comes the procrastinator realizes that the cost of W is c rather than βc, thus it might be that her second-period action (the one in round brackets) is no longer optimal. If the first-period action is W, in the second period the procrastinator chooses between WS and WW by comparing $p_{WS}V > p_{WW}V - c$ and thus s^{A2} does not change. Instead, if the first-period action is S, in the second period the procrastinator chooses between SW and SS by comparing $p_{WS}V - c > p_{SS}$, and thus SW is chosen if $V > \frac{c}{p_{WS}-p_{SS}} \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right)$. Thus, the actual action under $A2$ is

$$s^{A2} = \begin{cases} SS & \text{if } V < \frac{c}{p_{WS}-p_{SS}} \\ SW & \text{if } V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right) \\ WW & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

Under U, *when and if* the second period comes the procrastinator might depart from her initial plan of action, by realizing that the cost of working is actually higher than she believed it would be. We list here the pairwise comparisons of all the possible actions, which will be useful for the **(Conv)** case too.

$$\begin{aligned} W(\tilde{S}) \quad \succ \quad & W(\tilde{W}) \iff (1-q)p_{WS}V + qp_W V - c > (1-q)\left(p_{WW}V - \beta c\right) + qp_W V - c \\ \iff \quad & p_{WS}V > p_{WW}V - \beta c \\ \iff \quad & V < \frac{\beta c}{p_{WW} - p_{WS}} \end{aligned}$$

$$\begin{aligned} W(\tilde{S}) \quad \succ \quad & S(\tilde{W}) \iff (1-q)p_{WS}V + qp_W V - c > (1-q)\left(p_{WS}V - \beta c\right) + qp_S V \\ \iff \quad & V > \frac{c\left[1 - \beta(1-q)\right]}{q(p_W - p_S)} \end{aligned}$$

$$\begin{aligned} W(\tilde{W}) \quad \succ \quad & S(\tilde{W}) \iff (1-q)\left(p_{WW}V - \beta c\right) + qp_W V - c > (1-q)\left(p_{WS}V - \beta c\right) + qp_S V \\ \iff \quad & V > \frac{c}{(1-q)\left(p_{WW} - p_{WS}\right) + q\left(p_W - p_S\right)} \end{aligned}$$

[21] Notice that this ranking of thresholds of V relies on **(Conc)**; $p_{WW} - p_{WS} < p_{WS} - p_{SS}$.

$$W(\tilde{S}) \succ S(\tilde{S}) \iff (1-q)p_{WS}V + qp_W V - c > (1-q)p_{SS}V + qp_S V$$
$$\iff V > \frac{c}{(1-q)(p_{WS}-p_{SS}) + q(p_W - p_S)}$$

$$S(\tilde{W}) \succ S(\tilde{S}) \iff (1-q)(p_{WS}V - \beta c) + qp_S V > (1-q)p_{SS}V + qp_S V$$
$$\iff V > \frac{\beta c}{p_{WS}-p_{SS}}$$

$$W(\tilde{W}) \succ S(\tilde{S}) \iff (1-q)(p_{WW}V - \beta c) + qp_W V - c > (1-q)p_{SS}V + qp_S V$$
$$\iff V > \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS}) + q(p_W - p_S)}$$

Optimality of $W(\tilde{W})$.

$(s^U) = W(\tilde{W}) \iff V > \max\left\{\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}, \frac{\beta c}{p_{WW}-p_{WS}}, \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}\right\}$.
We refer to this as simply $\max\{.\}$. We compare pairs of these three thresholds,

$$\begin{aligned}
\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} &> \frac{\beta c}{p_{WW}-p_{WS}} \\
&\iff \\
\beta &< \frac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})} = \beta_1
\end{aligned}$$

$$\begin{aligned}
\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} &> \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \\
&\iff \\
(1-q)(p_{WW}-p_{SS})+q(p_W-p_S) &> (1-q)(p_{WW}-p_{WS})+q(p_W-p_S)+ \\
&\quad +\beta(1-q)[(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)] \\
&\iff \\
p_{WS}-p_{SS} &> \beta[(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)] \\
&\iff \\
\beta &< \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})} = \beta_3
\end{aligned}$$

$$\begin{aligned}
\frac{\beta c}{p_{WW}-p_{WS}} &> \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \iff \\
\beta[(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)] &> (p_{WW}-p_{WS})+\beta(1-q)(p_{WW}-p_{WS}) \iff \\
\beta[(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)] &> p_{WW}-p_{WS} \iff \\
\beta &> \frac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)(p_{WS}-p_{SS})} = \beta_4
\end{aligned}$$

Thus, $\max\{.\} = \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}$ requires $\beta > \beta_3$ and $\beta < \beta_4$, but we know that $\beta_3 > \beta_4$ (see Lemma 6). Thus,

$$(s^U) = W(\tilde{W}) \iff V > \begin{cases} \frac{\beta c}{p_{WW} - p_{WS}} & \text{if } \beta > \beta_1 \\ \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} & \text{if } \beta < \beta_1 \end{cases}$$

Optimality of $S(\tilde{S})$.

$(s^U) = S(\tilde{S}) \iff V < \min\left\{\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}, \frac{\beta c}{p_{WS}-p_{SS}}, \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}\right\}$. We refer to this as simply $\min\{.\}$. We compare pairs of these three thresholds,

$$\begin{aligned} \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} &> \frac{\beta c}{p_{WS}-p_{SS}} \\ &\iff \\ \beta &< \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WS}-p_{SS})} = \beta_2 \end{aligned}$$

$$\begin{aligned} \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} &> \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \\ &\iff \\ (1-q)(p_{WW}-p_{SS})+q(p_W-p_S) &> (1-q)(p_{WS}-p_{SS})+q(p_W-p_S)+ \\ &\quad +\beta(1-q)\left[(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)\right] \\ &\iff \\ p_{WW}-p_{WS} &> \beta\left[(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)\right] \\ &\iff \\ \beta &< \frac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)(p_{WS}-p_{SS})} = \beta_4 \end{aligned}$$

$$\begin{aligned} \frac{\beta c}{p_{WS}-p_{SS}} &> \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \\ &\iff \\ \beta\left[(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)\right] &> (p_{WS}-p_{SS})+\beta(1-q)(p_{WS}-p_{SS}) \\ &\iff \\ \beta\left[(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)\right] &> p_{WS}-p_{SS} \\ &\iff \\ \beta &> \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})} = \beta_3 \end{aligned}$$

Thus, $\min\{.\} = \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}$ requires $\beta > \beta_3$ and $\beta < \beta_4$, but we know that $\beta_3 > \beta_4$ (see Lemma 6). Thus,

$$(s^U) = S(\tilde{S}) \iff V < \begin{cases} \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} & \text{if } \beta > \beta_2 \\ \frac{\beta c}{p_{WS}-p_{SS}} & \text{if } \beta < \beta_2 \end{cases}$$

The optimality of $W(\tilde{W})$ and $S(\tilde{S})$ is wrapped up in the three specifications of (s^U) in Lemma 7, according to whether $\beta > \beta_2$, $\beta \in (\beta_1, \beta_2)$ or $\beta < \beta_1$.

Optimality of $W(\tilde{S})$ and $S(\tilde{W})$.

$(s^U) = W(\tilde{S}) \implies W(\tilde{S}) \succ S(\tilde{W}) \iff V > \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. In order to have a non-empty range of V's where $W(\tilde{S})$ is optimal we need $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$ to be below $\max\{.\}$ which characterizes $W(\tilde{W})$ and above $\min\{.\}$ which characterizes $S(\tilde{S})$.[22]

[22] In fact, $W(\tilde{S})$ is optimal if, not only $W(\tilde{S}) \succ S(\tilde{W})$, but also $W(\tilde{W})$ and $S(\tilde{S})$ are not optimal.

Since $\max\{.\}$ and $\min\{.\}$ take different values according to the value of β, we have to distinguish cases;

Case I; $\beta > \beta_1$.

For $W(\tilde{S})$ to be optimal,

$$\begin{array}{ccc}
\frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & < & \max\{.\} = \frac{\beta c}{p_{WW}-p_{WS}} \\
& \Longleftrightarrow & \\
(p_{WW}-p_{WS})\left[1-\beta(1-q)\right] & < & \beta q(p_W-p_S) \\
& \Longleftrightarrow & \\
\beta & > & \frac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)\left(p_{WW}-p_{WS}\right)} = \beta_1
\end{array}$$

which holds true. Finally, for $S(\tilde{W})$ to be chosen, we also need $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} > \min\{.\}$. Two subcases might occur;

Case I.I; $\beta > \beta_2$**.** Then,

$$\begin{array}{ccc}
\frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & < & \max\{.\} = \frac{c}{(1-q)\left(p_{WS}-p_{SS}\right)+q\left(p_W-p_S\right)} \\
& \Longleftrightarrow & \\
p_{WS}-p_{SS} & < & \beta\left[(1-q)\left(p_{WS}-p_{SS}\right)+q\left(p_W-p_S\right)\right] \\
& \Longleftrightarrow & \\
\beta & > & \beta_2
\end{array}$$

which holds true. Thus if $\beta > \beta_2$, $S(\tilde{W})$ is never chosen because $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \min\{.\}$ and the results we have so far are enough to fully characterize the strategy; that is, (s^U) in Lemma 7 under $\beta > \beta_2$.

Case I.II; $\beta \in (\beta_1, \beta_2)$**.** Then,

$$\begin{array}{ccc}
\frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & > & \max\{.\} = \frac{\beta c}{p_{WS}-p_{SS}} \\
& \Longleftrightarrow & \\
p_{WS}-p_{SS} & > & \beta\left[(1-q)\left(p_{WS}-p_{SS}\right)+q\left(p_W-p_S\right)\right] \\
& \Longleftrightarrow & \\
\beta & < & \beta_2
\end{array}$$

which holds true. Thus, if $\beta \in (\beta_1, \beta_2)$, $W(\tilde{S})$ is chosen in $\left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{\beta c}{p_{WW}-p_{WS}}\right)$ and by exclusion $S(\tilde{W})$ is chosen in $\left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right)$. Once again this is a full characterization; that is, (s^U) in Lemma 7 under $\beta \in (\beta_1, \beta_2)$.

Case II; $\beta < \beta_1$.

For $W(\tilde{S})$ to be optimal,

$$\begin{array}{rcl}
\dfrac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & > & \max\{.\} = \dfrac{c}{(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)} \\
& \Longleftrightarrow & \\
q(p_W-p_S) & < & \left[1-\beta(1-q)\right]\left[(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)\right] \\
& \Longleftrightarrow & \\
p_{WW}-p_{WS} & > & \beta\left[(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)\right] \\
& \Longleftrightarrow & \\
\beta & < & \beta_1
\end{array}$$

which holds true. Thus, if $\beta<\beta_1$, $W(\tilde{W})$ is chosen above $\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}$ and $W(\tilde{S})$ never chosen. Is $S(\tilde{W})$ chosen? $S(\tilde{W})\succ S(\tilde{S}) \iff V>\frac{\beta c}{p_{WS}-p_{SS}}$ and $\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}>\frac{\beta c}{p_{WS}-p_{SS}} \iff \beta<\frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})}=\beta_3$ which holds by $\beta<\beta_1<\beta_3$. This concludes the characterization; that is, (s^U) in Lemma 7 under $\beta<\beta_1$.

To end the proof we have to check whether the procrastinator sticks to the first-period (s^U) when the second-period comes, as we checked for (s^{A2}). A procrastinator and a rational agent face the same second-period action maximization problem, *given* the first-period action and the occurence of the second period. Thus, we differentiate between a procrastinator who shirked or worked in the first period, and check whether the second period action coincides with the planned one (in the brackets).

First-period action: S.

The second-period choice is between SW and SS; in particular, $SW\succ SS \iff p_{WS}V-c>p_{SS} \iff V>\frac{c}{p_{WS}-p_{SS}}$.

1. If $\beta>\beta_2$, having played S as a first-period action requires $V<\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}$.[23] Since $\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}<\frac{c}{p_{WS}-p_{SS}}$, the second-period action is always S. Thus, $S(\tilde{S})$ is confirmed in s^U.

2. If $\beta\in(\beta_1,\beta_2)$, having played S as a first-period action requires $V<\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. Since $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}>\frac{c}{p_{WS}-p_{SS}} \iff \beta<\frac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)(p_{WS}-p_{SS})}\equiv\beta_7$. We need to check if $\beta<\beta_7$ when $\beta\in(\beta_1,\beta_2)$. First,

$$\begin{array}{rcl}
\beta_7 & < & \beta_2 \\
& \Longleftrightarrow & \\
\dfrac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)\left(p_{WS}-p_{SS}\right)} & < & \dfrac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)\left(p_{WS}-p_{SS}\right)} \\
& \Longleftrightarrow & \\
(p_{WS}-p_{SS}) & < & q(p_W-p_S)+(1-q)\left(p_{WS}-p_{SS}\right)
\end{array}$$

[23] See, (s^U) in Lemma 7 under $\beta>\beta_2$.

which holds true. However,

$$\begin{array}{ccc}
\beta_7 & > & \beta_1 \\
& \Longleftrightarrow & \\
\dfrac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)\,(p_{WS}-p_{SS})} & > & \dfrac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)\,(p_{WW}-p_{WS})} \\
& \Longleftrightarrow & \\
p_{WS}-p_{SS} & > & q(p_W-p_S)+(1-q)\,(p_{WW}-p_{WS}) \\
& \Longleftrightarrow & \\
q & < & \bar{q}=\dfrac{(p_{WS}-p_{SS})-(p_{WW}-p_{WS})}{(p_W-p_S)-(p_{WW}-p_{WS})}\in(0,1)
\end{array}$$

Notice that this is the same $\bar{q}$ threshold of the **(Conv)** case in the basic model. Thus, when $\beta \in (\beta_1, \beta_2)$ there are subcases:

(a) if $q > \bar{q}$, then $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}}$ and thus SS is chosen;

(b) if $q < \bar{q}$, then there are two subsubcases,

  i. $\beta \in (\beta_1, \beta_7)$ where $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} > \frac{c}{p_{WS}-p_{SS}}$ and thus, SS is chosen if $V < \frac{c}{p_{WS}-p_{SS}}$ and SW is chosen if $V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right)$

  ii. $\beta \in (\beta_7, \beta_2)$ where $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}}$ and thus, SS is chosen.

3. If $\beta < \beta_1$, having played S as a first-period action requires $V < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}$. Since $\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}} \iff q > \bar{q} = \frac{(p_{WW}-p_{WS})-(p_{WS}-p_{SS})}{(p_{WW}-p_{WS})-(p_W-p_S)}$, then there are two subcases:

(a) if $q > \bar{q}$, then SS is better than SW;

(b) if $q < \bar{q}$, then SS is better if $V < \frac{c}{p_{WS}-p_{SS}}$ and SW is better if $V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right)$.

First-period action: W.

The second-period choice is between WS and WW; in particular, $WW \succ WS \iff V > \frac{c}{p_{WW}-p_{WS}}$.

1. If $\beta > \beta_2$, having played W as a first-period action requires $V > \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}$. Since $\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, there are two regions;

   if $V \in \left(\frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right)$ then the action is WS, while if $V > \frac{c}{p_{WW}-p_{WS}}$ then the action is WW.

2. If $\beta \in (\beta_1, \beta_2)$, having played W as a first-period action requires $V > \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. Notice that $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, since it is equivalent to $\beta > \beta_5 \equiv \frac{(p_{WW}-p_{WS})-q(p_W-p_S)}{(1-q)(p_{WW}-p_{WS})}$ which holds since $\beta_5 < \beta_1 < \beta_2$.[24] Thus, since $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, WS is chosen in $V \in \left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right)$, and WW is chosen if $V > \frac{c}{p_{WW}-p_{WS}}$.

[24] In fact, $\beta_5 < \beta_1 \iff \frac{(p_{WW}-p_{WS})-q(p_W-p_S)}{(1-q)(p_{WW}-p_{WS})} < \frac{p_{WW}-p_{WS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})} \iff (p_{WW}-p_{WS}) < q(p_W-p_S)+(1-q)\,(p_{WW}-p_{WS})$

3. If $\beta < \beta_1$, having played W as a first-period action requires $V > \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}$. Since $\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, WW is chosen if $V > \frac{c}{p_{WW}-p_{WS}}$, and WS is chosen if $V \in \left(\frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right)$.

■

Proof of Theorem 2. The characterization of Lemma 7 suffices to rank p^A and p^U. In particular, notice that s^U can take one of five different specifications according to β and q;

1. If $\beta > \beta_2$, the ranking of key-thresholds is $\frac{c}{p_W-p_S} < \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}} < \frac{c}{p_{WW}-p_{WS}}$, so there are three regions to analyze.[25] From the lowest to the highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{W, SS\}$, thus $p^A > p^U$; in the second, $W\tilde{S}$ or $\{W, SS\}$, thus $p^A < p^U$; in the third, $W\tilde{S}$ or $\{W, SW\}$, thus $p^A = p^U$. Thus, the result of Theorem 1 holds and all the thresholds of V are identical to the case $\beta = 1$.

2. If $\beta \in (\beta_1, \beta_2)$, $\beta < \beta_7$ **and** $q < \bar{q}$, the ranking of key-thresholds is $\frac{c}{p_W-p_S} < \frac{c}{p_{WS}-p_{SS}} < \frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, so there are three regions to analyze. From the lowest to the highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{W, SS\}$, thus $p^A > p^U$; in the second, $S\tilde{W}$ or $\{W, SW\}$, thus $p^A > p^U$; in the third, $W\tilde{S}$ or $\{W, SW\}$, thus $p^A = p^U$. Thus, $p^A \geq p^U$ holds everywhere.

3. If $\beta \in (\beta_1, \beta_2)$, $\beta > \beta_7$ **or** $q > \bar{q}$, the ranking of key-thresholds is $\frac{c}{p_W-p_S} < \frac{c[1-\beta(1-q)]}{q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}} < \frac{c}{p_{WW}-p_{WS}}$, so there are three regions to analyze. From the lowest to the highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{W, SS\}$, thus $p^A > p^U$; in the second, $W\tilde{S}$ or $\{W, SS\}$, thus $p^A < p^U$; in the third, $W\tilde{S}$ or $\{W, SW\}$, thus $p^A = p^U$. Thus, the result of Theorem 1 holds.

4. If $\beta < \beta_1$ and $q > \bar{q}$, the ranking of key-thresholds is $\frac{c}{p_W-p_S} < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} < \frac{c}{p_{WS}-p_{SS}} < \frac{c}{p_{WW}-p_{WS}}$, so there are three regions to analyze. From the lowest to the highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{W, SS\}$, thus $p^A > p^U$; in the second, $W\tilde{S}$ or $\{W, SS\}$, thus $p^A < p^U$; in the third, $W\tilde{S}$ or $\{W, SW\}$, thus $p^A = p^U$. Thus, the result of Theorem 1 holds.

5. If $\beta < \beta_1$ and $q < \bar{q}$, the ranking of key-thresholds is $\frac{c}{p_W-p_S} < \frac{c}{p_{WS}-p_{SS}} < \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}}$, so there are three regions to analyze. From the lowest to the highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{W, SS\}$, thus $p^A > p^U$; in the second, $S\tilde{W}$ or $\{W, SW\}$, thus $p^A > p^U$; in the third, $W\tilde{S}$ or $\{W, SW\}$, thus $p^A = p^U$. Thus, $p^A \geq p^U$ holds everywhere.

■

[25] We exclude the two extreme regions where always S or always W are chosen, because $p^A = p^U$.

Appendix B.2: The (Conv) case

Lemma 8 *Fix* $\{c, q\}$. *Assume* ***(Conv)*** *and that the agent is a procrastinator. The agent's optimal* ***planned*** *action is* (s^{A1}) *as* s^{A1} *in Lemma 5, and*

$$\text{If } \beta > \beta_5,\ (s^{A2}) = \begin{cases} S(S) & \text{if } V < \frac{(\beta+1)c}{p_{WW}-p_{SS}} \\ W(W) & \text{if } V > \frac{(\beta+1)c}{p_{WW}-p_{SS}} \end{cases}$$

$$\text{If } \beta < \beta_5,\ (s^{A2}) = \begin{cases} S(S) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(W) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W(W) & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta > \beta_3,\ (s^{U}) = \begin{cases} S(\tilde{S}) & \text{if } V < \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \\ W(\tilde{W}) & \text{if } V > \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} \end{cases}$$

$$\text{If } \beta < \beta_3,\ (s^{U}) = \begin{cases} S(\tilde{S}) & \text{if } V < \frac{\beta c}{p_{WS}-p_{SS}} \\ S(\tilde{W}) & \text{if } V \in \left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right) \\ W(\tilde{S}) & \text{if } V \in \left(\frac{c[1-\beta(1-q)]}{q(p_W-p_S)}, \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right) \\ W(\tilde{W}) & \text{if } V > \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} \end{cases}$$

The agent's ***actual*** *action is* $s^{A1} = (s^{A1})$, *and*

$$\text{If } \beta > \beta_5,\ s^{A2} = (s^{A2})$$

$$\text{If } \beta < \beta_5,\ s^{A2} = \begin{cases} SS & \text{if } V < \frac{c}{p_{WW}-p_{WS}} \\ WW & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta > \beta_3,\ s^{U} = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{p_{WS}-p_{SS}} \\ S\tilde{W} & \text{if } V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}\right) \\ W\tilde{S} & \text{if } V \in \left(\frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}, \frac{c}{p_{WW}-p_{WS}}\right) \\ W\tilde{W} & \text{if } V > \frac{c}{p_{WW}-p_{WS}} \end{cases}$$

$$\text{If } \beta < \beta_3,\ s^{U} = \begin{cases} S\tilde{S} & \text{if } V < \frac{c}{p_{WS}-p_{SS}} \\ S\tilde{W} & \text{if } V \in \left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right) \\ W\tilde{W} & \text{if } V > \frac{c[1-\beta(1-q)]}{q(p_W-p_S)} \end{cases}$$

where $\beta_5 \equiv \frac{p_{WS}-p_{SS}}{p_{WW}-p_{WS}}$.

Proof. Under $A1$ the optimal strategy s^{A1} coincides with the one under **(Conc)**. Under $A2$, the four possible actions WW, WS, SW and SS yield respectively $p_{WW}V - c - \beta c$, $p_{WS}V - c$, $p_{WS}V - \beta c$ and $p_{SS}V$, thus action WS is never chosen as first-period plan of action since dominated by SW. We have three thresholds in V, $WW \succ SW \iff V > \frac{c}{p_{WW}-p_{WS}}$, $SW \succ SS \iff V > \frac{\beta c}{p_{WS}-p_{SS}}$, and $WW \succ SS \iff V > \frac{(\beta+1)c}{p_{WW}-p_{SS}}$. According to the ranking of first, second and third threshold we have two cases;

1. If $\beta > \beta_5 = \frac{p_{WS}-p_{SS}}{p_{WW}-p_{WS}}$ then $\frac{c}{p_{WW}-p_{WS}} < \frac{\beta c}{p_{WS}-p_{SS}}$, and thus SW is never chosen. This yields the (s^{A2}) of Lemma 8. Then the second period comes and

the procrastinator realizes that W is more costly. If the first-period action was S, $SS \succ SW \iff V < \frac{c}{p_{WS}-p_{SS}}$, but since

$$\begin{array}{ccc} \dfrac{c}{p_{WS}-p_{SS}} & > & \dfrac{(\beta+1)c}{p_{WW}-p_{SS}} \\ & \iff & \\ (p_{WW}-p_{WS})+(p_{WS}-p_{SS}) & > & (p_{WS}-p_{SS})+\beta\,(p_{WS}-p_{SS}) \\ & \iff & \\ p_{WW}-p_{WS} & > & \beta\,(p_{WS}-p_{SS}) \end{array}$$

then SS is always preferred. If instead the first-period action was W, $WW \succ WS \iff V > \frac{c}{p_{WW}-p_{WS}}$, but since $\frac{c}{p_{WW}-p_{WS}} < \frac{2c}{p_{WW}-p_{SS}}$, then WW is always chosen. Thus, if $\beta > \beta_5$, $s^{A2} = (s^{A2})$.

2. If $\beta < \beta_5$, then $\frac{c}{p_{WW}-p_{WS}} > \frac{\beta c}{p_{WS}-p_{SS}}$, and thus actions WW, SW and SS are chosen. This yields the (s^{A2}) of Lemma 8. Then the second period comes and the procrastinator realizes that W is more costly. If the first-period action was S, $SS \succ SW \iff V < \frac{c}{p_{WS}-p_{SS}}$, but since $\frac{c}{p_{WS}-p_{SS}} > \frac{c}{p_{WW}-p_{WS}}$, then SS is always preferred. If instead the first-period action is W, $WW \succ WS \iff V > \frac{c}{p_{WW}-p_{WS}}$, thus WW is always chosen. This yields the s^{A2} of Lemma 8.

We are left to characterize the optimal actions under U. All the possible pairwise comparisons of actions yield the same thresholds of V's as in the case of (**Conc**), thus we do not report them here again (see, the proof of Lemma 7).

Optimality of $W(\tilde{W})$.

$(s^U) = W(\tilde{W}) \iff V > \max\{.\}$.[26] We already discussed the value of $\max\{.\}$ in the proof of Lemma 7, however under **(Conv)** from Lemma 6 we know that $1 < \beta_2 < \beta_1 < \beta_4$. Since $\beta < 1$ by assumption, $\max\{.\} \neq \frac{\beta c}{p_{WW}-p_{WS}}$. Thus, $(s^U) = W(\tilde{W}) \iff V > \max\{.\} = \begin{cases} \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} & \text{if } \beta > \beta_3 \\ \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)} & \text{if } \beta < \beta_3 \end{cases}$

Optimality of $S(\tilde{S})$.

$(s^U) = S\tilde{S} \iff V < \min\{.\}$. We already discussed the value of $\min\{.\}$ in the proof of Lemma 7, however under **(Conv)** from Lemma 6 we know that $1 < \beta_2 < \beta_1 < \beta_4$. Since $\beta < 1$ by assumption, $\min\{.\} \neq \frac{c}{(1-q)(p_{WS}-p_{SS})+q(p_W-p_S)}$. Thus, $(s^U) = S\tilde{S} \iff V < \min\{.\} = \begin{cases} \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} & \text{if } \beta > \beta_3 \\ \frac{\beta c}{p_{WS}-p_{SS}} & \text{if } \beta < \beta_3 \end{cases}$

Optimality of $S(\tilde{W})$ and $W(\tilde{S})$.

1. If $\beta > \beta_3 \in (0,1)$, $\max\{.\} = \min\{.\}$. This yields the (s^U) of Lemma 8.

2. If $\beta < \beta_3$, in the range $\left(\frac{\beta c}{p_{WS}-p_{SS}}, \frac{c}{(1-q)(p_{WW}-p_{WS})+q(p_W-p_S)}\right)$ neither $W(\tilde{W})$ nor $S(\tilde{S})$ are optimal, and $W(\tilde{S}) \succ S(\tilde{W}) \iff V > \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. This threshold of

[26] $\max\{.\}$ and $\min\{.\}$ have been defined in the proof of Lemma 7.

V turns out to be above min{.} and below max{.}. In fact,

$$\begin{array}{rcl} \frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & > & \frac{\beta c}{p_{WS}-p_{SS}} \\ & \Longleftrightarrow & \\ p_{WS}-p_{SS} & > & \beta\left[(1-q)\left(p_{WS}-p_{SS}\right)+q(p_W-p_S)\right] \\ & \Longleftrightarrow & \\ \beta & < & \frac{p_{WS}-p_{SS}}{(1-q)\left(p_{WS}-p_{SS}\right)+q(p_W-p_S)} \in (1,\infty) \end{array}$$

and

$$\begin{array}{rcl} \frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & < & \frac{c}{(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)} \\ & \Longleftrightarrow & \\ (1-q)\left(p_{WW}-p_{WS}\right) & > & \beta(1-q)\left[(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)\right] \\ & \Longleftrightarrow & \\ \beta & < & \frac{p_{WW}-p_{WS}}{(1-q)\left(p_{WW}-p_{WS}\right)+q\left(p_W-p_S\right)} \in (1,\infty) \end{array}$$

This yields the (s^U) of Lemma 8.

Notice that if $\beta = 1$, the condition $\beta > \beta_3$ coincides with $q < \bar{q}$ (see, Lemma 7). This simple fact shows that the above results generalizes the $\beta = 1$ analysis.

To end the proof we have to check whether the procrastinator sticks to the first-period (s^U) when the second-period comes, as we checked for (s^{A2}). A procrastinator and a rational agent face the same second-period action maximization problem, *given* the first-period action and the occurence of the second period. Thus, we differentiate between a procrastinator who shirked or worked in the first period, and check whether the second period action coincides with the planned one (in the brackets).

First-period action: S.

The second-period choice is between the $S\tilde{W}$ and $S\tilde{S}$; in particular, $S\tilde{W} \succ S\tilde{S} \iff V > \frac{c}{p_{WS}-p_{SS}}$.

1. If $\beta < \beta_3$, having played S as a first-period action requires $V < \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. Notice that,

$$\begin{array}{rcl} \frac{c\left[1-\beta(1-q)\right]}{q(p_W-p_S)} & > & \frac{c}{p_{WS}-p_{SS}} \\ & \Longleftrightarrow & \\ (p_{WS}-p_{SS})-q(p_W-p_S) & > & \beta(1-q)\left(p_{WS}-p_{SS}\right) \\ & \Longleftrightarrow & \\ \beta & < & \frac{(p_{WS}-p_{SS})-q(p_W-p_S)}{(1-q)\left(p_{WS}-p_{SS}\right)} \in (1,\infty) \text{ by } \textbf{(Conv)} \end{array}$$

Thus, in $\left(0, \frac{c}{p_{WS}-p_{SS}}\right)$ the procrastinator plays $S\tilde{S}$ while in $\left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}\right)$ the procrastinator plays $S\tilde{W}$.

2. If $\beta > \beta_3$, having played S as a first-period action requires $V < \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}$. Notice that,

$$\begin{aligned}
\frac{c\left[1+\beta(1-q)\right]}{(1-q)\left(p_{WW}-p_{SS}\right)+q\left(p_{W}-p_{S}\right)} &> \frac{c}{p_{WS}-p_{SS}} \\
&\Longleftrightarrow \\
\left(p_{WS}-p_{SS}\right)+\beta(1-q)\left(p_{WS}-p_{SS}\right) &> (1-q)\left(p_{WW}-p_{SS}\right)+q\left(p_{W}-p_{S}\right) \\
&\Longleftrightarrow \\
\beta(1-q)\left(p_{WS}-p_{SS}\right) &> (1-q)\left(p_{WW}-p_{SS}\right)+q\left(p_{W}-p_{S}\right)-\left(p_{WS}-p_{SS}\right) \\
&= (1-q)\left(p_{WW}-p_{WS}\right)+q\left[\left(p_{W}-p_{S}\right)-\left(p_{WS}-p_{SS}\right)\right] \\
&\Longleftrightarrow \\
\beta &> \frac{(1-q)\left(p_{WW}-p_{WS}\right)+q\left[\left(p_{W}-p_{S}\right)-\left(p_{WS}-p_{SS}\right)\right]}{(1-q)\left(p_{WS}-p_{SS}\right)} \\
&\equiv \beta_6
\end{aligned}$$

But since in what follows we prove that $\beta_6 < \beta_3$, then $\beta > \beta_6$ when $\beta > \beta_3$ and thus there are two regions; in $\left(0, \frac{c}{p_{WS}-p_{SS}}\right)$ the action chosen is $S\tilde{S}$, while in $\left(\frac{c}{p_{WS}-p_{SS}}, \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}\right)$ the action chosen is $S\tilde{W}$, as specified in the s^U of Lemma 8.

$$\beta_6 < \beta_3$$

$$\Longleftrightarrow$$

$$\frac{(1-q)\left(p_{WW}-p_{WS}\right)+q\left[\left(p_{W}-p_{S}\right)-\left(p_{WS}-p_{SS}\right)\right]}{(1-q)\left(p_{WS}-p_{SS}\right)} < \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)\left(p_{WW}-p_{WS}\right)}$$

$$\Longleftrightarrow$$

$$\begin{aligned}
&(1-q)q\left(p_{WW}-p_{WS}\right)\left(p_{W}-p_{S}\right)+q^2\left(p_{W}-p_{S}\right)^2-q^2\left(p_{WS}-p_{SS}\right)\left(p_{W}-p_{S}\right)+ \\
&+(1-q)^2\left(p_{WW}-p_{WS}\right)^2+(1-q)q\left(p_{WW}-p_{WS}\right)\left(p_{W}-p_{S}\right)-(1-q)q\left(p_{WW}-p_{WS}\right)\left(p_{WS}-p_{SS}\right)+ \\
&-(1-q)(p_{WS}-p_{SS})^2<0
\end{aligned}$$

$$\Longleftrightarrow$$

$$\begin{aligned}
&q\left(p_{WW}-p_{WS}\right)\left(p_{W}-p_{S}\right)-q^2\left(p_{WW}-p_{WS}\right)\left(p_{W}-p_{S}\right)+q^2\left(p_{W}-p_{S}\right)^2+(1-q)q\left(p_{WS}-p_{SS}\right)\left(p_{W}-p_{S}\right) \\
&-q\left(p_{WS}-p_{SS}\right)\left(p_{W}-p_{S}\right)+(1-q)\left(p_{WW}-p_{WS}\right)^2-(1-q)q\left(p_{WW}-p_{WS}\right)^2+ \\
&+(1-q)q\left(p_{WW}-p_{WS}\right)\left(p_{W}-p_{S}\right)-(1-q)q\left(p_{WW}-p_{WS}\right)\left(p_{WS}-p_{SS}\right)-(1-q)(p_{WS}-p_{SS})^2+ \\
&+(1-q)\left(p_{WW}-p_{WS}\right)\left(p_{WS}-p_{SS}\right)-(1-q)\left(p_{WW}-p_{WS}\right)\left(p_{WS}-p_{SS}\right)<0
\end{aligned}$$

The above expression is the sum of 12 elements. Take as a common factor $(1-q)\left(p_{WW}-p_{WS}\right)$ from elements 6-7-8-12, $(1-q)\left(p_{WS}-p_{SS}\right)$ from elements 4-9-10-11, and $q(p_W-p_S)$ from elements 1-2-3-5. Each of this common factor multiplies the same term, $\{(p_{WW}-p_{WS})-(p_{WS}-p_{SS})+q\left[(p_W-p_S)-(p_{WW}-p_{WS})\right]\}$, which thus cancels out, so $\beta_6 < \beta_3$ is equivalent to

$$\begin{aligned}
&\left\{(1-q)\left(p_{WW}-p_{WS}\right)+(1-q)\left(p_{WS}-p_{SS}\right)+q(p_W-p_S)\right\}* \\
&*\left\{\left(p_{WW}-p_{WS}\right)-\left(p_{WS}-p_{SS}\right)+q\left[\left(p_{W}-p_{S}\right)-\left(p_{WW}-p_{WS}\right)\right]\right\}<0
\end{aligned}$$

The term in the first curly brackets is positive. It is easy to see that the second is negative if and only if $q < \bar{q}$. If $\beta > \beta_3$, $\beta_3 < 1$, and thus $q < \bar{q}$ (see, Lemma 6). This concludes the proof that $\beta_6 < \beta_3$.

First-period action: W.
The second-period choice is between the $W\tilde{W}$ and $W\tilde{S}$; in particular, $W\tilde{W} \succ W\tilde{S} \iff V > \frac{c}{p_{WW}-p_{WS}}$.

1. If $\beta > \beta_3$, having played W as a first-period action requires $V > \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)}$. Since $\frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} > \frac{c}{p_{WW}-p_{WS}} \iff \beta > \frac{(1-q)(p_{WS}-p_{SS})+q[(p_W-p_S)-(p_{WW}-p_{WS})]}{(1-q)(p_{WW}-p_{WS})} \equiv \beta_8$, and since routine algebra which we omit shows that $\beta > \beta_3 \implies \beta > \beta_8$, we obtain the s^U of Lemma 8, where both $W\tilde{W}$ and $W\tilde{S}$ can be chosen.

2. If $\beta < \beta_3$, having played W as a first-period action requires $V > \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$. Since $\frac{c[1-\beta(1-q)]}{q(p_W-p_S)} > \frac{c}{p_{WW}-p_{WS}} \iff \beta < \frac{(p_{WW}-p_{WS})-q(p_W-p_S)}{(1-q)(p_{WW}-p_{WS})} \in (1,\infty)$ by **(Conv)**, then $W\tilde{W}$ is always chosen.

■

Proof of Theorem 3. The characterization of Lemma 8 suffices to rank p^A and p^U. Additionally, it is immediate to see that $\beta_5 < \beta_3$ since $\frac{p_{WS}-p_{SS}}{p_{WW}-p_{WS}} < \frac{p_{WS}-p_{SS}}{q(p_W-p_S)+(1-q)(p_{WW}-p_{WS})}$. Thus,

1. If $\beta > \beta_3$, the ranking of key-thresholds is $\frac{(\beta+1)c}{p_{WW}-p_{SS}} < \frac{c}{p_{WS}-p_{SS}} < \frac{c[1+\beta(1-q)]}{(1-q)(p_{WW}-p_{SS})+q(p_W-p_S)} < \frac{c}{p_{WW}-p_{WS}} < \frac{c}{p_W-p_S}$, so there are four regions to analyze. From lowest to highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{S, WW\}$, thus $p^A > p^U$; in the second, $S\tilde{W}$ or $\{S, WW\}$, thus $p^A > p^U$; in the third, $W\tilde{S}$ or $\{S, WW\}$, thus $p^A \gtrless p^U$, but it does not matter for the final result; in the fourth, $W\tilde{W}$ or $\{S, WW\}$, thus $p^A < p^U$. Thus, the result of Theorem 1 holds.

2. If $\beta < \beta_3$,[27] the ranking of key-thresholds is $\frac{(\beta+1)c}{p_{WW}-p_{SS}} < \frac{c}{p_{WS}-p_{SS}} < \frac{c}{p_W-p_S} < \frac{c[1-\beta(1-q)]}{q(p_W-p_S)}$, so there are three regions to analyze. From lowest to highest, we compare the actions under U and under A; in the first, $S\tilde{S}$ or $\{S, WW\}$, thus $p^A > p^U$; in the second, $S\tilde{W}$ or $\{S, WW\}$, thus $p^A > p^U$; in the third, $S\tilde{W}$ or $\{W, WW\}$, thus $p^A > p^U$. Thus, $p^A \geq p^U$ holds everywhere.

■

[27] Notice that it does not matter whether $\beta < \beta_5$ or $\beta \in (\beta_5, \beta_3)$.

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