Optimal mixed taxation, credit constraints and the timing of income tax reporting[*]

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Abstract

We study optimal income and commodity tax policy with credit-constrained low-income households. Workers are assumed to receive an even flow of income during the tax year, but make tax payments or receive transfers at the end of the year. They use their disposable income to purchase multiple commodities over the year. We show that differentiated subsidies on commodities can be optimal even if the Atkinson-Stiglitz Theorem conditions apply. When the optimal policy leaves low-income households with binding credit constraints, it is optimal to subsidize the good that is consumed in higher proportion by them. We show that this involves subsidizing more goods that fulfill basic needs, such as food or dwelling. The benefits of such subsidies have to be balanced with the costs of financing them, since unconstrained households also benefit from the rebate early in the fiscal year.

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1 Introduction

Many government transfer programs are income-tested and delivered through the tax system. Examples include refundable tax credits that decline in income, such as the Earned Income Tax Credit, the Additional Child Tax Credit and the Health Coverage Tax Credit in the U.S.; the Working Income Tax Benefit, the Canada Child Benefit and the Goods and Services Tax Credit in Canada; and the Working Tax Credit, the Child Tax Credit, and the Universal Credit in the U.K. A key feature of these transfer programs is that entitlements cannot be fully determined until the taxpayer's income tax form has been filed and approved by the tax authorities. In the above examples, transfer payments are paid periodically in a given year based on taxable income (or family income) of the previous year. In some cases, adjustments can occur while the transfers are being received if the taxpayer's circumstances change in a way that can be verified by the government, such as childbirth or change in employment or disability status.

The consequence is that transfer recipients' income flow is lumpy. Those with low enough income to be eligible for a transfer from the government will have low—possibly zero—income during the year and a large transfer starting after the year ends. Individuals who anticipates a transfer would like to smooth their consumption stream over the year by borrowing. However, they may be precluded from doing so by a credit constraint. Financial institutions may be unwilling to lend to them except at exorbitant interest rates, especially if they do not have a credit rating or if the financial institution cannot verify the expected transfer.

We adopt an optimal income and commodity tax perspective to study policy responses to this issue. The informational assumptions of optimal taxation accord well with the problem. The model we use is stylized and meant to capture the essential features of the information constraint faced by the government and the credit constraint faced by transfer recipients. Unlike in the standard optimal income tax setting, we assume that individuals receive an even flow of income during the tax year, but make tax payments or receive transfers at the

end of the year.[1] Individuals use their disposable income to purchase a flow of multiple commodities over the tax year. The government knows only the workers' labor incomes at the end of the year. However, following Guesnerie (1995), we assume that the government observes all anonymous transactions on commodity markets and can impose a set of linear commodity taxes or subsidies at the time the purchases occur. Therefore, if the government wants to undertake some redistribution before the end of the fiscal year, implicit transfers can be made through commodity subsidies and could be targeted to the intended individuals by a differential rate structure.

Our main focus is on the case where individuals are credit constrained which can prevent them from smoothing their consumption over the fiscal year. The credit constraint becomes especially relevant when the government's redistribution scheme implies paying transfers at the end of the year. With perfectly functioning credit markets, those anticipating transfers would borrow throughout the year to smooth the consumption financed by their future transfer. Then, the standard results of optimal tax theory would hold, including the well-known Atkinson & Stiglitz (1976) theorem when labor and consumption are weakly separable. However, when transfer recipients face a binding credit constraint that precludes them from smoothing their consumption, giving transfers at the end of the year does not achieve the government's redistributive objectives earlier in the year. And, the government cannot provide optimal transfers before the end of the tax year since it does not have the required information to determine who is entitled to them. We show that differentiated subsidies on commodities can be welfare-improving even if the Atkinson-Stiglitz Theorem conditions apply.

The idea that consumption tracks income due to credit constraints is well established. For example, in the buffer-stock model of Deaton (1991), consumers' inability to borrow and impatience predicts that consumption will track income and that credit constraints can

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[1]In practice, tax remittance are often made throughout the year by employers through payroll deductions, but this only applies for taxpayers and not transfer recipients. Ignoring these remittances will have no effect on our analysis since those who pay taxes face no credit constraint.

be binding. Various studies using U.S. data confirm this. Using evidence on caloric intake of food stamp recipients, Shapiro (2005) finds that the short-term discount rate of these individuals is very high and hardly reconciliable with geometric discounting. Studying the effect of stimulus payments from the 2001 tax cut episode to explain the phenomenon of 'wealthy hand-to-mouth' who own mostly illiquid assets, Gruber (1997) finds evidence that unemployment insurance, which is paid on a frequent basis, significantly smooths household consumption. Parker (1999) finds that consumers do not perfectly smooth their demand for goods when they expect a change in their income (although, in their case, the complexity in the tax code may be at stake). More recently, Aguila et al. (2017) found in a natural experiment that smoothing cash-transfers over the year facilitates consumption smoothing. In particular, they find that more frequent cash-transfer programs are associated with more consistent spending on basic needs, such as food and doctor appointments.

Another source of evidence comes from household behavior during the months when the Earned Income Tax Credit (EITC) is received. McGranahan & Schanzenbach (2013) find that households who are eligible for the EITC spend relatively more on healthy items during the months when most refunds are paid. Among these healthy items one finds vegetables, meat, poultry and dairy products. In a recent survey paper, Nichols & Rothstein (2016) stress that "[households] are often unable to borrow at reasonable interest rates (as evidenced by the high take-up of extremely high interest refund anticipation loans). If credit constraints are binding, a lump-sum payment has a smaller effect on the household's utility than would a series of smaller payments throughout the year." They also note that until 2010, EITC recipients could apply for a partial advance payment throughout the year. Although a small proportion of individuals opted-in, the most plausible explanation for taking up the credit would be that individuals are severely credit constrained.

In a recent work, Baker (2017) finds that the income elasticity of consumption is significantly higher for highly indebted households (after controlling for net assets). He concludes that "credit constraints play a dominant role in driving differential household consumption

responses across households with varying levels of debt." Also, using data from households who experienced a temporary income reduction during the U.S. federal government shutdown in 2013, Baker & Yannelis (2015) find indications that households who have better access to credit or who have accumulated more savings exhibit significantly smaller spending reductions during the transitional shock.

In the following sections, we study optimal income and commodity tax policy with credit-constrained low-income households in a standard nonlinear income taxation setting. The model features several skill-types of households who supply labor and consume two commodities. To simplify matters, we assume that transactions can occur at two discrete points: in the middle of the period and at the end. Preferences are weakly separable so in the absence of credit constraints, optimal commodity taxes will be uniform at indeterminate rates given that proportional commodity taxation is equivalent to proportional income taxation. The two commodities are not consumed in the same proportions by different skill-types, and this will lead to differential commodity subsidization in the presence of credit constraints. The credit constraint will take the simplest of forms. As well, for reasons to be explained, it will be costly for the government to make budgetary expenditures before the end of the period. Doing so requires it to borrow against its end-of-period tax revenues.

In principle, the government could make a uniform lump-sum payment to all persons at the beginning of the period. Combining a lump-sum transfer with non-differentiated commodity taxes would be equivalent to a linear progressive tax system and would allow the government to redistribute at the beginning of the period even if it had no information on individuals incomes. If preferences were weakly separable in goods and labor and quasi-homothetic in good — the Deaton (1979) conditions — non-differentiated commodity taxes would be optimal, and this would have implications for our analysis. In our analysis, we assume that the government does not use a uniform lump-sum transfer at the beginning of the period. In particular, we assume that all components of the direct tax-transfer system are implemented at the end of the period. We return to a discussion of beginning-of-period

lump-sum taxes at the end.

2   Model

There are N types of individuals who are indexed by $i \in \{1, ..., N\}$. The number of type$-i$ individuals is n_i, each of whom has exogenous productivity w^i. The whole population is normalized to one so that $\sum_{i=1}^{N} n_i = 1$. The economy lasts for one period, which we can think of as a tax year. We divide the period into two sub-periods $t = 1, 2$, and assume that each individual works with the same intensity in both sub-periods and earns a gross income $Y^i/2$ in each. At the end of $t = 2$, a type$-i$ individual pays an income tax T^i (or receives a transfer if it takes a negative value). When individuals choose their labor supplies ex-ante, they know their end-of period income tax liability and therefore their disposable income over both sub-periods.

We use the methodology of Christiansen (1984) to introduce consumption of commodities into the model. In each sub-period t, type$-i$ individuals choose a consumption bundle consisting of two goods (c_t^i, d_t^i). The producer prices of goods c and d are set to unity, and the consumer prices can include a commodity tax, which can equivalently be either per unit or ad valorem: $q_c \equiv 1 + t_c$ and $q_d \equiv 1 + t_d$. Commodity taxes t_c and t_d are the same for both sub-periods and for all individuals since otherwise arbitrage opportunities would exist. An individual's utility function is assumed for simplicity to take the following additive form:

$$U^i(c_t^i, d_t^i, Y_i) = \sum_t u(c_t^i - \bar{c}, d_t^i) - h\left(\frac{Y^i}{w^i}\right) \tag{1}$$

where Y^i/w^i is labor supply in each of the two sub-periods, and $h(\cdot)$ is a strictly convex cost or disutility function. The function $u(\cdot, \cdot)$ is the per-period utility of consuming the bundle of goods. To ensure that commodity tax differentiation is not a by-product of nonlinear Engel curves, we sometimes assume that $u(\cdot, \cdot)$ is quasi-homothetic in c_t^i and d_t^i by introducing a

basic need $\bar{c}$ on good c and letting $u(\cdot, \cdot)$ be homothetic in $c_t^i - \bar{c}$ and d_t^i. The quantity $\bar{c}$ could stand for a minimal quantity of food or shelter. For simplicity, we assume that individuals do not discount their utility across periods, which does not restrict our results. Note that although individuals supply labor in both sub-periods, the disutility of labor supply is defined over total (annual) labor supply. Since commodities are separable from labor or leisure in the utility function (1), the Atkinson-Stiglitz Theorem would apply in this model in the absence of a credit constraint, as we confirm below.[2]

We introduce imperfections in the credit market in the form of a credit constraint. The credit constraint applying in the first sub-period is

$$q_c c_1^i + q_d d_1^i \leqslant \frac{Y^i}{2} + \phi, \tag{2}$$

where ϕ is exogenously given. In what follows, we assume $\phi = 0$ so individuals are precluded from borrowing. Individuals have access to a competitive credit market if they want to save or are able to borrow. Those who save do so at rate $\underline{r}$ and those who borrow do so at rate $\bar{r}$, with $\underline{r} \leqslant \bar{r}$. This reflects the cost of financial intermediation. For an individual i, we denote by $r_i \in \{\underline{r}, \bar{r}\}$ depending on whether, in the optimum, he is respectively a net saver or borrower at $t = 1$. If the government borrows, it can do so at rate $r_g > \underline{r}$, meaning that it borrows at a higher rate than the risk-free rate at which individuals can invest their short-term savings.[3]

Under these assumptions, we shall see that the two sub-period setting gives the same solution as a standard Mirrlees problem when there is no credit spread, that is, when $\underline{r} = \bar{r} = r_g$. This is our benchmark case which we study first. Then, we introduce a borrowing constraint that prevents individuals from using more than ϕ dollars of their end-of-year transfers as a collateral when applying for a loan. As mentioned, a simple case is when

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[2]The model assumes that individuals commit to their labor supply and that labor supply is the same across periods. This does not drive the results and simplifies the analysis.

[3]In particular, this prevents the fiscal policy from being a Ponzi scheme and eliminates arbitrage opportunities.

$\phi = 0$, which mimics the corner solution one would obtain if borrowers faced an interest rate $\bar{r}$ that is prohibitively high. Given that our model abstracts from solvency issues and financial risks related to lending to individuals, this is a simple way to introduce credit market frictions without explicitly modeling solvency risks.[4]

To be precise, in the case where there is no credit constraint, the annual budget constraint for a type$-i$ individual is (from an end-of-year standpoint)

$$(q_c c_1^i + q_d d_1^i)(1 + r_i) + q_c c_2^i + q_d d_2^i \leqslant \frac{Y^i}{2}(1 + r_i) + \frac{Y^i}{2} - T^i.$$

Since individuals earn $Y^i/2$ every sub-period and only pay their taxes (get their transfers) T^i at the end of the year and they can make transactions in the financial markets, the nonlinear tax problem amounts to choosing annual disposable income defined as

$$I^i \equiv \left(\frac{2 + r_i}{2} \right) Y^i - T^i. \tag{3}$$

Therefore, one can rewrite the individual's annual budget constraint as

$$(q_c c_1^i + q_d d_1^i)(1 + r_i) + q_c c_2^i + q_d d_2^i \leqslant I^i. \tag{4}$$

2.1 Tax normalizations

In the standard static optimal income and commodity tax analysis, uniform commodity taxes are equivalent to a proportional income tax. This implies that the absolute level of commodity taxes is indeterminate: reducing commodity tax rates proportionately and increasing the income tax rate by the same amount will have no effect on equilibrium outcomes. Commodity taxes can then be normalized by, for example, setting one commodity tax rate

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[4]A more complex model would involve risk. Then, it would be costlier to banks to lend to individuals and the interest rate for borrowers would be high. This would give us the same intuition, but would significantly complicate the problem.

to zero. In our setting, this is not possible if credit constraints are binding. That is because while commodity taxes are paid on purchases in both sub-periods, income taxes apply only at the end of the periods.

To illustrate, suppose the government imposes undifferentiated commodity taxes $t_c = t_d$. In the absence of binding credit constraints and assuming no interest rate spread between borrowing and lending, it can reach the same allocation by taxing everyone's yearly income at the proportional rate $t_Y = t_c/(1 + t_c) = t_d/(1 + t_d)$. In this case, we can normalize one consumption tax to zero and let the flat revenue-collection component be captured by the proportional tax on income (leisure). Recall, however, that income taxes are collected at the end of the period, while commodity taxes apply in each sub-period. Thus, the time stream of tax liabilities will differ under the two systems. A uniform commodity tax system will generate tax liabilities in both sub-periods while income tax revenues will be paid at the end of the period. This difference in timing has no real effect in the absence of credit constraints and interest rate spreads. The analogous result applies in the case where a uniform commodity subsidy is applied.

However, when an individual's borrowing constraint binds, this equivalence does not hold. The income tax is not paid in the first period, so with $\phi = 0$ the binding credit constraint (2) becomes

$$(1 + t_c)c_1^i + (1 + t_d)d_1^i = \frac{Y^i}{2}. \tag{5}$$

A proportional increase in commodity tax rates will tighten the credit constraint in (5), while a corresponding proportional decrease in the income tax rate will not undo this tightening. Therefore, proportional commodity taxes or subsidies are not equivalent to proportional income taxes or subsidies. The absolute level of commodity tax rates matters so we cannot normalize one rate to zero.

Note further that (5) does not contain a tax on its right-hand side. Therefore, if the government want to tax income specifically in the first period, it has to do it through the

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taxation of goods. Similarly, if he wants to redistribute in the first period, it has to do it either through a subsidy on goods or through a uniform lump-sum subsidy to all individual in the first sub-period (since it cannot identify individuals by type then).

In what follows, we treat the absolute levels of commodity tax rates as government policy variables along with the nonlinear income tax system. Unlike in the standard models of optimal income and commodity taxation, our analysis yields a well-defined tax mix.

2.2 Government's budget constraint

The government's budget constraint in absolute terms in end-of-period values is

$$\sum_i \left(\left(\frac{2 + r_i}{2} \right) Y^i - I^i \right) + (1 + r_g)(q_c - 1) \sum_i c_1^i + (q_c - 1) \sum_i c_2^i + (1 + r_g)(q_d - 1) \sum_i d_1^i$$

$$+ (q_d - 1) \sum_i d_1^i = R. \tag{6}$$

where R is an exogenous revenue requirement. Note that the discount factor r_g is used to obtain the end-of-period values of commodity tax receipts in the first sub-period. That is because we are assuming that the government is a net borrower. If it subsidizes commodities in the first sub-period, it must borrow at the rate r_g to finance those subsidies. Some of the benefit of the subsidies accrues to high-income individuals who are savers and obtain a return $\underline{r}$ on their savings. The fact that $r_g > \underline{r}$ makes it socially costly to transfer resources to them in $t = 1$. By the same token, if the government taxes commodities in the first sub-period, it reduces its borrowing and the saving of high-income individuals also decreases, which again saves resources since $r_g > \underline{r}$. However, the credit constraint is tightened for low-income individuals for whom it binds.

3 Optimal tax mix

We derive the government's optimal tax structure using a standard mechanism design problem for income taxes augmented by a choice of commodity tax rates. The government offers bundles of income and disposable income (Y^i, I^i) intended for types i, where income is earned equally over the two sub-periods. Then, using (3) taxes paid at the end of the period are residually given by $T^i = (2 + r_i)Y^i/2 - I^i$, where T^i can be negative for low-productivity types. The government also chooses t_c and t_d, or equivalently q_c and q_d. As we shall see, when an individual is credit constrained in the optimum, the optimal price ratio q_c/q_d will generally differ from unity. We begin by characterizing individual behavior, and then turn to the government's problem.

3.1 Individual behavior

We solve the type-i individual's problem in two steps in reverse order. In the second step, knowing Y^i, I^i, q_c and q_d, the individual chooses bundles (c_t^i, d_t^i) for $t = 1, 2$. In the first step and anticipating the outcomes of the second step, the individual chooses from the bundles of income and disposable income (Y^i, I^i) offered by the government.[5]

3.1.1 Step 2: Choice of commodity bundles

Given Y^i, I^i, q_c, q_d, individuals of type i choose commodity bundles (c_t^i, d_t^i) to maximize utility (1) subject to the annual budget constraint (4) and the credit constraint (2). The value function for this problem is:

$$\psi^i(Y^i, I^i, q_c, q_d) = \max_{c_t^i, d_t^i} \sum_{t=1,2} u(c_t^i - \bar{c}, d_t^i) + \theta^i \left[I^i - \sum_{t=1,2} (1 + r_i)^{t-1} (q_c c_t^i + q_d d_t^i) \right]$$

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[5]For a similar approach, see Edwards et al. (1994)

$$-\mu^i \left[q_c c_1^i + q_d d_1^i - \left(\frac{Y^i}{2} + \phi \right) \right], \tag{7}$$

where the credit constraint takes the values $\phi \in \{0, \infty\}$, depending on the specific case under study. Applying the envelope theorem to the value function $\psi^i(\cdot)$,

$$\psi_I^i = \theta^i, \quad \psi_Y^i = \frac{\mu^i}{2}, \quad \psi_{q_c}^i = -\theta^i \sum_{t=1,2} (1+r_i)^{t-1} c_t^i - \mu^i c_1^i, \quad \psi_{q_d}^i = -\theta^i \sum_{t=1,2} (1+r_i)^{t-1} d_t^i - \mu^i d_1^i. \tag{8}$$

Note that $d\psi^i / d\phi = \mu^i$. Since consumer utility is non-decreasing in the size of the credit constraint ϕ, that implies $\mu^i \geqslant 0$ with the inequality applying when the constraint is binding. Note also that, by definition, $\mu^i = 0 \; \forall i$ when $\phi \to \infty$.

3.1.2 Step 1: Choice of income and net income bundles

Given commodity tax rates (t_c, t_d) and anticipating step 2 above, the government on behalf of individuals of the two types offers income-consumption bundles (Y^i, I^i). In an optimum, individuals choose the bundles intended for them. This yields total utility for a type-i person:

$$V^i(Y^i, I^i, q_c, q_d) = \psi^i(Y^i, I^i, q_c, q_d) - h\left(\frac{Y^i}{w^i} \right). \tag{9}$$

Using the envelope results (8) on ψ^i, $V^i(\cdot)$ satisfies the following properties:

$$V_Y^i = \frac{\mu^i}{2} - \frac{1}{w^i} h'\left(\frac{Y^i}{w^i} \right), \quad V_I^i = \psi_I^i, \quad V_{q_c}^i = \psi_{q_c}^i, \quad V_{q_d}^i = \psi_{q_d}^i. \tag{10}$$

Preferences of an individual of type i in (Y, I)-space have a slope:

$$\frac{dI^i}{dY^i} = -\frac{V_Y^i}{V_I^i} = \frac{1}{\theta^i} \left[\frac{1}{w^i} h'\left(\frac{Y^i}{w^i} \right) - \frac{\mu^i}{2} \right]$$

Finally, denote $\widehat{V}^i$ as the total indirect utility of a type i who mimics a type $-i$. The mimicker will have the same income stream so will face the same credit constraint as the

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individual being mimicked. Analogously to V^i in (9), indirect utility is given by:

$$\widehat{V}^i(q_c, q_d, Y^{-i}, I^{-i}) = \psi^i(Y^{-i}, I^{-i}, q_c, q_d) - h\left(\frac{Y^{-i}}{w^i}\right). \tag{11}$$

Similar envelope properties to (10) apply, and the slope of the mimicker's indifference curves will be:

$$\frac{d\widehat{I}^i}{d\widehat{Y}^i} = -\frac{\widehat{V}_Y^i}{\widehat{V}_I^i} = \frac{1}{\theta^{-i}}\left[\frac{1}{w^i}h'\left(\frac{\widehat{Y}^i}{w^i}\right) - \frac{\mu^{-i}}{2}\right]$$

3.2   Tax implementation

Tax implementation involves finding marginal tax rates that implement the optimality conditions derived using mechanism design analysis. Doing so involves relating marginal tax rates to individual behavior as follows. The government implements a nonlinear tax function $T(Y^i)$. Using (3), we can rewrite the expression for indirect utility in (9) as

$$V^i(\cdot) = \psi^i\left(Y^i, \left(\frac{2+r_i}{2}\right)Y^i - T(Y^i), q_c, q_d\right) - h\left(\frac{Y^i}{w^i}\right).$$

The individual chooses income Y^i to maximize $V^i(\cdot)$. Using the envelope conditions (8), the first-order condition can be written

$$\psi_Y^i + \psi_I^i\frac{\partial I^i}{\partial Y^i} - h_Y\left(\frac{Y^i}{w^i}\right) = \frac{\mu^i}{2} + \theta^i \cdot \left(\frac{2+r_i}{2} - T'(Y)\right) - \frac{1}{w^i}h'\left(\frac{Y^i}{w^i}\right) = 0.$$

Isolating the marginal tax rate gives

$$T'(Y^i) = \frac{2+r_i}{2} - \frac{1}{\theta^i w^i}h'\left(\frac{Y^i}{w^i}\right) + \frac{1}{2}\frac{\mu^i}{\theta^i}. \tag{12}$$

This expression for $T'(Y^i)$ defines the marginal tax wedge facing a type-i individual in terms of the individual's preferences. It implies that an individual supplies more labor when credit is constrained for a given marginal tax rate since then $\mu^i > 0$, and θ^i is smaller than in the

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benchmark case. Below we use (12) to characterize the marginal tax rates that implement the solution to the government's optimal tax problem.

3.3 Government's problem

In our problem, the government redistributes from more productive to less productive individuals. We use the methodology developed by Hellwig (2007) — also applied by Bastani (2015) — to derive optimal tax schedules with a finitely large number of types. The government maximizes social welfare:

$$W = \sum_i n_i \Phi(V^i)$$

subject to the budget constraint (6) and to $N - 1$ incentive compatibility (IC) constraints that take the form of downward adjacent constraints,

$$V^i(Y^i, I^i, q_c, q_d; w^i) \geqslant \widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, q_d; w^i) \qquad \forall i. \qquad (\gamma^i)$$

where $\Phi(V^i)$ is a concave social utility function, with $\Phi'(V^i) > 0$ and $\Phi''(V^i) \leqslant 0$. The function $\widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, q_d; w^i)$ in the IC constraints is the indirect utility obtained by a type i who mimics the adjacent lower type $i-1$, so is given by (11). The equation indicators γ^i represent the Lagrangian multipliers of the incentive constraints in the government's problem, and λ is the Lagrangian multiplier for the budget constraint (6). Note that for R small enough, at least one type (the lowest) receives a transfer.

Given our assumption about preferences, all individuals would smooth their consumption across sub-periods 1 and 2 in the absence of credit constraints, albeit imperfectly. Credit constraints will be binding only for those expecting a transfer at the end of the period since then they will want to borrow in sub-period 1. Those who pay positive taxes will save at $t = 1$ to spread their tax liabilities across sub-periods.

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We consider the government problem in three successive settings of increasing complexity. We begin with the benchmark case in which no one is credit-constrained and there is no credit spread. We then assume a credit constraint with $\phi = 0$ that is binding on at least one type (the lowest), but restrict the government to using a nonlinear income tax. The credit spread is irrelevant in this case since no individuals will borrow, and the government gets all its revenues at the end of the period. In the final case, both credit constraints and a credit spread apply, and we let the government choose differentiated commodity taxes or subsidies alongside the nonlinear income tax. We denote by $\mathcal{L}(\cdot)$ the Lagrangian function of the government. The first-order conditions for the government's problem in the third, most general, setting where the credit constraint is binding for at least one type and the government chooses commodity tax rates are listed in Appendix A.

3.3.1   Benchmark case: unconstrained individuals and no credit spread

This case corresponds to the standard optimal nonlinear income tax problem with linear commodity taxes. All individuals and the government can borrow and lend at the common interest rate r. First, we establish that the government need not use commodity taxation at all, and then we characterize the optimal income tax system.

The government can normalize commodity taxes by setting $q_c = 1$, and then optimize on relative commodity prices which we denote by $\alpha = q_d/q_c$. Choosing α is equivalent to choosing $q_d = 1 + t_d$. Since individuals are not credit-constrained, $\mu^i = 0$, in the individual's value function (7). Using the envelope properties for the individuals in (8) and (10), the government's first-order conditions shown in Appendix A lead to the standard Atkinson-Stiglitz theorem:

Proposition 1. *When $i = 1, ..., N$ are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.*

Proof: See Appendix B.

Thus, the Atkinson-Stiglitz theorem continues to apply even though consumption and labor supply occur sequentially over the tax year. The theorem stipulates only that commodity taxes should be uniform if used, but since uniform commodity taxes are equivalent to a proportional income tax in this benchmark case, they are redundant and thus unnecessary.

Since credit constraints are not binding in this case, $\mu^i = 0$, so the definition of the marginal tax wedge in (12) simplifies to

$$T'(Y^i) = \left(\frac{2+r_i}{2}\right) - \frac{1}{\theta^i w^i} h'\left(\frac{Y^i}{w^i}\right). \tag{13}$$

Using the first-order conditions in Appendix A to rewrite the right-hand side of (13), we obtain the following marginal tax formulas in an optimum:

$$T'^*(Y^i) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i}\left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}}\right). \tag{14}$$

As shown by Hellwig (2007), the term in parentheses is always positive when the single-crossing condition is satisfied and when leisure is an normal good. Moreover, $\gamma^{N+1} = 0$ since there is no downward incentive constraint at the top. Therefore, marginal tax rates are everywhere positive except at Y^N, for which $T'(w^N) = 0$ so there is no distortion. These are the standard optimal income tax results.

3.3.2 Case with binding credit constraints and no commodity taxes

Suppose now that transfers to the lowest types are sufficiently large that the credit constraint on at least one type is binding, so $\mu^i > 0$ for at least some i. Those whose credit constraint does not bind pay taxes at the end of the period and save for it, whereas the poorest ones would have liked to borrow using future transfers as collateral but they cannot. Since there

are no commodity taxes in this case, the government has no revenues and no expenses in the first sub-period so r_g does not need to be specified. Those who save do so at rate $r_i = \underline{r}$. In the optimum, the marginal income tax rate for individual i is again given by (14), which we can rewrite as:

$$T'^*(Y^i) = \frac{\gamma^{i+1}}{\lambda n_i}\left(\frac{h'(Y^i/w^i)}{w^i} - \frac{h'(Y^i/w^{i+1})}{w^{i+1}}\right). \tag{15}$$

Substituting the first-order condition of the government with respect to I^i, which is (27) in Appendix A, into the first-order condition with respect to Y^i, (28), we obtain

$$\frac{\partial \mathcal{L}}{\partial Y^i} = \quad -n_i\Phi'(V^i)\frac{1}{w^i}h'\left(\frac{Y^i}{w^i}\right) - \gamma^i\frac{1}{w^i}h'\left(\frac{Y^i}{w^i}\right) + \gamma^{i+1}\frac{1}{w^{i+1}}h'\left(\frac{Y^i}{w^{i+1}}\right)$$
$$+\lambda\left(\frac{2+r_i}{2}\right)n_i + \lambda n_i\frac{\mu^i}{2} = 0, \qquad \forall i. \tag{16}$$

The last term in (16) is strictly positive for constrained individuals, and equals zero when individuals are unconstrained. It is the additional benefit of increasing constrained individuals' cash-on-hand in the first-period by making them work more and earn more income, and thereby relaxing their credit constraint. Everything else being equal, increasing Y^i means, from (15), that the optimal marginal tax rates will be pushed down for constrained agents whenever $h(\cdot)$ is strictly convex. Since mimicking becomes less attractive when credit constraints are binding, γ^{i+1} falls and this argument is reinforced. The Lagrange multiplier on the government's budget constraint, λ, must also be higher when credit constraints bind.

3.3.3 Case with binding credit constraints and income and commodity taxes

When the government has access to commodity taxes or subsidies, it can use them to relax the binding incentive constraints by subsidizing consumption in sub-period 1. But, this comes at a cost since it must borrow at the rate r_g to finance the subsidies. Given that commodity subsidization also benefits the unconstrained individuals, saving of the latter is increased. Since $r_g > \underline{r}$ government saving accompanied by private dissaving results in a resource cost.

In this case, two outcomes are possible. First, subsidization of commodities may be sufficient to eliminate the credit constraint of all low-income individuals. Alternatively, the cost of commodity subsidization may be sufficiently large that in an optimum, some low-income individuals remain credit-constrained. Since the qualitative results differ in the two cases, we consider them separately.

Case A. Credit constraints eliminated in the optimum

In this case, which happens when $r_g - \underline{r}$ is small enough, the government can use commodity taxes to undo the credit constraint of all individuals. The following proposition is proved in Appendix B.

Proposition 2: If in the optimum $\mu^i = 0$, $\forall i$, so policy relaxes all credit constraints in the economy, then $t_c^* = t_d^* < 0$.

Thus, both goods are subsidized at the same rate. The commodity subsidy system acts as a proportional subsidy on income. Since credit constraints are not binding, the equivalent of a second-best is recovered, although with interest rates higher for borrowers (the government) than for savers.

To ensure that the entire tax system maximizes social welfare in an incentive-compatible way, income tax rates are adjusted to reflect the fact that uniform commodity subsidies are equivalent to a subsidy on income. In the optimum, the effective marginal income tax rate of individual i taking account of both income tax and commodity subsidy distortions is identical to the one obtained in the benchmark case without credit constraints and commodity subsidies.

In particular, the optimal marginal tax rate faced by individual i is now

$$T'^*(Y^i) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right) - t^* \sum_t (1+r_g)^{2-t} \left(z_i - T'^*(Y^i) \right) \left(\frac{\partial c_t^i}{\partial I^i} + \frac{\partial d_t^i}{\partial I^i} \right),$$

(17)

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where $t^* \equiv t_c^* = t_d^*$ and $z_i = (2 + r_i)/2$. This tax formula, analogous to that derived by Edwards et al. (1994), shows that when the government subsidizes consumption proportionally, this creates purchasing power that is identical to an increase in net income. Therefore, a share $(z - T'(Y^i))$ of the "income value" of the subsidy has to be left in the individuals' pockets, adjusted for the funding cost of the subsidies r_g.

The tax formula in (17) reflects the fact that the wedge between labor and consumption must encompass the marginal incentives and disincentives generated by all tax instruments. This can be seen by rewriting (17) as a marginal effective tax rate:

$$
\begin{aligned}
T'^{E*}(Y^i) &\equiv T'^*(Y^i) + t^* \sum_t (1 + r_g)^{2-t} (z_i - T'^*(Y^i)) \left(\frac{\partial c_t^i}{\partial I^i} + \frac{\partial d_t^i}{\partial I^i} \right) \\
&= \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right),
\end{aligned}
\tag{18}
$$

where the righthand side is the optimal labor wedge in (14). This shows that subsidies on consumption goods must be clawed back by increases in marginal income tax rates. It also shows that the standard properties of the optimal tax systems still apply, including non-negative marginal tax rates at all income levels and a zero effective marginal tax rate at the top.

Finally, note that the most extreme case of such an optimum would be when subsidies can be funded at no opportunity cost for the government, or when $r_g = \underline{r}$. Then, transferring purchasing power from the second to the first period is done at no cost for the government, and it can always lower the prices of commodities so as to make all credit constraints in the economy slack. In this unrealistic example, the timing of income-tested payments has no effective consequence on the overall tax policy and social optimum. As soon as subsidies become costly, there is a threshold level of the cost beyond which the government will leave some individuals credit constrained.

Case B. Some credit constraints binding in the optimum

When the costs of funding commodity subsidies in the first period are high enough, it may be optimal for the government to leave some individuals' credit constraints binding. In this case, differential subsidy or tax rates should apply, but it may be optimal either to subsidize both goods, or to subsidize one and tax the other. Because the objective of subsidization is to transfer purchasing power to the first-period, commodity tax income (if any) is always smaller than subsidies expenses for the government.

Going back to the intuition that underlies the Atkinson-Stiglitz theorem helps us to understand this result. In a standard optimal taxation model without credit constraints, commodity taxes are a redundant policy instrument if labor and consumption are weakly separable in the utility function. If used, all commodity taxes (or subsidies) are undifferentiated, and they are equivalent to a proportional tax on income. Redundancy arises because differentiated commodity taxation generates both income and substitution effects. The income effects can be entirely cancelled out by adjusting the income tax schedule.[6] Therefore, differential commodity taxes or subsidies would be optimal if substitution effects were desirable. Since these distortions play no role in relaxing incentive-compatibility constraints, commodity taxes are proportional if used.

When the government finds it optimal to leave some credit constraints binding at the bottom of the skills distribution, this intuition fails. Because constrained individuals receive a transfer later in the fiscal year and cannot smooth consumption, a government that uses commodity taxation cannot adjust its optimal income tax schedule to offset income effects created by commodity taxes (subsidies) in the first period. When the government wants to increase constrained individuals' purchasing power in the first period, subsidization becomes a non-redundant, and potentially useful, policy tool. However, because of the opportunity cost of financing subsidies early in the year, the government wants to use this tool as economically as possible, which involves subsidizing more intensely the good that is proportionately consumed more by those who are constrained (good c).

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[6]In a way, the government can therefore repay itself for the subsidies granted, or reduce average tax rates to compensate individuals when commodities are taxed.

This may happen even when utility-of-consumption functions $u(\cdot)$ feature linear Engel curves. Our simple case in which there is a basic need for good c, $\bar{c}$, justifies either subsidizing good c at a higher rate, or simply subsidizing c and taxing d if $r_g - \underline{r}$ is very high. These results are illustrated in the numerical section below. Proposition 3, whose proof is given in Appendix B, gives the general condition under which differentiation happens under linear Engel curves.

Proposition 3: Define $B_i \equiv (\mu_i/\lambda)(\Phi'(V^i) + \gamma^i/n_i - \gamma^{i+1}/n_i) - (r_g - r_i)$. Also denote by $\mathcal{C} \neq \emptyset$ the subset of types whose credit constraint bind in the optimum: $i \in \mathcal{C} \Leftrightarrow \mu^i > 0$. Then, the optimal policy has $t_d > t_c$ if and only if

$$\left(\frac{\sum_i n_i B_i c_1^i}{\sum_{i \in \mathcal{C}} n_i c_1^i} - \frac{\sum_i n_i B_i d_1^i}{\sum_{i \in \mathcal{C}} n_i d_1^i} \right) > 0. \tag{19}$$

Overall, Proposition 3 states that the optimal policy involves differentiating commodity taxes, with $t_c < t_d$, if we find a higher benefit/cost ratio of subsidizing good c than good d. Re-expressing (19) enables us to interpret the proposition more intuitively. Denote the ratio of the quantities of each good consumed by all *constrained* type-i individuals to the aggregate quantities they consume as:

$$\xi_i^c \equiv \frac{n_i c_1^i}{\sum_{j \in \mathcal{C}} n_i c_1^i}; \quad \xi_i^d \equiv \frac{n_i d_1^i}{\sum_{j \in \mathcal{C}} n_i d_1^i}, \quad \forall i \in \mathcal{C}, \tag{20}$$

Note that for low-income individuals, $\xi_i^c > \xi_i^c$ when good c is subject to a minimum need level $\underline{c}$. Denote by $b_i \equiv \Phi'(V^i) + \gamma^i/n_i - \gamma^{i+1}/n_i$ the marginal social weight associated with giving one individual $i \in \mathcal{C}$ one dollar lump-sum in the first period. We presume that, at least at the bottom of the distribution, b_i is decreasing with i due to the redistributive objective of the social planner and strict concavity of utility functions. Then, substituting

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these expressions into (19), the optimal policy involves $t_c^* < t_d^*$ if

$$\underbrace{\frac{1}{\lambda} \sum_{i \in \mathcal{C}} \mu^i b_i (\xi_i^c - \xi_i^d)}_{\text{Equity}} - \underbrace{(r_g - \underline{r}) \left(\frac{\sum_{i \notin \mathcal{C}} n_i c_1^i}{\sum_{j \in \mathcal{C}} n_j c_1^j} - \frac{\sum_{i \notin \mathcal{C}} n_i d_1^i}{\sum_{j \in \mathcal{C}} n_j d_1^j} \right)}_{\text{Efficiency}} > 0. \qquad (21)$$

The equity term applies only to constrained individuals who have $\mu^i > 0$, confirming the redistributive role of differentiation only when some credit constraints bind at the optimum. For a type i, it is increasing with the marginal social weight b_i, with the marginal cost of being constrained μ^i, and good c is favored when poorer constrained individuals consume proportionately more of it. The efficiency term in (21) is the relative cost of favoring good c. It is proportional to the credit spread $r_g - \underline{r}$, and is greater than zero when all *unconstrained* individuals consume a larger share of good c in the economy than of good d.

The equity and efficiency terms go in opposite directions when $\bar{c}$ increases. Corollary 1, which is stated below and proven in Appendix A, shows that if Engel's curves are linear and there are no basic needs, then it is optimal for commodity tax rates to be undifferentiated. When $\bar{c} = 0$, all individuals consume goods c and d in same proportions. The Corollary implies that credit constraints or basic needs alone are not sufficient to justify differentiation.

Corollary 1: If $\bar{c} = 0$, then there is no differentiation and $t_c^* = t_d^* < 0$. This result is independent of the specific social welfare function $\Phi(\cdot)$ used by the social planner.

Finally, when commodity taxes are differentiated, the government must adjust its optimal income tax schedule in consequence. The optimal effective marginal tax rate, which is

obtained from the government's and agents' first-order conditions, is

$$T'^{E*}(Y^i) \equiv T'(Y^i) + t_c^* \left[\sum_t (1 + r_i)^{2-t} \left(\frac{\partial c_t^i}{\partial I^i}(z_i - T'^*(Y^i)) + \frac{\partial c_t^i}{\partial Y^i} \right) \right]$$
$$+ t_d^* \left[\sum_t (1 + r_i)^{2-t} \left(\frac{\partial d_t^i}{\partial I^i}(z_i - T'^*(Y^i)) + \frac{\partial d_t^i}{\partial Y^i} \right) \right] = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left(\frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right).$$

$$(22)$$

4 Numerical examples

We illustrate our results with some numerical simulations. We assume that utility of consumption and disutility of labor functions in (1) are constant elasticity, and the utility of consumption is quasi-homothetic. These functions are given by

$$u(c_t^i, d_t^i) = \kappa \frac{(c_t^i - \bar{c})^{1-\rho}}{1 - \rho} + \kappa \frac{d_t^{i \, 1-\rho}}{1 - \rho}; \tag{23}$$

$$h(\ell) = \frac{\ell^{1+\sigma}}{1 + \sigma}, \quad \ell \equiv Y/w. \tag{24}$$

We set the values of the parameters to $\rho = 0.9$ and $\sigma = 2$. Recall that the case without credit constraints is analogous to a standard second-best, but with the fiscal year divided into two subperiods. For convenience, we choose $\kappa = (1/4)^\rho$, which implies that with $\underline{r} = 0$ and $\bar{c} = 0$ the simulations give the same optimal tax system as if the fiscal year were not divided into subperiods.

The number of workers at each wage level, n_i, follows a lognormal distribution with parameters $(\mu, \sigma) = (2.757, 0.5611)$. It is taken from Mankiw et al. (2009), who estimate these parameters from the 2007 March wave of the Current Population Survey (CPS). A discrete version of the distribution is used to obtain 100 wage levels with a fixed distance between any two wage levels. The probability mass function is rescaled so that $\sum_i n_i = 1$. We initially set the interest rate spread to 15 percentage points (which is a bit lower than

borrowing rates on credit cards), with $\underline{r} = 0$ and $r_g = 0.15$, although sensitivity analyses are are also computed.

The year 2007 is used to calibrate our simulations to make them comparable with Mankiw et al. (2009). We use the basic need $\bar{c} = 5$, which at a unit consumer price approximately represents \$5.75 daily, or \$2,100 annually. This is a conservative amount, and compares with the average Supplemental Nutrition Assistance Program ("food stamp" program) payment of about \$3 a day in 2007. This is well below the Federal poverty line **of** \$10,210 (\$28 per day) for a single-person household.

We compute total taxes paid by individuals at given levels of income, and the effective marginal tax rates as in Edwards et al. (1994). The cumulative total tax payment by a worker with labor income Y^i evaluated from the government's point of view at the end of the period is

$$T^E(Y^i) = T(Y^i) + t_c \left[\sum_t (1 + r_g)^{2-t} c_t^i \right] + t_d \left[\sum_t (1 + r_g)^{2-t} d_t^i \right]. \tag{25}$$

Baseline simulations

We compute baseline simulations for each of the three scenarios considered in Section 3 and leading to Propositions 1–3. The first one is the *No credit constraint* scenario where individuals (and the government) can save and borrow at the same rate. The government can implement a nonlinear income tax as well as linear commodity taxes. This first scenario is a useful benchmark as it is analogous to a standard second-best optimal tax regime. Note that unlike the following cases, the government could reallocate funds costlessly between periods, although there is no need to do so since individuals have the same opportunity.

In the second scenario (*Credit constraint*) individuals are precluded from borrowing so $\phi = 0$. The government is restricted to nonlinear income taxation implemented at the end

of the period to achieve its redistributive objective. With $\underline{r} = 0$, individuals who expect to have negative tax liabilities at the end of the fiscal year $(T(Y^i) < 0)$ may want to borrow and will be unable to do so.

In the third scenario (*Credit constraint with subsidies*), the government can use commodity taxes or subsidies in addition to nonlinear income taxation. Commodity subsidization enables the government to reallocate funds from the end of the period to the beginning, but this entails government borrowing which is socially costly because of the credit spread.

In our baseline case, we use a utilitarian social welfare function, $W = \sum_i n_i V^i$ along with needs $\bar{c} = 5$ and credit spread $r_g - r_i = 0.15$. Besides our baseline calculations we examine the sensitivity of optimal tax policies to variations in the aversion to inequality, in needs $\bar{c}$, and in the credit spread $r_g - r_i$. The results are summarized in Tables 1–2 and Figures 1–4.

[*Table 1 about here*]

The key characteristics of the baseline case are shown in Table 1. In the third scenario, *Credit constraint with subsidies*, the two commodities are subsidized at rates between 12% and 14%, with the subsidy being greater for good c, which is consumed in higher proportions for low-skilled credit-constrained persons. The government is willing to distort consumption patterns of all persons in order to target subsidies more toward those who are credit-constrained. Compared with scenario two where commodity subsidization is ruled out, the proportion of individuals who are credit-constrained falls from about 62% to 43% when the government can deploy commodity subsidies. The utilitarian social welfare levels are reported for all three scenarios. The case with credit constraints and commodity taxes/subsidies dominates scenario two with credit constraints but no commodity subsidies. The table also presents as an alternate measure of social welfare the welfare gains starting from the laissez-faire allocation. This gain is calculated as the minimal percentage increase in consumption from the laissez-faire, required to attain the same welfare levels as in the

relevant scenarios ($\%\Delta^{LF}$). This percentage is fixed across individuals and time-periods.[7]. Once more, we observe that the case with commodity taxes is preferred to that without, though the difference is relatively small because the utilitarian social welfare function combined with limited curvature of the utility-of-consumption function restricts the weight put on lower-skilled workers.

[*Figure 1 about here*]

The characteristics of the optimal income tax systems in the baseline case are shown in Figure 1 for all three scenarios. The *No credit constraint* scenario is the standard Mirrleesian optimal income tax case. As the solid lines indicate, marginal income tax rates are high at the bottom and decline with incomes, initially very steeply. Average income tax rates are monotonically increasing although at a decreasing rate. Since there are no commodity taxes, actual and effective tax rates are the same. In scenario two where there are credit constraints but no commodity taxes, marginal tax rates at the bottom are very small and rising as shown by the dashed lines. The intuition is that the government wants to encourage labor supply and therefore subperiod 1 income in order to ameliorate the credit constraint. Note however that this does not translate into significant differences in average tax rates which remain very similar to the *No credit constraint* scenario.

The *Credit constraint with subsidies* scenario has a similar pattern of marginal income tax rates to scenario two, but the rates are much higher. The same applies for average tax rates. This reflects the fact that both goods are subsidized, which is equivalent to a subsidy to labor supply whose costs must be financed by the government. The income tax system compensates for this by increasing both marginal and average tax rates. The consequence is that effective marginal and average tax rates are quite similar for scenarios two and three. Looking at the optimal effective marginal tax rates, one sees that the *Credit constraint*

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[7]This way to account for welfare variations is documented in Farhi & Werning (2013) and Stantcheva (2017).

with subsidies scenario is an intermediate case between the *No credit constraint* and *Credit constraint* scenarios where commodity subsidies are not allowed.

Sensitivity analysis

We next consider how these results change when we vary the parameters of the model. Three sort of variations are considered: i) changes in the credit spread $r_g - r_i$, ii) changes the level of needs $\bar{c}$, and iii) changes in the weight put on lower-skilled persons. To achieve the latter, we replace the government's utilitarian social welfare function with a weighted utilitarian social welfare function $W \equiv \sum_i n_i \omega_i V^i$ with weights

$$\omega_i \equiv \frac{w_i^{-\eta}}{\sum_i n_i w_i^{-\eta}}, \quad \eta \geqslant 0; \quad \sum_i n_i \omega_i = 1 \tag{26}$$

This function puts more weight on workers with low wages as η, the aversion to inequality, increases. The utilitarian baseline case corresponds with the case where $\eta = 0$.

[Figure 4 about here]

Figure 4 shows how optimal commodity tax rates change with the credit spread, with needs and with various values for the aversion to inequality. In Figure 4a, needs are kept at the baseline level while the spread varies. For any given value of η, increases in the credit spread reduce the size of commodity subsidies while increasing their spread. This reflects the fact that higher values of $r_g - r_i$ increase the social cost of the government borrowing to finance subsidies in the first sub-period. The government responds by reducing the size of the subsidies and by targeting them more to good c which is consumed in greater proportions by low-income persons who are more likely to be credit-constrained. For small enough values of the spread, the two commodity tax rates are the same. As Proposition 2 indicates, when the subsidy rates are high enough, no individuals are credit-constrained and the government

deploys a uniform subsidy. Figure 4a also shows that as aversion to inequality increases from $\eta = 0$ to $\eta = 0.2$ and then to $\eta = 0.5$ so more weight is put on low-skilled individuals, the size of the subsidy increases thereby generating more redistribution.

Figure 4b considers variations in need holding the credit spread at its baseline level. As $\bar{c}$ increases, subsidy rates increase reflecting the fact that an increase in $\bar{c}$ increases the marginal utility of income relatively more for low-income individuals. The interactions of variations in $\bar{c}$ and η lead to further interesting insights. As in Figure 4a, increases in the aversion to inequality η increases subsidy rates as expected. Moreover, for $\eta = 0.5$ where subsidies are quite large, the subsidy rates t_c and t_d approach equality for high enough values of needs. This is another manifestation of Proposition 2. Large enough values of η and $\underline{c}$ cause the subsidy rates to be large enough to relax the credit constraint for all individuals in which case it is optimal to set $t_c = t_d$.

[*Table 2 about here*]

Table 2 shows how social welfare gains relative to the laissez-faire vary with η for the three scenarios. Naturally the gains are highest for the *No credit constraint* case, which is the standard second-best optimal income tax case. Welfare gains are higher in scenario three where commodity subsidies are used than in scenario two where they are not. But, it is striking that welfare gains with subsidies are relatively close to those in the *No credit constraint* case and much higher than in the *Credit constraint* case without subsidies. This indicates the importance of subsidies as a way of addressing credit constraints.

[*Figure 2 about here*]

Figure 2 illustrates how income tax schedules vary with the interest rate spread for the *Credit constraint with subsidies* case holding needs and aversion to inequality at their

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baseline levels ($\bar{c} = 5, \eta = 0$). When the differential is low so the cost to the government of subsidization is minimal, the optimal tax systems features large subsidies and requires higher labor income taxation as can been seen by both the marginal tax rates and average tax rates. As above, when the government can transfer more money in the first period, it can also distort the labor decision of lower skilled workers more and undertake more redistribution. This can be seen by looking at the effective marginal tax rates. The case with the highest spread, and thus the highest costs, also features the lowest effective marginal income tax rates since the government encourages work especially at the bottom of the distribution.

[*Figure 3 about here*]

Figure 3 illustrates the effect of variations in basic need on labor income taxes, holding $r_g - r_i$ and η at their baseline levels. Recall that with higher needs, consumption subsidies will be higher. The government then requires higher income tax levels to finance the subsidies. This is visible from the plots depicting marginal and average tax rates. The higher basic need also increases the level of redistribution, which results in higher effective marginal tax rates for all workers, but especially at the bottom of the income distribution. Average effective tax rates also increase with $\bar{c}$, since a higher basic need increases marginal utility of consumption of the necessity good at a given level of consumption. This reinforces the redistributive motive of the government.

5 Conclusions and an extension to lump-sum transfers

In this paper, we studied an optimal tax system when transactions on goods happen more frequently than the payment of income-tested transfers. Credit constraints arise because individuals cannot fully use future transfers as collateral. Our results show that when the optimal policy is able to relax the constraint on all individuals, it involves proportional subsidies on all goods. When the cost of providing the subsidies is too high, differentiation may

happen when constrained individuals spend a higher proportion of their disposable income on a good (for instance a necessity) than the general population. Then, the government can either subsidize all goods at differential rates, or subsidize some commodities while taxing others.

As mentioned in the Introduction, if the government could make lump-sum transfers in sub-period 1 separately from its income tax-transfer system at the end of the period, it could use them to ameliorate binding credit constraints. This opens the door to interesting interactions between commodity taxation and these payments.

Consider first the extreme case where $r_g = \bar{r} = \underline{r}$, so the lump-sum transfers could be made costlessly by the government borrowing against future tax revenues. The government will offer a universal lump-sum transfer at $t = 1$ sufficiently large to relax all credit constraints and reduce all tax liabilities at the end of the period by an equivalent amount. If this scheme is available to the planner, then the allocation would be identical to our benchmark case without credit constraints. There would be no need to use commodity subsidies and the Atkinson-Stiglitz Theorem would hold.

Suppose instead that $r_g > \underline{r}$ so the government faces a cost of transferring money from the second to the first sub-period. In this scenario, the government may no longer be able to front-load completely redistribution in the first sub-period and adjust the income tax schedule to leave all workers unconstrained. If the government uses only nonlinear income taxation along with the universal lump-sum transfer, the situation is similar to our second case above. Some individuals remain credit-constrained, and the standard form of the optimal marginal income tax rates but since some individuals remain credit-constrained social welfare is less than in the benchmark case.

When the government can use commodity taxes along with period-1 lump-sum transfers, things change potentially dramatically. Uniform commodity taxes combined with a lump-sum transfer is equivalent to a linear progressive tax system which transfers income from

high- to low-income individuals. Moreover, the commodity taxes generate tax revenue for the government to finance the lump-sum transfers thereby offsetting the need to borrow. If the government sets the commodity tax rate high enough, it would seem it could finance an amount of lump-sum transfers sufficient to relax all credit constraints of low-income individuals while at the same time avoiding the need to borrow. And, if the utility-of-consumption function $u(\cdot)$ satisfies the Deaton conditions, it would seem to be optimal to use undifferentiated commodity taxes.

In fact, the optimal allocation achieved in this manner appears likely to lead to more credit-constrained workers. This is because the high level of commodity taxes must apply in both periods. This induces the government to adjust the labor income tax schedule to make the allocation incentive compatible. To do this, the tax burden of workers is drastically reduced to the point where a majority of workers face net transfers from the labor income tax. This leads many workers to want to borrow but are unable to do so due to the credit constraint.

We conjecture that the uniform commodity tax result obtained will break down if Engel curves are nonlinear. Furthermore, the ability of the government to use commodity tax revenues obtained from period-1 commodity purchases to finance the lump-sum transfer requires that the government actually receive those revenues in period 1. In practice, the firms collecting commodity taxes will not remit them to the government until the end of the tax year and this will cause the above mechanism to break down. Even in the case if linear Engel curves, we conjecture that divorcing the timing of the collection of commodity tax revenues from the payment of the lump-sum transfer will also lead to differentiated commodity taxes. In fact, we should be able to recover many of the results found earlier in this paper. Proving these conjectures is left to further research.

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A First-order conditions of the general problem

The Lagrangian of the government is

$$\mathcal{L} = \sum_{i=1}^{N} n_i \Phi(V^i) + \sum_{i=2}^{N} \gamma^i [V^i(Y^i, I^i, q_c, q_d; w^i) - \widehat{V}^i(Y^{i-1}, I^{i-1}, q_c, q_d; w^i)]$$

$$+ \lambda \left[\sum_{i=1}^{N} n_i \left(\left(\frac{2+r_i}{2} \right) Y^i - I^i \right) + (q_c - 1) \sum_{i=1}^{N} n_i \sum_t (1 + r_g)^{2-t} c_t^i \right.$$

$$\left. + (q_d - 1) \sum_{i=1}^{N} n_i \sum_t (1 + r_g)^{2-t} d_t^i \right].$$

We present the first-order conditions in their most general form to keep the notation as compact as possible. Note that, by the definition of the problem, $\gamma^1 \equiv 0$ since the lowest type cannot mimic any lower adjacent type. Note, also, that all types that are not constrained in the optimum have $\mu^i = 0$. Those who are constrained have $\partial c_1^i / \partial I^i = \partial d_1^i / \partial I^i = 0$ and those who are not constrained have $\partial c_1^i / \partial Y^i = \partial d_1^i / \partial Y^i = 0$. The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial I^i} = n_i \Phi'(V^i)\theta^i + \gamma^i \theta^i - \gamma^{i+1}\theta^i - \lambda n_i + \lambda n_i (q_c - 1) \sum_t \frac{\partial c_t^i}{\partial I^i}(1 + r_g)^{2-t}$$

$$+ \lambda n_i (q_d - 1) \sum_t \frac{\partial d_t^i}{\partial I^i}(1 + r_g)^{2-t} = 0 \qquad \forall i, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial Y^i} = n_i \Phi'(V^i) \left(\frac{\mu^i}{2} - \frac{1}{w^i} h' \left(\frac{Y^i}{w^i} \right) \right) + \gamma^i \left(\frac{\mu^i}{2} - \frac{1}{w^i} h' \left(\frac{Y^i}{w^i} \right) \right)$$

$$- \gamma^{i+1} \left(\frac{\mu^i}{2} - \frac{1}{w^{i+1}} h' \left(\frac{Y^i}{w^{i+1}} \right) \right) + \lambda \left(\frac{2+r}{2} \right) n_i$$

$$+ \lambda n_i (q_c - 1) \sum_t \frac{\partial c_t^i}{\partial Y^i}(1 + r_g)^{2-t} + \lambda n_i (q_d - 1) \sum_t \frac{\partial d_t^i}{\partial Y^i}(1 + r_g)^{2-t} = 0 \qquad \forall i, \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial q_c} = \sum_i n_i \Phi'(V^i) \left(-\theta_i \sum_t (1+r_i)^{2-t} c_t^i - \mu^i c_1^i \right)$$

$$+ \sum_{i=1}^{N-1} (\gamma^i - \gamma^{i+1}) \left(-\theta^i \sum_t (1+r_i)^{2-t} c_t^i - \mu^i c_1^i \right)$$

$$+ \lambda (q_c - 1) \sum_i n_i \sum_t \frac{\partial c_t^i}{\partial q_c} (1+r_g)^{2-t} + \lambda (q_d - 1) \sum_i n_i \sum_t \frac{\partial d_t^i}{\partial q_c} (1+r_g)^{2-t}$$

$$+ \lambda \sum_i n_i \sum_t c_t^i (1+r_g)^{2-t} = 0. \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial q_d} = \sum_i n_i \Phi'(V^i) \left(-\theta_i \sum_t (1+r_i)^{2-t} d_t^i - \mu^i d_1^i \right)$$

$$+ \sum_{i=1}^{N-1} (\gamma^i - \gamma^{i+1}) \left(-\theta^i \sum_t (1+r_i)^{2-t} d_t^i - \mu^i d_1^i \right)$$

$$+ \lambda (q_c - 1) \sum_i n_i \sum_t \frac{\partial c_t^i}{\partial q_d} (1+r_g)^{2-t} + \lambda (q_d - 1) \sum_i n_i \sum_t \frac{\partial d_t^i}{\partial q_d} (1+r_g)^{2-t}$$

$$+ \lambda \sum_i n_i \sum_t d_t^i (1+r_g)^{2-t} = 0. \tag{30}$$

Effective marginal tax rates

By (27), obtain $\forall i$

$$\left(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} \right) \theta^i = \lambda n_i \left(1 - t_c \sum_t \frac{\partial c_t^i}{\partial I^i} (1+r_g)^{2-t} - t_d \sum_t \frac{\partial d_t^i}{\partial I^i} (1+r_g)^{2-t} \right). \tag{31}$$

Using $z_i \equiv (2 + r_i)/2$, $z_i - T' = \dfrac{h'(Y^i/w^i)}{w_i \theta^i} - \dfrac{\mu^i}{\theta^i} \dfrac{1}{2}$, and adding and subtracting

$$\gamma^{i+1} \theta^i \left(\frac{\mu^i}{\theta^i} \frac{1}{2} - \frac{h'(Y^i/w^i)}{w^i \theta^i} \right)$$

in (28), obtain

$$\left(n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}\right) \theta^i \left(T'(Y^i) - z_i\right) - \gamma^{i+i}\theta^i \left(\frac{h'(Y^i/w^i)}{w^i\theta^i} - \frac{h'(Y^i/w^{i+i})}{w^{i+i}\theta^i}\right)$$

$$+ \lambda n_i \left(z_i + t_c \sum_t \frac{\partial c_t^i}{\partial Y^i}(1 + r_g)^{2-t} + t_d \sum_t \frac{\partial d_t^i}{\partial Y^i}(1 + r_g)^{2-t}\right) = 0. \tag{32}$$

Substituting (31) into (32) and reorganizing, one obtains

$$T'(Y^i) + t_c \sum_t (1 + r_g)^{2-t} \left(\frac{\partial c_t^i}{\partial I^i}(z_i - T') + \frac{\partial c_t^i}{\partial Y^i}\right)$$

$$+ t_d \sum_t (1 + r_g)^{2-t} \left(\frac{\partial d_t^i}{\partial I^i}(z_i - T') + \frac{\partial d_t^i}{\partial Y^i}\right)$$

$$= \frac{\gamma^{i+1}\theta^i}{\lambda n_i} \left(\frac{h'(Y^i/w^i)q_c}{w^i\theta^i} - \frac{h'(Y^i/w^{i+i})}{w^{i+i}\theta^i}\right). \tag{33}$$

B   Proofs

For further use, we present the expressions used for compensated demands. We compensate demands by varying disposable income I^i but taking annual earnings Y^i as given. For an unconstrained individual and using a tilde to denote compensated demands, the Slutsky equations can be written

$$\frac{\partial c_t^i}{\partial q_c} = \frac{\partial \tilde{c}_t^i}{\partial q_c} - \frac{\partial c_t^i}{\partial I^i} \sum_{t=1,2}(1 + r_i)^{2-t}c_t^i, \qquad \frac{\partial d_t^i}{\partial q_c} = \frac{\partial \tilde{d}_t^i}{\partial q_c} - \frac{\partial d_t^i}{\partial I^i} \sum_{t=1,2}(1 + r_i)^{2-t}c_t^i, \tag{34}$$

$$\frac{\partial d_t^i}{\partial q_d} = \frac{\partial \tilde{d}_t^i}{\partial q_d} - \frac{\partial d_t^i}{\partial I^i} \sum_{t=1,2}(1 + r_i)^{2-t}d_t^i, \qquad \frac{\partial c_t^i}{\partial q_d} = \frac{\partial \tilde{c}_t^i}{\partial q_d} - \frac{\partial c_t^i}{\partial I^i} \sum_{t=1,2}(1 + r_i)^{2-t}d_t^i. \tag{35}$$

If an individual is constrained, then in the first period $\partial c_1^i/\partial I^i = \partial d_1^i/\partial I^i = 0$. Given time-separability we can make use of the fact that first-period expenditures satisfies $q_c c_1^i + q_d d_1^i =$

$Y^i/2 + \phi$, evaluated locally at $\phi = 0$. Then, compensated demands at $t = 1$ satisfy

$$\frac{\partial c_1^i}{\partial q_c} = \frac{\partial \tilde{c}_1^i}{\partial q_c} - c_1^i \frac{\partial c_1^i}{\partial \phi}, \qquad \frac{\partial d_1^i}{\partial q_c} = \frac{\partial \tilde{d}_1^i}{\partial q_c} - c_1^i \frac{\partial d_1^i}{\partial \phi}, \tag{36}$$

$$\frac{\partial d_1^i}{\partial q_d} = \frac{\partial \tilde{d}_1^i}{\partial q_d} - d_1^i \frac{\partial d_1^i}{\partial \phi}, \qquad \frac{\partial c_1^i}{\partial q_d} = \frac{\partial \tilde{c}_1^i}{\partial q_d} - d_1^i \frac{\partial c_1^i}{\partial \phi}. \tag{37}$$

Therefore, following a marginal change in one price, keeping labor effort constant, compensation can be achieved by allowing the constrained individual to borrow marginally more. In the second period,

$$\frac{\partial c_2^i}{\partial q_c} = \frac{\partial \tilde{c}_2^{\ i}}{\partial q_c} - \frac{\partial c_2^i}{\partial I^i} c_2^i, \qquad \frac{\partial d_2^i}{\partial q_c} = \frac{\partial \tilde{d}_2^{\ i}}{\partial q_c} - \frac{\partial d_2^i}{\partial I^i} c_2^i, \tag{38}$$

$$\frac{\partial d_2^i}{\partial q_d} = \frac{\partial \tilde{d}_2^{\ i}}{\partial q_d} - \frac{\partial d_2^i}{\partial I^i} d_2^i, \qquad \frac{\partial c_2^i}{\partial q_d} = \frac{\partial \tilde{c}_2^{\ i}}{\partial q_d} - \frac{\partial c_2^i}{\partial I^i} d_2^i. \tag{39}$$

Proposition 1: When $i = 1, ..., N$ are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.

Proof: First, define the price ratio $\alpha \equiv q_d/q_c$. Set $q_c = 1$, so the first-order condition (30) chooses α and (29) can be ignored. Take the first-order conditions with respect to I^i in (27), multiply them by $\sum_t (1 + r)^{2-t} d_t^i$, to obtain

$$n_i \Phi'(V^i) \theta^i \sum_t (1 + r)^{2-t} d_t^i + (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1 + r)^{2-t} d_t^i - \lambda n_i \sum_t (1 + r)^{2-t} d_t^i$$

$$+ \lambda n_i (\alpha - 1) q_c \sum_t \frac{\partial d_t^i}{\partial I^i} (1 + r)^{2-t} \sum_t (1 + r)^{2-t} d_t^i = 0 \qquad \forall i,$$

Rearranging the last term, one gets

$$n_i \Phi'(V^i) \theta^i \sum_t (1 + r)^{2-t} d_t^i + (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1 + r)^{2-t} d_t^i - \lambda n_i \sum_t (1 + r)^{2-t} d_t^i$$

$$+\lambda n_i(\alpha-1)\sum_t\left(q_c\frac{\partial d_t^i}{\partial I^i}\sum_t(1+r)^{2-t}d_t^i\right)(1+r)^{2-t}=0 \qquad \forall i.$$

Substituting the Slutsky equations (34) and (35) into this and summing over all i gives

$$n_i\sum_t\Phi'(V^i)\theta^i\sum_t(1+r)^{2-t}d_t^i+\sum_i(\gamma^i-\gamma^{i+1})\theta^i\sum_t(1+r)^{2-t}d_t^i-\lambda\sum_i n_i\sum_t(1+r)^{2-t}d_t^i$$

$$+\lambda\sum_i n_i(\alpha-1)\sum_t\left(\frac{\partial\tilde{d}_t^i}{\partial\alpha}-\frac{\partial d_t^i}{\partial\alpha}\right)(1+r)^{2-t}=0,$$

which, after substituting for (29) yields

$$\lambda\sum_i n_i(\alpha-1)\sum_t\frac{\partial\tilde{d}_t^i}{\partial\alpha}(1+r)^{2-t}=0.$$

This requires that $\alpha^*=0$. ∎

Proposition 2: If in the optimum $\mu^i=0$, $\forall i$, so policy relaxes all credit constraints in the economy, then $t_c^*=t_d^*<0$.

Proof: Multiply (27) by $\sum_t c_t^i(1+r_i)^{2-t}$, then substitute for compensated demands, and substitute the result it into (29). Using $r_i=\underline{r}$, $\forall i$, obtain

$$\lambda t_c\sum_i n_i\sum_t\frac{\partial\tilde{c}_t^i}{\partial q_c}(1+r_g)^{2-t}+\lambda t_d\sum_i n_i\sum_t\frac{\partial\tilde{d}_t^i}{\partial q_c}(1+r_g)^{2-t}$$

$$+\lambda\sum_i n_i(r_g-\underline{r})c_1^i=0. \tag{40}$$

Multiply (27) by $\sum_t c_t^i(1+r_i)^{2-t}$, then use compensated demands and substitute the result into (30). Obtain

$$\lambda t_c\sum_i n_i\sum_t\frac{\partial\tilde{c}_t^i}{\partial q_d}(1+r_g)^{2-t}+\lambda t_d\sum_i n_i\sum_t\frac{\partial\tilde{d}_t^i}{\partial q_d}(1+r_g)^{2-t}$$

$$+ \lambda \sum_i n_i (r_g - \underline{r}) d_1^i = 0. \tag{41}$$

Equations (40) and (41) characterize (t_c^*, t_d^*) when optimal income taxes are optimal and all individuals are unconstrained. In matrices, the system can be expressed as

$$\begin{pmatrix} \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1 + r_g)^{2-t} & \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1 + r_g)^{2-t} \\ \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_d} (1 + r_g)^{2-t} & \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} \end{pmatrix} \begin{pmatrix} t_c \\ t_d \end{pmatrix} = \begin{pmatrix} -(r_g - \underline{r}) \sum_i n_i c_1^i / \lambda \\ -(r_g - \underline{r}) \sum_i n_i d_1^i / \lambda \end{pmatrix}$$

where the leftmost matrix is $(2 + r_g)$ times the per-period Slutzky matrix. With time-separable utility, this matrix is necessarily negative semi-definite. We denote it by S and its determinant by $det(S) > 0$. By Cramer's rule

$$t_c = \frac{r_g - \underline{r}}{\lambda det(S)} \times \left(\sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_d} (1 + r_g)^{2-t} \sum_i n_i c_1^i - \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i d_1^i \right) < 0 \tag{42}$$

and

$$t_d = \frac{r_g - \underline{r}}{\lambda det(S)} \times \left(\sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i d_1^i - \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i c_1^i \right) < 0 \tag{43}$$

The properties of the Slutsky matrix directly imply that (42) and (43) are negative since goods c and d are net substitutes.

To prove differentiation differentiation, let us convert per-unit taxes into ad-valorem tax rates $(\tau_c, \tau_d) \equiv (t_c/q_c, t_d/q_d)$. Thus, we are interested in the sign of $\tau_c - \tau_d \equiv t_c/q_c - t_d/q_d$, or in the difference between $t_c q_d$ and $t_d q_c$. Use the identity $r_g \equiv r_g \pm \underline{r}$, the the homogeneity of the Slutzky matrix, and divide all terms by $r_g - \underline{r}$, which result tell us that differentiation occurs if

$$\sum_i c_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_d} - \sum_i d_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_c} \neq \sum_i d_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_t^i}{\partial q_c} - \sum_i c_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_1^i}{\partial q_d}.$$

Cancelling similar terms and regrouping terms of the same sign together, differentiation

occurs if

$$\sum_i c_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_d} + \sum_i c_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_1^i}{\partial q_d} \neq \sum_i d_1^i \sum_i n_i \frac{q_c \partial \tilde{c}_t^i}{\partial q_c} + \sum_i d_1^i \sum_i n_i \frac{q_d \partial \tilde{d}_1^i}{\partial q_c}$$

where both sides equal zero, again, by the per-period homogeneity properties of Slutzky sub-matrices under time-separability. Therefore, the proof is completed and taxes are undifferentiated and negative. ∎

Proposition 3: Let us denote $B_i \equiv (\mu_i/\lambda)(\Phi'(V^i) + \gamma^i/n_i - \gamma^{i+1}/n_i) - (r_g - r_i)$. Also denote by $\mathcal{C} \neq \emptyset$ the subset of types whose credit constraint bind in the optimum: $i \in \mathcal{C} \Leftrightarrow \mu^i > 0$. Then, the optimal policy has $t_d > t_c$ if and only if

$$\left(\frac{\sum_i n_i B_i c_1^i}{\sum_{i \in \mathcal{C}} n_i c_1^i} - \frac{\sum_i n_i B_i d_1^i}{\sum_{i \in \mathcal{C}} n_i d_1^i} \right) > 0.$$

Proof: Let us characterize the optimal commodity tax system when nonlinear income taxes are optimal. Let us suppose that, in the optimal tax system, $i \in \mathcal{C}$ if the individual is constrained and $i \in U$ if he is unconstrained, with $\mathcal{C} \neq \emptyset$. To characterize q_c^*, multiply (27) by $\sum_t (1 + r_i)^{2-t} c_t^i$, substitute compensated demands into it, aggregate over all i, and substitute the result into (29). We obtain that q_c^* is characterized by:

$$\lambda \sum_{i \in \mathcal{U}} n_i \left[t_c \sum_t (1 + r_g)^{2-t} \frac{\partial \tilde{c}_t^i}{\partial q_c} + \lambda t_d \sum_t (1 + r_g)^{2-t} \frac{\partial \tilde{d}_t^i}{\partial q_c} \right]$$

$$+ \lambda \sum_{i \in \mathcal{C}} n_i \left[t_c \left(\frac{\partial c_1^i}{\partial q_c}(1 + r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_c} + \frac{\partial c_2^i}{\partial I} c_1^i (1 + r_i) \right) \right.$$

$$\left. + t_d \left(\frac{\partial d_1^i}{\partial q_c}(1 + r_g) + \frac{\partial \tilde{d}_2^i}{\partial q_c} + \frac{\partial d_2^i}{\partial I} c_1^i (1 + r_i) \right) \right]$$

$$+ \lambda \sum_i n_i (r_g - r_i) c_1^i - \sum_i (n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}) c_1^i = 0$$

40

Using compensated demands to simplify terms for $i \in \mathcal{C}$, we use

$$
\begin{aligned}
\frac{\partial c_1^i}{\partial q_c}(1+r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_c} + \frac{\partial c_2^i}{\partial I}c_1^i(1+r_i) &= \frac{\partial \tilde{c}_1^i}{\partial q_c}(1+r_g) - c_1\frac{\partial c_1^i}{\partial \phi}(1+r_g) + \frac{\partial \tilde{c}_2^i}{\partial q_c} + \frac{\partial c_2^i}{\partial I^i}c_1^i(1+r_i) \\
&= \sum_i (1+r_g)^{2-t}\frac{\partial \tilde{c}_t^i}{\partial q_d} + \frac{\partial c_2^i}{\partial I^i}d_1^i(1+r_i) - d_1\frac{\partial c_1^i}{\partial \phi}(1+r_g) \\
&= \sum_i (1+r_g)^{2-t}\frac{\partial \tilde{c}_t^i}{\partial q_c} + \left(\frac{\partial c_2^i}{\partial I^i} - \frac{\partial c_1^i}{\partial \phi}\right)c_1^i(1+r_i) - (r_g - r_i)c_1^i\frac{\partial c_1^i}{\partial \phi}.
\end{aligned}
$$

Using this last expression and its equivalent for good d, and substituting into the first-order condition with respect to q_c,

$$
\lambda t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c}(1+r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c}(1+r_g)^{2-t}
$$

$$
+\lambda t_c \sum_{i \in \mathcal{C}} n_i \left[\left(\frac{\partial c_2^i}{\partial I^i} - \frac{\partial c_1^i}{\partial \phi}\right)c_1^i - r_g c_1^i \frac{\partial c_1^i}{\partial \phi.}\right] + \lambda t_d \sum_{i \in \mathcal{C}} n_i \left[\left(\frac{\partial d_2^i}{\partial I^i} - \frac{\partial d_1^i}{\partial \phi}\right)c_1^i - r_g c_1^i \frac{\partial d_1^i}{\partial \phi.}\right]
$$

$$
+ \lambda \sum_i n_i(r_g - r_i)c_1^i - \sum_i \mu^i[n_i\Phi'(V^i) + \gamma^i - \gamma^{i+1}]c_1^i = 0. \tag{44}
$$

Then making use of the linearity of Engel's curves,

$$
\lambda t_c \sum_i n_i \sum_t \frac{\partial \tilde{c}_t^i}{\partial q_c}(1+r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_t^i}{\partial q_c}(1+r_g)^{2-t}
$$

$$
-\lambda t_c r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} - \lambda t_d r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi}
$$

$$
+\lambda \sum_i n_i(r_g - r_i)c_1^i
$$

$$
-\sum_i \mu^i[n_i\Phi'(V^i) + \gamma^i - \gamma^{i+1}]c_1^i = 0. \tag{45}
$$

Performing the same operations on the good d and expressing the system in matrices, get

$$\begin{pmatrix} s_{cc} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} & s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi} \\ s_{dc} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi} & s_{dd} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} \end{pmatrix} \begin{pmatrix} t_c \\ t_d \end{pmatrix}$$

$$= \begin{pmatrix} \sum_i \mu^i [n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}] c_1^i / \lambda - \sum_i n_i (r_g - r_i) c_1^i \\ \sum_i \mu^i [n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1}] d_1^i / \lambda - \sum_i n_i (r_g - r_i) d_1^i \end{pmatrix}$$

Call the lefmost, 2×2 matrix of the system, S'. Its determinant is unambiguously positive:

$$det(S') = [s_{cc} s_{dd} - s_{cd}^2]$$

$$-r_g s_{cc} \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} - r_g s_{dd} \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} + r_g s_{cd} \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi} + r_g s_{cd} \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi}$$

where the first line above is positive (since S is negative semi-definite and of size 2×2) and the second line is also positive, since c and d are normal and net substitutes.

Optimal commodity taxes

Using Cramer's rule,

$$t_c^* = \frac{1}{det(S')} \left[\sum_i n_i B_i c_1^i \left(s_{dd} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} \right) - \sum_i n_i B_i d_1^i \left(s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi} \right) \right] \tag{46}$$

and

$$t_d^* = \frac{1}{det(S')} \left[\sum_i n_i B_i d_1^i \left(s_{cc} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} \right) - \sum_i n_i B_i c_1^i \left(s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi} \right) \right] \tag{47}$$

Differentiation

To extract some intuition out of these formulas, we first want to find a way to use

the homogeneity of the Slutzky Matrices. Make use of ad valorem taxes $\tau_c = t_c/q_c$ and $\tau_d = t_c/q_d$:

$$\tau_c^* = \frac{1}{q_c \times det(S')} \left[\sum_i n_i B_i c_1^i \left(s_{dd} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} \right) - \sum_i n_i B_i d_1^i \left(s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi} \right) \right]$$

and

$$\tau_d^* = \frac{1}{q_d \times det(S')} \left[\sum_i n_i B_i d_1^i \left(s_{cc} - r_g \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} \right) - \sum_i n_i B_i c_1^i \left(s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi} \right) \right].$$

We know that $\tau_d > \tau_c$ iff $t_d q_c - t_c q_d > 0$. Making use of the two equations above, obtain the condition:

$$det(S')(t_d q_c - t_c q_d) =$$

$$\left[\sum_i n_i B_i d_1^i \left(q_c s_{cc} - r_g q_c \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} \right) - \sum_i n_i B_i c_1^i \left(q_c s_{cd} - r_g q_c \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi} \right) \right]$$

$$- \left[\sum_i n_i B_i c_1^i \left(q_d s_{dd} - r_g q_d \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} \right) - \sum_i n_i B_i d_1^i \left(q_d s_{cd} - r_g q_d \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi} \right) \right].$$

Using the properties of Slutsky matrices, which implies that $\sum_i n_i B_i d_1^i [q_c s_{cc} + q_d s_{cd}] = 0$; $\quad \sum_i n_i B_i c_1^i [q_d s_{dd} + q_c s_{cd}] = 0$, the expression reduces to

$$det(S')(t_d q_c - t_c q_d) =$$

$$- \sum_i n_i B_i d_1^i r_g q_c \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial c_1^i}{\partial \phi} + \sum_i n_i B_i c_1^i r_g q_c \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial c_1^i}{\partial \phi}$$

$$+ \sum_i n_i B_i c_1^i r_g q_d \sum_{i \in \mathcal{C}} n_i d_1^i \frac{\partial d_1^i}{\partial \phi} - \sum_i n_i B_i d_1^i r_g q_d \sum_{i \in \mathcal{C}} n_i c_1^i \frac{\partial d_1^i}{\partial \phi}.$$

Walras' law tells us that $q_c \frac{\partial c_1^i}{\partial \phi} + q_d \frac{\partial d_1^i}{\partial \phi} = 1$ for constrained individuals. Linear Engel's curves

tell us that $\dfrac{\partial c_1^i}{\partial \phi} = \dfrac{\partial c_1}{\partial \phi}, \forall i \in \mathcal{C}$ and the same for good d. Therefore, re-expressing again,

$$det(S')(t_d q_c - t_c q_d) = \sum_i n_i B_i c_1^i r_g \sum_{i \in \mathcal{C}} n_i d_1^i - \sum_i n_i B_i d_1^i r_g \sum_{i \in \mathcal{C}} n_i c_1^i.$$

$$\Rightarrow t_d q_c - t_c q_d = \frac{r_g}{det(S')} \left(\sum_i n_i B_i c_1^i \sum_{i \in \mathcal{C}} n_i d_1^i - \sum_i n_i B_i d_1^i \sum_{i \in \mathcal{C}} n_i c_1^i \right);$$

Therefore, $t_d > t_c$ if and only if

$$\left(\frac{\sum_i n_i B_i c_1^i}{\sum_{i \in \mathcal{C}} n_i c_1^i} - \frac{\sum_i n_i B_i d_1^i}{\sum_{i \in \mathcal{C}} n_i d_1^i} \right) > 0.$$

$\blacksquare$

Corollary 1: If $\bar{c} = 0$, then there is no differentiation and $t_c^* = t_d^* < 0$. This result is independent of the specific social welfare function $\Phi(\cdot)$ used by the social planner.

Proof: Denote by $\alpha_i \equiv c_1^i / d_1^i$ in the optimum. Since Engel curves are linear and affine through the origin, $\alpha_i = \alpha, \forall i$. Thus,

$$\left(\frac{\sum_i n_i B_i c_1^i}{\sum_i n_i B_i d_1^i} - \frac{\sum_{i \in \mathcal{C}} n_i c_1^i}{\sum_{i \in \mathcal{C}} n_i d_1^i} \right) = \left(\alpha \frac{\sum_i n_i B_i d_1^i}{\sum_i n_i B_i d_1^i} - \alpha \frac{\sum_{i \in \mathcal{C}} n_i d_1^i}{\sum_{i \in \mathcal{C}} n_i d_1^i} \right) = 0.$$

This result is independent of the specific form of $\Phi(\cdot)$, which only appears in $B_i, \forall i$. $\blacksquare$

C   Tables and Figures

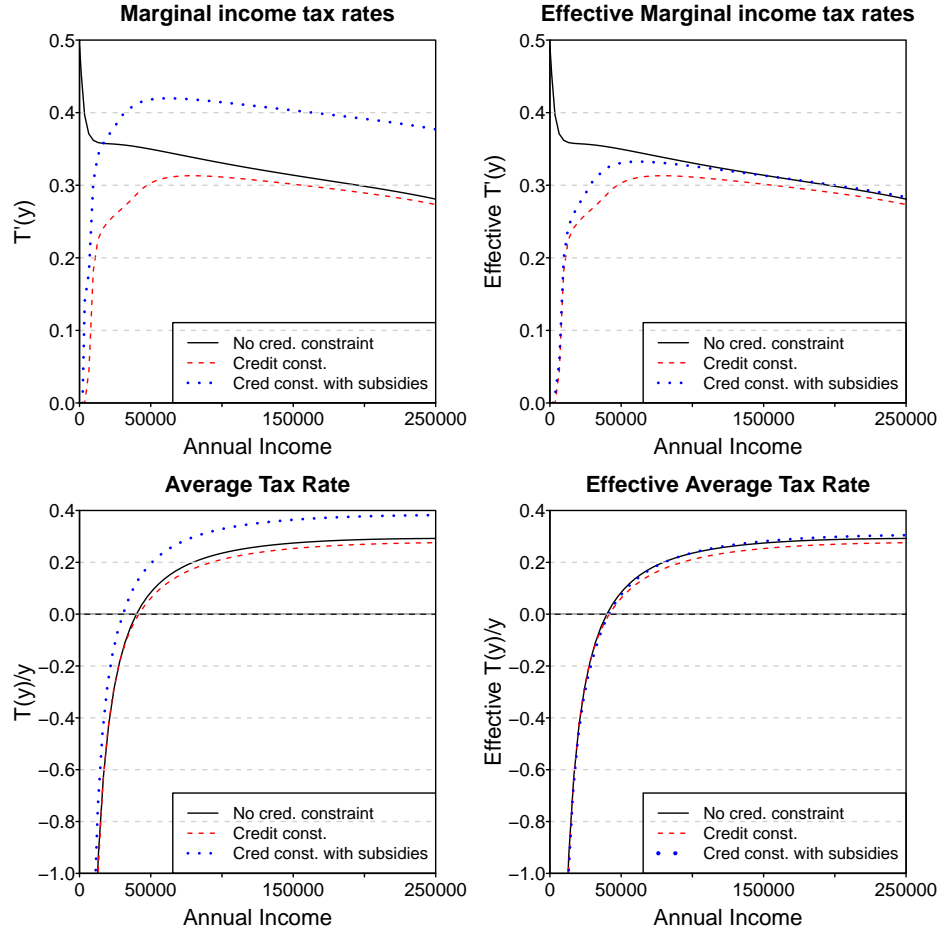Figure 1: Optimal Labor Income Tax Rates under Different Scenarios

Table 1: Characteristics of Optimal Allocation under Different Scenarios

Scenario	q_c	q_d	q_d/q_c	% Constrained	$\sum_i n_i V^i$	$\%\Delta^{LF}$
No Credit Constraint	1	1	1	0%	16.399	10.22%
Credit Constraint	1	1	1	62.23%	16.346	7.01%
Cred. Const. and Subsidies	0.858	0.881	1.028	42.96%	16.35	7.28%

Source: Authors' calculations. Numbers rounded.

Table 2: Welfare gains ($\%\Delta^{LF}$) of the difference scenarios under different levels of η

η	No Credit Constraint	Credit Constraint	Cred. Const. and Subsidies
0.2	16.36%	9.95%	11.9%
0.5	51.53%	15.23%	43.07%
0.7	217.72%	19.19%	206.8%

Figure 2: Optimal Labor Income Tax Rates under Different Interest Rate Spreads
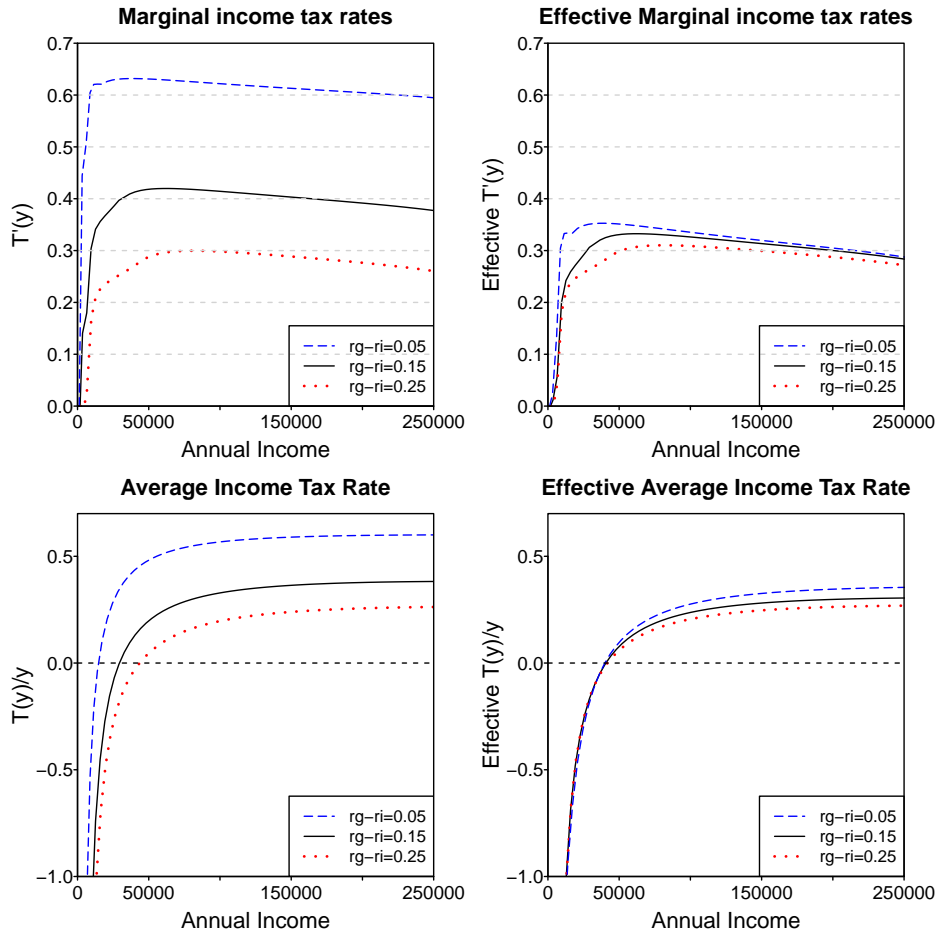
Figure 3: Optimal Labor Income Tax Rates under Different levels of Basic Need
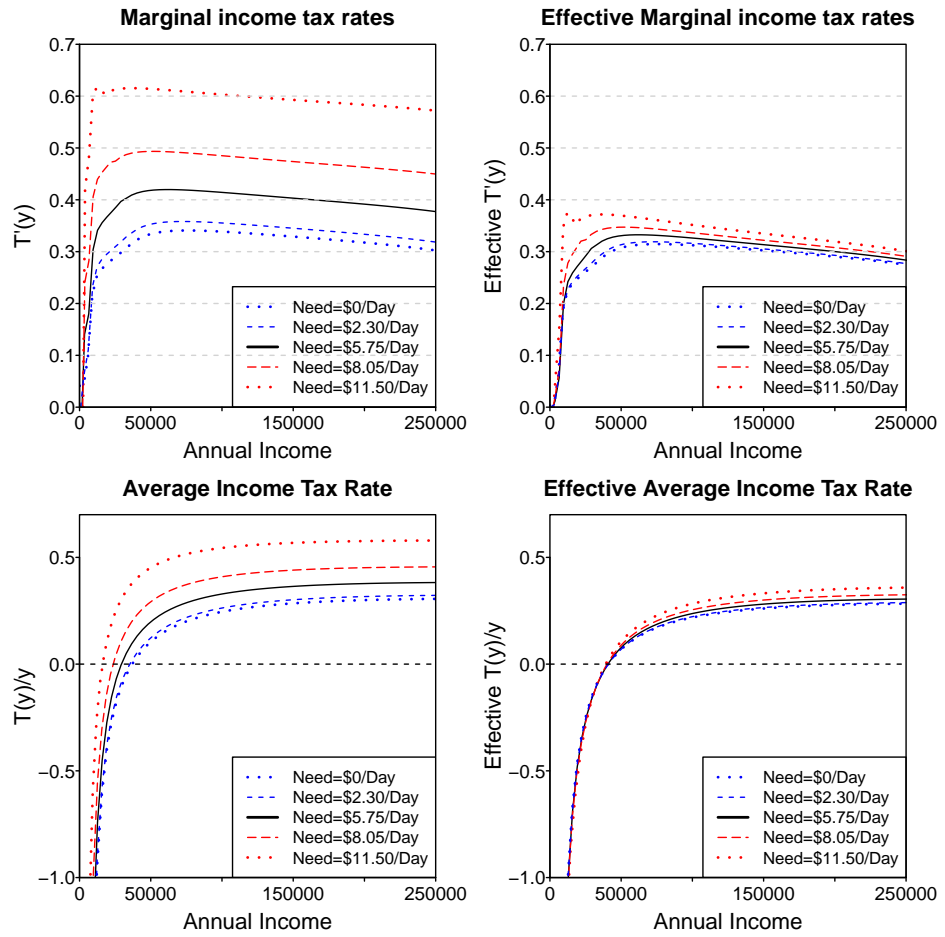
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Figure 4: Optimal Commodity Taxes under Different Interest Rate Spreads and levels of Basic Need