Effects of foreign aid on the recipient country's economic growth

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Abstract

We introduce an infinite-horizon endogenous growth framework for studying the effects of foreign aid on the economic growth in a recipient country. Aid is used to partially finance the recipient's public investment. We point out that the same rule of aid may have very different outcomes, depending on the recipient's circumstances in terms of level of initial capital, domestic investment, efficiency in the use of aid and in public investment, etc. Foreign aid may promote growth in the recipient country, but the global dynamics of equilibrium are quite complex (because of the non-monotonicity and steady state multiplicity). The economy may converge to a steady state or grow without bounds. Moreover, there are rooms for the divergence and a two-period cycle. We fully characterize conditions under which each scenario takes place. Our analysis contributes to the debate on the nexus between aid and economic growth and in particular on the conditionality of aid effects.

Keywords: Aid effectiveness, economic growth, cycle, poverty trap, public investment.
JEL Classification: H50, O19, O41

1   Introduction

Since the United Nations Summit in September 2000 at which the Millennium Development Goals (MDGs) were agreed, foreign aid, in particular Official Development Assistance (ODA) has been continually increasing. For example, in 2015, development aid provided by the donors in the OECD Development Assistance Committee (DAC) was 131.6 billion USD, increased by 6.9% in real terms from 2014, and by 83% from 2000. At the same time, bilateral aid, provided by one country to another, risen by 4% in real terms.[1]

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[1]For more information, see http://www.oecd.org/development/development-aid-rises-again-in-2015-spending-on-refugees-doubles.htm

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Many issues are under debate regarding the effectiveness of aid in terms of economic growth. Extensive empirical investigations using different data samples show conflicting results. On the one hand, some studies show that aid may exert a positive and conditional effect on economic growth. Indeed, in a seminal paper Burnside and Dollar (2000) use a database on foreign aid developed by the World Bank and find that foreign aid has a positive effect on growth only in recipient countries which have good fiscal, monetary and trade policies. Collier and Dollar (2001, 2002) use the World Bank's Country Policy and Institutional Assessment (CPIA) as a measure of policy quality and show that aid may promote economic growth and reduce the poverty in recipient countries if the quality of their policies is sufficiently high. The findings in Guillaumont and Chauvet (2001), Chauvet and Guillaumont (2003, 2009) indicate that the marginal effect of aid on growth is contingent on the recipient countries' economic vulnerability. While the economic vulnerability is negatively associated with growth, the marginal effect of aid on growth is an increasing function of vulnerability.

On the other hand, other empirical studies, not rejecting the conditionality of aid effects, show a certain fragility of results and underline a non-linear effect of aid on growth (Hansen and Tarp, 2001; Easterly et al., 2004; Islam, 2005; Roodman, 2007; Clemens et al., 2012; Guillaumont and Wagner, 2014). For example, Islam (2005) shows an aid Laffer curve in recipient countries with political stability. The effect of aid on growth may be negative at a high level of aid inflows. Hansen and Tarp (2001) find that the effectiveness of aid is conditional on investment and human capital in recipient countries and aid has no effect on growth when controlling for these variables. Their findings shed light on the link between aid, investment and human capital and show that aid increases economic growth through its impact on capital accumulation. Using the same empirical specification as that in Burnside and Dollar (2000), but expanding the data set sample, Easterly et al. (2004) nuance the claim from that of these authors. The results on aid effectiveness seem to be fragile when varying the sample and the definition of different variables such as aid, growth and good policy (Easterly, 2003).

While empirical studies on the aid effectiveness are abundant, there are quite few theoretical analysis on this issue. Our paper aims to fill this gap by presenting a framework to investigate the interplay between foreign aid and economic growth of recipient countries. We also explain why the effects of foreign aid are significant for some countries but not for others. To do so, we consider a simple discrete-time infinite-horizon growth model where public investment, which is financed by foreign aid and capital tax, may improve the total factor productivity (TFP) if it is large enough. Inspired by Dalgaard (2008) and the empirical literature, we formulate aid flows taking into account the donor's rules and the recipient's need represented by its initial capital stock. In the case of a poor country, we also consider the efficiency in the use of aid and in public investment, then examine their impacts on the aid effectiveness. Our simple model allows us to find explicitly the dynamics of capital stocks, and provide the full analysis of equilibrium transitional dynamics.

We show that if the initial circumstances of the recipient are good enough (high productivity and initial capital), the country does not need foreign aid to achieve its development goals. This result is in line with the findings in growth models with increasing return to scale. Consequently, our analysis focuses on the case in which the recipient country's initial capital and productivity are not high. The main results can be described as follows:

First, when the foreign aid is very generous and/or the use of aid is efficient, and the recipient country has a high quality of circumstances (high and efficient public investment and/or low fixed cost in public investment), then the economy will grow without bounds for any level of initial capital stock. Consequently, the country will no longer receive aid from some date on.

The second case, corresponding to the richest dynamics of capital, is found when foreign aid is not very generous and the recipient has an intermediate quality of circumstances. In this case, we distinguish 2 regimes: (R1) the recipient country focuses on its domestic investment, characterized by a remarkable level of capital tax financing public investment, and (R2) it focuses on foreign aid, characterized by the fact that the use of foreign aid is sufficiently efficient. In the first regime (domestic investment focus), if the country has sufficient initial capital or/and the foreign aid is quite high, the economy can grow. Otherwise, it would collapse (i.e., the capital level tends to zero) or stay at the unique steady state. In the second regime, the transitional dynamics are complex because of the non-monotonicity of the capital dynamics. The non-monotonicity is due to the fact that the country focuses on foreign aid which decreases when the economy gets better. In this regime, there are two steady states: the lower one is interpreted as a poverty trap while the second one as a middle-income trap. Let us present our findings under this regime R2.

R2.1. We prove that any poor country, receiving a middle-level aid flow and using it efficiently, always grows at the first stage of its development process and hence will never collapse. This is intuitive because if the capital stock is very low, the country receives a significant aid flow which improves its investment. Under mild conditions, we show that the economy can increasingly converge to the low steady state.

R2.2. However, the convergence may fail in some cases and there may exist a two-period cycle capital path. The intuition is the following: When a poor country receives aid at an initial date (date 0), its economy may grow at date 1. By the rule of aid, the aid flow for date 1 may decrease, leading to a decrease of total public investment at date 1. Hence, the capital at date 2 may decrease, and so on. Thus, a two-period cycle may arise.

R2.3. Last, we point out that a poor country, having strong dynamics of capital, can take advantage of foreign aid to finance its public investment. This may lead it to overcome the middle-income trap in a finite number of periods and to obtain growth in the long run. In this case, the recipient will no longer receive aid from some date on.[2]

Our results appear to corroborate with empirical findings concerning a conditionality of aid effectiveness. Effects of aid in different recipient countries may also illustre our analyses. In particular, South Korea offers an illustration for our results in regime R1 and regime R2.3. Indeed, this country was a recipient country during the period of 1960-1990 (after the Korean War 1950-1953) and experienced a high domestic investment during this period. South Korea is now a developed country, and has become a member of the OECD-DAC (since 2010). The average aid flows have decreased during the period of 1960-1980, from 6.3% to 0.1% of GDP. It became negative at the beginning of the

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[2]Strong dynamics of capital are defined in our paper in the sense that there is some value of capital under the middle-income trap, which produces an output higher than the middle-income trap.

1990s.[3] Our analysis in the regime R2.1 may be illustrated by the Tunisia case where aid flows have also decreased during the 1960-2003, from 8.1% of GDP in the 1960s to 1.5% during 1990-2003.[4]

From a theoretical point of view, our paper is closely related to Dalgaard (2008) who considers that aid flows depend on the recipient's income per capita and on the donor's exogenous degree of inequality aversion. However, there are some differences. First, Dalgaard (2008) considers an OLG growth model while we use an infinite-horizon model à la Ramsey. Second, in Dalgaard (2008), public investments are fully financed by foreign aid while in our paper public investments are financed, not only by foreign aid, but also by capital tax. Third, in Dalgaard (2008) the transitional dynamics of capital stock are totally determined by the degree of inequality aversion on the part of the donor while in our framework they depend not only on the aid rule but also on the country's characteristics. In particular, our framework allows us to study whether a poor country can surpass, not only the low steady state, but also the high one and then achieve growth in the long run.

Our theoretical results, which are complemented by a number of numerical simulations, indicate that the effects of aid (in the short run and in the long run) are complex, non-linear and conditional on recipient countries' characteristics. By the way, our paper is related to Charterjee et al. (2003), Charteerjee and Tursnovky (2007). Indeed, Charterjee et al. (2003) examine the effects of foreign transfers on economic growth of the recipient country given that foreign transfers are positively proportional to the recipient's GDP but are not subject to conditions. They show that their effects on growth and welfare are different according to the type of transfers, untied or tied to investment in public infrastructures. Charteerjee and Tursnovky (2007) underly the role of endogeneity of labor supply as a crucial transmission mechanism for foreign aid.

Our paper is likewise related to the literature on optimal growth with increasing returns (Jones and Manuelli, 1990; Kamihigashi and Roy, 2007; Bruno et al., 2009). However, different from numerous papers in this literature, we consider a decentralized economy while these authors study centralized economies. Besides, we point out the role of aid which can provide investment for the least developed country, this help the recipient country to evade poverty and potentially obtain positive growth in the long run.

The remainder of the paper is organized as follows. Section 2 characterizes the case of a small recipient country. Section 3 presents the dynamics of capital, in particular the poverty trap without international aid. In Section 4, we emphasize the role of international aid by analyzing the conditions for the effectiveness of aid. Section 5 concludes. Appendices gathering technical proofs are presented in Section A.

2   A small economy with foreign aid

This section presents our framework. We consider an economy with infinitely-lived identical consumers and a representative firm. The population size is constant over time and normalized to unity. Labor is exogenous and inelastic. The representative firm produces a single good, which can be used for either consumption or investment. The government uses capital tax and foreign aid to finance public investment (including R&D investment)

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[3]See also Marx and Soares (2013) and Guillaumont and Guillaumont Jeanneney (2010).
[4]See also Guillaumont and Guillaumont Jeanneney (2010).

which can improve the total factor productivity. The waste in aid spending is considered by the presence of unproductive aid. The latter has no direct effect on the household's welfare, nor on the production process. The fraction of wasteful aid may reflect the degree of inefficiency (including corruption) in the use of aid.

2.1 Foreign aid and public investment

Literature on aid conditionalities has a large consensus on the recipient's need as a significant criterion of aid allocation: countries with a high need should receive a high amount of aid (Alesina and Dollar, 2000; Guillaumont and Chauvet, 2001; Berthelemy and Tichit, 2004; Dalgaard, 2008; Carter, 2014). This criterion, among others, is used in several bilateral and multilateral aid policies. In this sense, we assume that aid flows a_t are given by:

$$a_t = (\bar{a} - \phi k_t)^+ \equiv \max\{\bar{a} - \phi k_t, 0\} \tag{1}$$

where $\bar{a} > 0$, the maximal aid amount that the recipient country can receive, and ϕ, independent of the per capita capital, may be referred to all exogenous rules imposed by the donor.

Equation (1) means that the higher the capital k_t, the lower the country ranks in its need, then the lower the aid received. A similar assumption may be found in Carter (2014) and Dalgaard (2008).[5] The form of equation (1) implies that a decrease in ϕ and/or an increase in $\bar{a}$ lead(s) to a higher aid flow. Moreover, from a threshold $(\bar{a}/\phi)$ of capital, the recipient country no longer receives aid.

The positive couple $(\bar{a}, \phi)$ is interpreted as the aid rule imposed by the donor. It is taken as given by the recipient country and represents aid conditionalities. Allocation of aid may be conditional on the policy performance as underlined in Burnside and Dollar (2000), Collier and Dollar (2001, 2002). According to these authors, a country with a high policy quality is more able to use aid in an efficient way. Guillaumont and Chauvet (2001) focus on a fairness argument when they focus on the recipient's economic vulnerability: more aid should be provided to countries with a high economic vulnerability since in these countries aid would be more efficient. This argument also fits in a philosophy of fairness which proposes that aid should compensate the recipient country for its vulnerable initial situation (in macroeconomic conditions or lack of human capital), so that all countries can begin at the same initial opportunities. McGillivray and Pham (2017), Guillaumont et al. (2017) consider the lack of human capital as a determinant criterion. Other analyses underline the link between aid and political variables (strategic allies, former colonial status, and the ability to use aid effectively), between aid and macroeconomic conditions (trade openness, commercial allies, etc.) (Alesina and Dollar, 2000; Berthelemy and Tichit, 2004). All these factors are exogenous for the recipient country

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[5]Carter (2014) considers that aid flow received by country i is positively correlated with country performance rating as underlined in Collier and Dollar (2001, 2002) (with index Country Policy and Institutional Assessment) and is negatively associated with income per capita. Dalgaard (2008) assumes that per capita flow of aid at time t is also a reversed function of income of per capita at period $(t-1)$, $a_t = \theta y_{t-1}^\lambda$, where $\theta > 0$ and $\lambda < 0$. In this aid function, λ reflects the degree of inequality aversion of the donor. Parameter θ represents exogenous determinants of aid. Although Appendix C.2 in Dalgaard (2008) presents a generalized aid allocation rule, it seems that this rule does not cover the form of (1) because the function $(\bar{a} - \phi x)^+$ is not differentiable and the analyses of effects of threshold $(\bar{a}/\phi)$ is an added-value of our paper.

as they are chosen by donors and may be considered as different interpretations of the parameter ϕ. Equation (1) may reflect a trade-off between needs (low initial capital) and country-selectivity (high ϕ) or a compatibility between needs (low initial capital) and country-selectivity (low ϕ) based on aid performance or other criteria.

The recipient country uses aid and tax on capital to finance public investment, which improves the private capital productivity. As some spending of aid is wasted in most recipient countries, there is a significant part of the unproductive activity, noted as a_t^u. This is potentially explained by the corruption, administrative fees, etc. Then, the attribution of aid may be written as:

$$a_t = a_t^i + a_t^u \tag{2}$$

where a_t^i represents the part of aid which contributes to the public investment of the recipient country. If we consider a fixed fraction of aid for each activity, we can rewrite equation (2) as follows:

$$a_t = \alpha_i a_t + \alpha_u a_t \tag{3}$$

with $\alpha_u = 1 - \alpha_i$. Parameter $\alpha_u \in (0,1)$ reflects the degree of inefficiency (including the corruption) in the use of aid while α_i represents the efficiency in the use of aid.

Let us denote B_t the public investment financed by tax on capital and by aid, B_t may be written as:

$$B_t = T_{t-1} + a_t^i \tag{4}$$

where T_{t-1} is the tax at period $t - 1$. We assume that $T_{t-1} = \tau K_t$. τK_t is a part of capital at date $(t - 1)$ used to finance public investment which will have its effect at date t. Since, all capital tax is used to fund public investment, τ may be interpreted as the government effort in financing public investment. The positive effect of foreign aid on public investment is an obvious finding in empirical studies (Khan and Hoshino, 1992; Franco-Rodrigez et al., 1998; Ouattara, 2006; Feeny and McGillivray, 2010).

2.2 Production with endogenous productivity

At each date, the representative firm maximizes its profit. The production function at date t is given by:

$$F_t(K_t) = A_t K_t$$

where K_t represents the capital while A_t represents the total factor productivity. Capital K may be referred to a broad concept of capital including, for example, human capital. In the spirit of Barro (1990), we assume that A_t is endogenous and depends on public investment B_t as follows:

$$A_t := A \left[1 + (\sigma B_t - b)^+ \right]$$

Parameter $A \in (0, \infty)$ is interpreted as the autonomous productivity. When $B_t \leq b/\sigma$, we have $(\sigma B_t - b)^+ = 0$, and then the production function $F_t(K_t)$ recovers the standard AK model. This means that the positive effect of public investment in technology is observed only from the level b/σ. Parameter $\sigma \in (0, \infty)$ measures the extent to which the public investment translates into technology, and the production process. In this sense, σ may reflect the efficiency of public investment and b/σ is considered as a threshold from

which the public investment improves the technology. The existence of this threshold is supported by some empirical evidences, for example Azariadis and Drazen (1990).

Combining the threshold for a positive effect of public investment with equation (4), we observe that for a significant effect of aid on the capital productivity and production, aid should verify the following condition:

$$a_t^i \geq \frac{b}{\sigma} - \tau K_t \tag{5}$$

As in Charterjee et al. (2003), Charteerjee and Tursnovky (2007), Dalgaard (2008), we consider that aid is used to finance public expenditures. However, for countries with low capital (in the sense that $b > \sigma \tau K_t$), aid should be higher than a critical level to improve the technology, and this is necessary for positive growth in the long run.[6] This assumption may be referred to the big push concept of aid supported in Sachs (2005) and discussed in Guillaumont and Guillaumont Jeanneney (2010). Our setup is also supported by Wagner (2014) who uses a data including 89 recipient countries and identified the existence of a critical level above which aid is effective in terms of economic growth.

At each period t, given public investment B_t, the representative firm maximizes its profit:

$$P_{ft}: \qquad \pi_t \equiv \max_{K_t \geq 0} \left(F_t(K_t) - r_t K_t \right) \tag{6}$$

It is straightforward to obtain r_t and π_t for a competitive economy:

$$r_t = A\left[1 + \left(\sigma B_t - b\right)^+\right] \text{ and } \pi_t = 0. \tag{7}$$

2.3 Household

Let us consider the representative consumer's optimization problem. She maximizes her intertemporal utility by choosing consumption and capital sequences (c_t, k_t):

$$P_c: \qquad \max_{(c_t, k_{t+1})_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \beta^t U(c_t) \tag{8}$$

$$\text{s.t: } c_t + k_{t+1} + T_t \leq (1-\delta)k_t + r_t k_t + \pi_t \tag{9}$$

where $k_0 > 0$ is given, β is the rate of time preference and $u(\cdot)$ is the consumer's instantaneous utility function. T_t is the tax, r_t is the capital return while π_t is the firm's profit at date t.

For the sake of tractability, we assume that the consumer knows that $T_t = \tau k_{t+1}$ and instantaneous utility function is logarithmic, $U(c_t) = \ln(c_t)$. According to Lemma 5 in Appendix A.1, we establish the relationship between k_{t+1} and k_t

$$k_{t+1} = \beta \frac{1 - \delta + r_t}{1 + \tau} k_t. \tag{10}$$

By the concavity of the utility function, this solution is unique.

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[6]Our setup is different from that in Dalgaard (2008). Indeed, Dalgaard (2008) considers a production function: $y_t = k_t^\alpha g_t^{1-\alpha}$, where g_t represents government services, entirely financed by international aid a_t. This means that the first dollars received from donors have a positive effect on the recipient's production.

2.4 Intertemporal equilibrium

Definition 1 (Intertemporal equilibrium). *Given capital tax rate τ, a list $(r_t, c_t, k_t, K_t, a_t)$ is an intertemporal equilibrium if:*

1. *(c_t, k_t) is a solution of the problem P_c, given a_t^i, r_t, π_t.*

2. *(K_t) is a solution of the problem P_{ft}, given B_t and r_t.*

3. *Market clearing conditions are satisfied, i.e., $K_t = k_t$ and $c_t + k_{t+1} + T_t = (1-\delta)k_t + Y_t$.*

4. *The government budget is balanced: $T_t = \tau k_{t+1}$.*

5. *(Rule of Aid) $a_t = \max\{\bar{a} - \phi k_t, 0\}$ and $a_t^i = \alpha_i a_t$.*

Combined with (7), the dynamics of capital stock may be rewritten as follows:

$$k_{t+1} = G(k_t) \equiv f(k_t)k_t \tag{11}$$

$$\text{where} \quad f(k_t) \equiv \beta \frac{1 - \delta + A\left[1 + \left(\sigma(\tau k_t + \alpha_i(\bar{a} - \phi k_t)^+) - b\right)^+\right]}{1 + \tau} \tag{12}$$

This dynamic system is non-linear and non-monotonic. The next sections analyze the global dynamics of capital stocks (k_t) and the effects of international aid on the recipient's economic growth. Before doing this, it is useful to introduce some notions of growth and collapse.

Definition 2. *(Growth, collapse, and poverty trap)*

1. *The economy collapses if $\lim_{t \to \infty} k_t = 0$. It grows without bounds if $\lim_{t \to \infty} k_t = \infty$.*

2. *A value $\bar{k}$ is called a trap if, for any initial capital stock $k_0 < \bar{k}$, we have $k_t < \bar{k}$ for any t high enough.*

Our formal definition of trap means that a poor country $(k_0 \leq k)$ continues to be poor. It is in line with the notion of poverty trap in Azariadis and Stachurski (2005): A poverty trap is a self-reinforcing mechanism which causes poverty to persist.

3 Equilibrium dynamics without foreign aid

This section considers an economy which does not receive foreign aid. Its public investment B_t is entirely financed by tax revenue. We will analyze the dynamics of capital in the long run. From equation (11), we have:

$$k_{t+1} = f_b(k_t)k_t, \text{ where } f_b(k_t) \equiv \beta \frac{1 - \delta + A\left[1 + \left(\sigma \tau k_t - b\right)^+\right]}{1 + \tau} \tag{13}$$

Let us denote:

$$r_a \equiv \beta \frac{1 - \delta + A}{1 + \tau}. \tag{14}$$

We observe $f_b(k_t) \geq r_a$ for any t. Therefore, we have:

Remark 1 (Role of technology). *Consider an economy without aid. If $r_a > 1$, the economy will grow without bounds.*[7]

Condition $r_a > 1$ is equivalent to $A > \frac{1+\tau}{\beta} + \delta - 1$. Our result indicates that when the autonomous technology A is sufficiently high, it may generate growth whatever the levels of other factors such as: initial capital or efficiency of public investment. In this case, the country is not eligible to receive aid. Since our purpose is to look at the impacts of public investment and foreign aid, from now on, we will work under the following assumption.

Assumption 1 (For the rest of the paper). *$r_a < 1$ or equivalently,*
$A < \frac{1+\tau}{\beta} + \delta - 1$.[8]

Under this assumption, the economy would never reach economic growth in the long run without public investment B_t (in infrastructure, in R&D program, etc.). Public investment B_t is then required to improve technology, and this is necessary for positive economic growth in the long run.

According to equation (13) and the fact that $f_b(k_t)$ is an increasing function, we get the following analysis concerning the dynamics of capital stock:

Proposition 1 (Poverty trap and growth: role of public investment). *Consider an economy with a low level of autonomous technology (Assumption 1 holds), and without foreign aid. The public investment in technology is entirely financed by tax revenue. The dynamics of capital, characterized by equation (13), are as follows:*

1. *If $f_b(k_0) > 1$, i.e., $\sigma\tau k_0 > b + D$, then (k_t) increases and the economy grows without bounds.*

2. *If $f_b(k_0) < 1$, i.e., $\sigma\tau k_0 < b + D$, then (k_t) decreases and the economy collapses.*

3. *If $f_b(k_0) = 1$, i.e., $\sigma\tau k_0 = b + D$, then $k_t = k_0$ for any t.*

There exists a unique steady state k^{**}, and $k^{**} = \frac{b+D}{\tau\sigma}$ where

$$D \equiv \frac{1}{A}\Big(\frac{1+\tau}{\beta} + \delta - 1\Big) - 1 > 0. \tag{15}$$

We may interpret b as a fixed cost of public investment. If the return of public investment $(\sigma B_t \equiv \sigma\tau k_0)$ is less than $b + D$, public investment τk_0 does not make any change to the total factor productivity. Following this interpretation, $b + D$ can be viewed as the threshold so that if the return of public investment in R&D (σB_t) is less than this level, there is no growth of capital stock, i.e. $k_{t+1} < k_t$ for all t.

Figure 1 illustrates Proposition 1.[9] The point of interaction between the convex curve and the first bisector corresponds to the unstable steady state k^{**} which is considered as a poverty trap for this economy (see Definition 2). For all initial capital k_0 higher than k^{**} (corresponding to $\sigma\tau k_0 > b + D$), the economy will grow without bounds while it will collapse if the initial capital is lower than k^{**}. It should be noticed that k^{**} is decreasing in A, σ while it is increasing in b. This means that an economy having a high autonomous technology A, high efficiency σ and low fixed cost b in public investment, obtains a higher probability to surpass its poverty trap as the condition $\sigma\tau k_0 > b + D$ is more likely to be satisfied.

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[7]In this case, $f_b(k_t) \geq r_a > 1$, then $k_{t+1} > k_t$ for any t.

[8]We ignore the case $r_a = 1$ because this case is not generic.

[9]In Figure 1, parameters are $\beta = 0.8, \delta = 0.2, A = 0.5, \tau = 0.4, \sigma = 2, \bar{a} = 0, b = 2$.
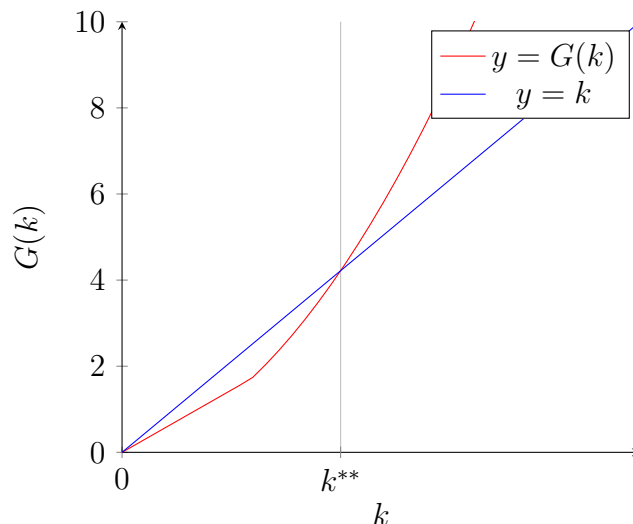
Figure 1: Transition dynamics without foreign aid and $r_a < 1$.

4   Equilibrium dynamics with foreign aid

Point 2 of Proposition 1 shows that the economy collapses without international aid if $\sigma \tau k_0 < b + D$. Since we want to investigate the effectiveness of aid, we will work under the following assumption in Section 4:

Assumption 2 (For the whole Section 4).

$$\sigma \tau k_0 < D + b \tag{16}$$

where D is defined by (15)

Given this pessimistic initial situation of the recipient country, we examine how international aid could generate positive perspectives in the short run as well as in the long run. Recall that $k_{t+1} = G(k_t)$. Before exploring the dynamics of capital stock, it is essential to underline properties of function $f(k)$ and $G(k)$. To do so, we introduce some notations

$$x_1 \equiv \bar{a}/\phi \tag{17}$$

$$x_2 \equiv \frac{\sigma \alpha_i \bar{a} - b}{\sigma(\alpha_i \phi - \tau)} \quad \text{i.e. } x_2 \text{ such that} \quad \sigma\big(\tau k + \alpha_i(\bar{a} - \phi k)\big) - b = 0 \tag{18}$$

$$x_3 \equiv \frac{1 - \delta + A(1 + \sigma \alpha_i \bar{a} - b)}{2A\sigma(\alpha_i \phi - \tau)}. \tag{19}$$

Let us explain the meaning of x_1, x_2, x_3. First, x_1 is the maximum level of capital stock so that the recipient country does not receive international aid. When the country receives aid, x_2 is the critical threshold from which public investment B_t (financed by aid and tax revenue) has a positive impact on productivity.

When the country receives aid $(\bar{a} - \phi k > 0)$ and public investment has positive impact on productivity $(\sigma\big(\tau k + \alpha_i(\bar{a} - \phi k)\big) - b > 0)$, x_3 is a local-maximum point of function G

(because $f_3'(x_3) = 0$) where

$$f_3(x) \equiv \beta \frac{1 - \delta + A\left[1 + \left(\sigma(\tau x + \alpha_i(\bar{a} - \phi x)) - b\right)\right]}{1 + \tau} x. \qquad (20)$$

Lemma 1 (Monotonicity of G)**.** *The function G is increasing on $[0, \infty)$ if one of the following conditions is satisfied.*

1. *$\tau \geq \alpha_i \phi$. (The government effort is high.)*

2. *$\tau < \alpha_i \phi$ and $\sigma \alpha_i \bar{a} < b$ (this implies that $x_2 < 0$). (The government effort is low, the fixed cost b is high with respect to the maximum level of aid $\bar{a}$, or/and the efficiency of public investment σ is low.)*

3. *$\tau < \alpha_i \phi$, $\sigma \alpha_i \bar{a} > b$ and $x_3 > \min(x_1, x_2)$. (The government effort is low, the maximum level of aid $\bar{a}$ and/or the efficiency of public investment σ is high with respect to the fixed cost b, and the local-maximum point of output is high enough.)*

Lemma 2 (Non-monotonicity of G)**.** *Assume that $\tau < \alpha_i \phi$, $\sigma \alpha_i \bar{a} > b$, and $x_3 < \min(x_1, x_2)$. Then G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$.*

Proofs of Lemmas 1, and 2 are presented in Appendix A.2.

We can compute no-trivial fixed points (capital steady states), i.e. strictly positive solutions of the equation $G(k) = k$. So, we have to find k such that

$$f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+ = \frac{D + b}{\sigma}.$$

The following result is obtained using Lemmas 1 and 2.

Lemma 3 (Steady states)**.**

1. *If $\sigma \bar{a} \min(\alpha_i, \tau/\phi) > D + b$, then there is no fixed point.*

2. *Consider the case where $\sigma \bar{a} \min(\alpha_i, \tau/\phi) \leq D + b$.*

   (a) *If $\tau > \alpha_i \phi$, which implies $\sigma \bar{a} \alpha_i \leq D + b$, then the unique fixed point is*

   $$\begin{cases} k^* \equiv \frac{\frac{D+b}{\sigma} - \bar{a}\alpha_i}{\tau - \alpha_i \phi} \in (0, \bar{a}/\phi) & \text{if } \sigma \bar{a} \tau/\phi > D + b \\ k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty) & \text{if } \sigma \bar{a} \tau/\phi < D + b. \end{cases} \qquad (21)$$

   (b) *If $\tau < \alpha_i \phi$, which implies $\sigma \bar{a} \tau/\phi \leq D + b$, then*

   i. *If $\sigma \bar{a} \alpha_i < D + b$, then the unique fixed point is $k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.*

   ii. *If $\sigma \bar{a} \alpha_i > D + b$, then there are two fixed points $k^* \equiv \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i \phi - \tau} \in (0, \bar{a}/\phi)$ and $k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.[10]*

Proof. See Appendix A.2. □

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[10]Condition $\sigma \bar{a} \alpha_i > D + b$ ensures that $k^* > 0$.

4.1 Growth under high quality circumstances

This section investigates effects of aid on the recipient prospects when the recipient country has high quality circumstances in terms of efficiency in the use of aid, fixed cost and efficiency in public investment, autonomous technology, etc.

Proposition 2 (Growth without bounds thanks to foreign aid). *Considering an aid recipient under a poverty trap without aid, characterized by condition (16). The dynamics of capital with foreign aid are characterized by (11).*
   If

$$r_d \equiv \frac{\beta}{1+\tau}\Big[1 - \delta + A\Big(1 + \big(\sigma\bar{a}\min(\alpha_i, \tau/\phi) - b\big)^+\Big)\Big] > 1 \tag{22}$$

$$\text{or equivalently, } \sigma\bar{a}\min(\alpha_i, \tau/\phi) > D + b, \tag{23}$$

then we have that,

1. *the economy will grow without bounds for any level of initial capital k_0,*

2. *international aid $a_t = (\bar{a} - \phi k_t)^+$ decreases in t. Consequently, there exists a time T such that aid flows $a_t = 0$ for any $t \geq T$.*

Proposition 2 can be proved by using point 4 of Lemma 1 and point 1 of Lemma 3. Notice that in this case, G is increasing and a steady state does not exist.

Condition (23) inf Proposition 2 may be written as follows

$$\sigma\bar{a}\frac{\tau}{\phi} > D + b \quad \text{and} \quad \sigma\alpha_i\bar{a} > D + b, \tag{24}$$

where D is given by equation (15). Two conditions in (24) mean that the foreign aid is generous (high $\bar{a}$ and low ϕ) and/or the recipient country has high quality circumstances (that is, a high efficiency σ and low fixed cost b in public investment, and/or a high level of autonomous technology A). In particular, the first condition in (24) may be associated with a high government effort (high τ) in financing public investment while the second condition may be associated with a high efficiency in the use of aid (high α_i). In other words, given aid flows and the donor's rules characterized by the couple $(\bar{a}, \phi)$, condition (24) is more likely to be satisfied if the recipient country has high quality circumstances, decisive for the effectiveness of aid.

Proposition 2 presents the best and ideal scenario since whatever the initial capital, generous aid combined with high quality circumstances could help the recipient country to grow without bounds in the long run. Figure 2 illustrates this proposition under condition (equivalently 23) (or (24)).[11] The graph on the left corresponds to the case $\alpha_i < \tau/\phi$ and that on the right corresponds to the case $\alpha_i > \tau/\phi$. We observe that, without exogenous aid (corresponding to $\bar{a} = 0$), the dynamics of capital correspond to that in Figure 1 and there is one poverty trap. Thanks to development aid, the dynamics of capital change and they are represented by the curve above the first bisector.

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[11] Parameters in Figure 2 are $\beta = 0.8; \tau = 0.4; \delta = 0.2; A = 0.4; \sigma = 2; \alpha_i = 0.8; \bar{a} = 17, b = 2, \phi = 2$ verifying conditions $r_a < 1$ and (23). On the left: $\phi = 0.4$. On the right: $\phi = 2$.
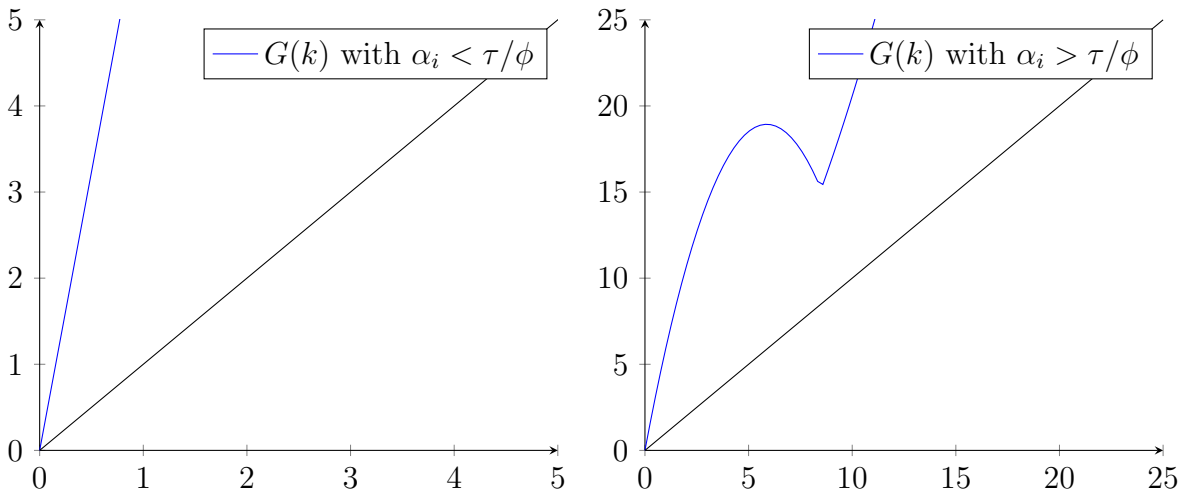
Figure 2: (Growth without bounds): $r_a < 1$ and (23) holds

4.2 Growth or collapse? The role of aid

We are now interested in the case where condition (23) is not satisfied: recipient countries do not have high quality circumstances and/or aid flows, subject to conditions, are bounded, due to the budget constraint from the donors. In the next sections, we consider the following condition:

$$\text{Assumption for the rest of the paper:} \quad \sigma \bar{a} \min(\alpha_i, \tau/\phi) < D + b. \quad (25)$$

From (25), we can identify three cases:

$$\text{Low circumstances: } \frac{\sigma\tau}{\phi} < \frac{D+b}{\bar{a}} \text{ and } \sigma\alpha_i < \frac{D+b}{\bar{a}} \quad (26)$$

$$\text{Intermediate circumstances 1 (domestic investment focus): } \sigma\alpha_i < \frac{D+b}{\bar{a}} < \frac{\sigma\tau}{\phi} \quad (27)$$

$$\text{Intermediate circumstances 2 (aid focus): } \frac{\sigma\tau}{\phi} < \frac{D+b}{\bar{a}} < \sigma\alpha_i \quad (28)$$

In both (27) and (28), we have $\sigma\bar{a}\min(\alpha_i, \tau/\phi) < D+b < \sigma\bar{a}\max(\alpha_i, \tau/\phi)$. However, we distinguish two intermediate circumstances: one with domestic investment focus when $\alpha_i\phi < \tau$, that is the government investment (measured by τ) is quite high with respect to the efficiency degree in the use of aid (measured by α_i); and another with aid focus when $\alpha_i > \tau/\phi$, that is the use of aid is quite efficient.

We firstly consider the cases of low circumstances and the domestic investment focus. According to Lemma 1, we have the following result.

Proposition 3 (Growth or collapse? The role of aid)**.** *Consider an aid recipient under poverty trap without aid, characterized by condition (16): $\sigma\tau k_0 < D+b$. Assume that one of three conditions in Lemma 1 holds. Then (k_t) is monotonic in t and the transitional dynamics of (k_t) are characterized as follows.*

1. *If $f(k_0) > 1$, i.e., $\left(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\right)^+ > D$, then (k_t) increases and the economy grows without bounds. Consequently, there exists a time T such that aid flows $a_t = 0$ for any $t \geq T$.*

2. If $f(k_0) < 1$, i.e., $\left(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\right)^+ < D$, then (k_t) decreases and the economy collapses. Consequently, there exists a time T_1 such that aid flows $a_t > 0$ for any $t \geq T_1$.

3. If $f(k_0) = 1$, then $k_t = k_0$ for any t.

Moreover, following Lemma 3, we have:

$$\text{The unique steady state } = \begin{cases} k^{**} = \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty) \text{ if } (26) \text{ holds (low circumstances)} \\ k^* = \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau}(0, \bar{a}/\phi) \text{ if } (27) \text{ holds (intermediate circumstances 1).} \end{cases}$$
(29)

We are considering a country with low initial capital stock in the sense that $\sigma\tau k_0 < b + D$ (Assumption 2). According to Proposition 3, we observe that: given such an initial capital stock k_0, if the aid rule is generous (in the sense that $\bar{a}$ is high and/or ϕ is low) and the use of aid is efficient (α_i is high) so that $\left(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\right)^+ > D$, then the economy will grow without bounds. Otherwise, the economy will collapse or stay at the steady state. In other words, the development aid might help the recipient to surpass its poverty trap while this is impossible without foreign assistance. Our result indicates that low-income and vulnerable countries need not only a large scaling-up of aid but also the efficiency in the use of aid (parameter α_i) to help them to get out of the poverty trap. Our finding may be considered as a theoretical illustration for the argument evoked in Kraay and Raddatz (2007) using a Solow model.[12]

We observe that the poverty trap in the intermediate circumstances 1 (with domestic investment focus) is k^*, which is lower that k^{**}, i.e. the poverty trap in the low circumstances. This means that the intermediate circumstances give a better outcome than the low circumstances as the recipient's probability of escaping its poverty trap is higher in the intermediate circumstances.

Our result is related to the literature on optimal growth with increasing returns (Jones and Manuelli, 1990; Kamihigashi and Roy, 2007; Bruno et al., 2009). Our added-value is two-fold. First, we consider a decentralized economy while these authors studied centralized economies. Second, we point out the role of aid which can provide investment for recipient countries, and this help them to obtain positive growth in the long run.

4.3 Stability, fluctuations or take-off? The complexity of aid's effects

We have so far analyzed three circumstances (high, low and intermediate circumstances 1 with domestic investment focus) in which the capital path (k_t) is monotonic. In these cases, the recipient country may or may not fully exploit the same flow of aid following its initial situation. This section focuses on the remaining cases characterized by the following assumption:

Assumption 3 (Assumptions for the whole Section 4.3).

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[12]In a Solow model with two exogenous saving rates, there are two steady states which are locally stable. Kraay and Raddatz (2007) indicate that in such a model, if the saving rate is low, foreign aid could help the recipient to accumulate capital. Saving rate might jump to the higher level, and then, the economy would converge to the high steady state.

1. $\sigma\tau/\phi < \frac{D+b}{\bar{a}} < \sigma\alpha_i$ (condition (28) - intermediate circumstances 2 with aid focus)

2. $0 < x_3 < \min(x_1, x_2)$ where x_1, x_2, x_3 are given by (17), (18) and (19).

Assumption 3 means that: (1) the government investment is low (i.e. τ is low) but the use of aid is quite efficient (i.e. α_i is quite hight); (2) the maximum level of aid $\bar{a}$ and/or the efficiency of public investment σ are quite high with respect to the fixed cost b, but the local-maximum point x_3 of output is not high enough to surpass thresholds x_1, x_2. Notice that if Assumption 3 is violated, we recover analyses in the previous sections.

Under Assumption 3, G is not monotoninc. It is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$. By combining Assumption 3, Lemma 2 and point $(2.b.i)$ of Lemma 3, there exists two steady states

$$\text{low steady state:} \quad k^* = \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi)$$

$$\text{high steady state:} \quad k^{**} = \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty).$$

It is easy to see that the high steady state k^{**} is unstable. The main question in this section is whether the recipient country can encompass the high steady state and attain an economic take-off. It is also about to investigate whether the capital stock converges to the middle income trap or fluctuates around it.

Let us start by considering a poor country (k_0 is low).

Proposition 4. *Let Assumption 3 be satisfied. When the initial capital stock k_0 is low enough, the capital stock at the next period will be higher than k_0: $k_1 > k_0$.*

Proof. See Appendix A.3. $\qquad\qquad\square$

Proposition 4 leads to an important implication: any poor country (characterized by Assumption 3) receiving foreign aid always grows at the first stage of its development process (see Figure 3 for an illustration, with k_0 sufficiently far from the low steady state). In this case, aid may not promote growth but the economy never collapses: this is an important difference between the case of intermediate circumstances with aid focus and the cases of low circumstances or intermediate circumstances with domestic investment focus (which may rise a collapse). It follows that we should provide development aid for such poor countries.

However, our result does not mean that we should provide more development aid for any country at any stage of its development. A natural question arises: What happens to poor or developing countries (having low of middle value of k_0)? We will address this question in next subsections.

4.3.1 Stability and fluctuations

We start this section by considering the stability of capital path.

Proposition 5 (Stability of low steady state)**.** *Let Assumption 3 be satisfied.*

1. *Considering the case where $\sigma\bar{a}\alpha_i < D + b + \frac{1}{A}\left(\frac{1+\tau}{\beta}\right)$, or equivalently $x_3 > k^*$. We have that: if $k_0 \in (0, k^*)$, then $k_t \in (0, k^*)$ for any t and $\lim_{t\to\infty} k_t = k^*$.*

2. *Considering the case where $\sigma\bar{a}\alpha_i > D + b + \frac{1}{A}\left(\frac{1+\tau}{\beta}\right)$, or equivalently $x_3 < k^*$. The steady state k^* is locally stable[13] if and only if*

$$\sigma\bar{a}\alpha_i < D + b + \frac{2}{A}\left(\frac{1+\tau}{\beta}\right) \tag{30}$$

Proof. See Appendix A.4. □

Recall that we are considering $\sigma\tau k_0 < D + b$, i.e., $k_0 < k^{**}$ the country is in a situation sufficiently vulnerable to have a probability of collapse if there is no aid (according to point 1 of Proposition 1). Point 1 of Proposition 5 shows a role of aid: a country receiving development aid may converge to some point. This may happen under Assumption 3 and $x_3 > k^*$, that is the low steady state is lower than the local-maximum point (x_3) of output. This finding complements Proposition 2 and Proposition 3: foreign aid may promote growth in the recipient country. It should be noticed that Propositions 2, 3 and 5 consider different circumstances (high, low, intermediate 1 and intermediate 2 circumstances) which are not overlapped.

Figure 3 illustrates Proposition 5.[14] On the left we have $x_3 > k^*$, and $\lim_{t\to\infty} k_t = k^*$ for any $k_0 \in (0, k^*)$. However the convergence of capital stock may fail when $x_3 < k^*$. Indeed, point 2 of Proposition 5 shows that there may be room for local instability when k_0 is around the low steady state.
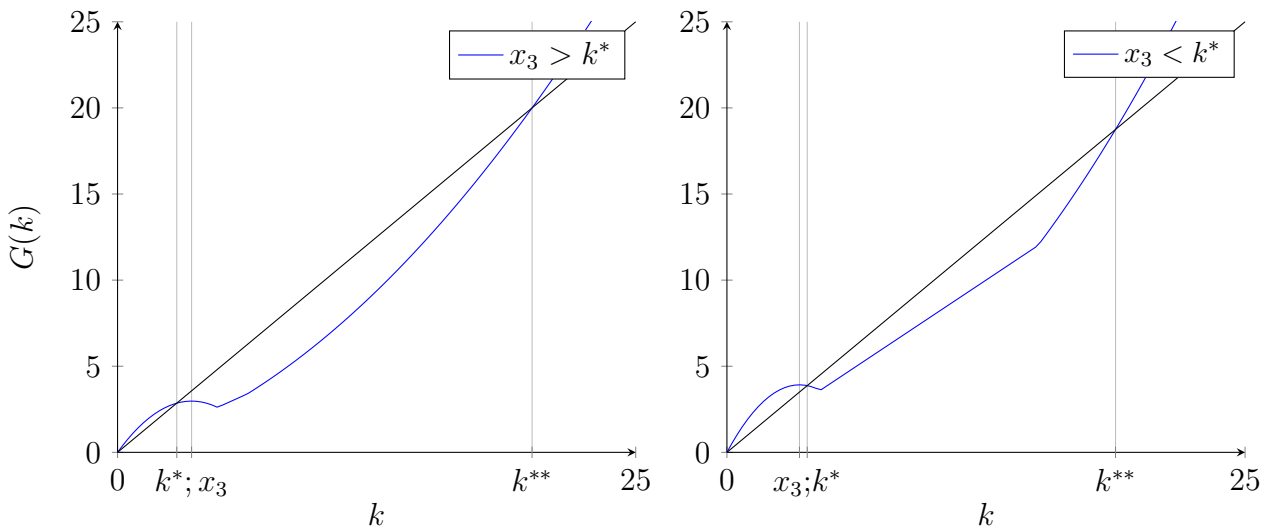

Figure 3: Assumption 3 is satisfied. On the left: $x_3 > k^*$. On the right: $x_3 < k^*$ and (30) holds.

Another question arises: is there fluctuation of capital paths or cycle around the low steady state? Our analysis is based on the following intermediate result.

Lemma 4. *Assume conditions in Assumption 3 hold and $x_3 < k^*$. Assume also that*

$$\sigma\bar{a}\alpha_i > D + b + \frac{2}{A}\left(\frac{1+\tau}{\beta}\right). \tag{31}$$

---

[13]It means that there exists $\epsilon > 0$ such that $\lim_{t\to\infty} k_t = k^*$ for any $k_0 \in (k^* - \epsilon, k^* + \epsilon)$.

[14]Parameters in Figure 3. On the left: $\beta = 0.5; \tau = 0.2; \delta = 0.2; A = 0.5; \sigma = 0, 8; \alpha_i = 0.8; \bar{a} = 10, \phi = 2, b = 1$. On the right: $\beta = 0.8; \tau = 0.2; \delta = 0.2; A = 0.4; \sigma = 1; \alpha_i = 0.7; \bar{a} = 12, \phi = 2, b = 3$.

Then, there exists $y_1 \in (x_3, k^*)$ and $y_2 > 0$ in $(0, x_2)$ such that

$$y_1 \neq y_2, \quad f_3(y_1) = y_2, \quad f_3(y_2) = y_1. \tag{32}$$

Moreover, if we add assumption that $G(y_1) < x_2$, then such values y_1, y_2 satisfy

$$y_1 \neq y_2, \quad G(y_1) = y_2, \quad G(y_2) = y_1. \tag{33}$$

Proof. See Appendix A.5. □

Considering y_1, y_2 determined by (33) of Lemma 4, let us denote

$$\mathcal{F}_0 \equiv \{y_1, y_2\}, \quad \mathcal{F}_{t+1} \equiv G^{-1}(\mathcal{F}_t) \quad \forall t \geq 0, \quad \mathcal{F} \equiv \cup_{t \geq 0} \mathcal{F}_t.$$

The following result is a direct consequence of Lemma 4 and definition of $\mathcal{F}$.

Proposition 6 (A two-period cycle around the low steady state). *Under Assumption 3 and conditions in Lemma 4, we have: if $k_0 \in \mathcal{F}$, then there exists t_0 such that $k_{2t} = y_1$, $k_{2t+1} = y_2$ for any $t \geq t_0$.*
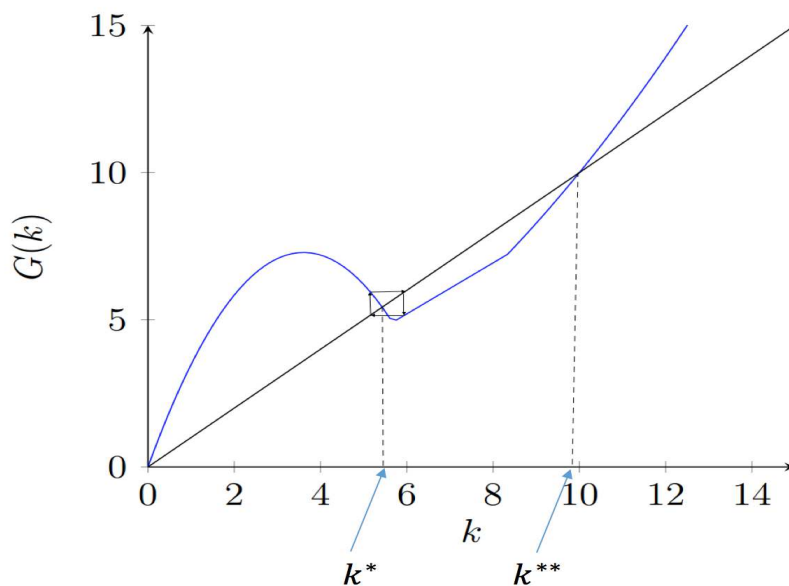

Figure 4: *Fluctuation around the low steady state.* Condition (31) holds and $x_3 < k^*$.

Proposition 6 indicates that if the initial capital belongs to $\mathcal{F}$ of $\mathbb{R}^+$, there is neither possibility for the recipient country to converge to the low steady state, nor the possibility of reaching an economic take-off.[15] The key for obtaining Proposition 6 is condition (31) which is equivalent to $3\frac{1+\tau}{\beta} - (1-\delta) < A(1 + \sigma\alpha_i\bar{a} - b)$. This holds if and only if

$$1 + \sigma\alpha_i\bar{a} > b \text{ and } A > \frac{3\frac{1+\tau}{\beta} - (1-\delta)}{1 + \sigma\alpha_i\bar{a} - b} \tag{34}$$

---

[15]However, it should be noticed that the fluctuation around the low steady state is not necessarily worse than the convergence towards this level.

It means that the maximum of aid $\bar{a}$ and the efficiency in the use of aid and the TFP are quite high.

The intuition of Proposition 6 is the following: Consider a country having a middle-level of initial capital and satisfying condition (34), when it receives aid at the initial date, its economy may grow at date 1 (according to Proposition 4). When the economy grows, its capital at date 1 increases. By the rule of aid, the aid flow for date 1 may decrease, leading to a decrease of total investment at date 1. Hence, the capital at date 2 may decrease, and so on. It follows that a two-period cycle may arise. Figure 4 illustrates this cycle.[16]

4.3.2 Lucky growth

As shown above, a country having intermediate circumstances 2 with aid focus can converge to the low stead state or fluctuate around it. In this section, we wonder whether such a country can achieve growth in the long run. Notice that we continue to consider Assumption 3, under which we have $x_3 < k^{**}$.

We distinguish two subcases: (1) $G(x_3) \le k^{**} = G(k^{**})$ corresponding to low dynamics of capital; and (2) $G(x_3) > k^{**} = G(k^{**})$ strong dynamics of capital, meaning that with some value in $(0, k^{**})$ (here it is x_3), the output can overcome the critical threshold k^{**}. Condition $G(x_3) > k^{**}$ is equivalent to

$$\frac{\beta}{1+\tau} \frac{\left(1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)\right)^2}{4A(\alpha_i\phi - \tau)} > \frac{D+b}{\tau}. \tag{35}$$

Under Assumption 3, the left hand side of (35) increases in $\bar{a}, \sigma, \alpha_i$ but decreasing in ϕ.[17] The right hand side depends neither on $(\bar{a}, \phi)$ nor on (σ, α_i). Hence, condition $G(x_3) > k^{**}$ is more likely to hold if $\bar{a}, \sigma, \alpha_i$ are high and/or ϕ is low.

Let us denote

$$U_0(k^{**}) \equiv \{x \in [0, k^{**}] : G(x) > k^{**}\}, \quad U_{t+1}(k^{**}) \equiv G^{-1}(U_t(k^{**})), \quad \forall t \ge 0$$
$$U(k^{**}) \equiv \cup_{t\ge 0} U_t(k^{**}).$$

Note that $k^* \notin U(k^{**})$ and $k^* > x_3$. Here, k^{**} is the high steady state. It is easy to see that k_t tends to infinity if $k_0 > k^{**}$. The following result shows the asymptotic property of equilibrium capital path (k_t) for the case $k_0 < k^{**}$.

Proposition 7 (Lucky growth). *Assume that conditions in Assumption 3 hold.*

1. *If $G(x_3) \le k^{**}$, then $k_t \le k^{**}$ for any $k_0 \le k^{**}$.*

2. *If $G(x_3) > k^{**}$, then we have: $U(k^{**}) \ne \emptyset$, and $\lim_{t\to\infty} k_t = \infty$ for any $k_0 \in U(k^{**})$. Consequently, aid flows $a_t = 0$ for t high enough.*

   Moreover, if $k_0 \in U_T(k^{**})$, then $k_t < k^{**}$ for any $t < T$, $k_t > k^{**}$ for any $t > T$.

Proof. See Appendix A.6. $\qquad\qquad\square$

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[16]Parameters in Figure 4 are $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.5, \sigma = 1.2, \alpha_i = 0.8; \bar{a} = 12, b = 2, \phi = 2$.

[17]It is easy to see that the left hand side increases in $\bar{a}, \sigma$ but decreasing in ϕ. It is increasing in α_i because $x_3 < x_1$.
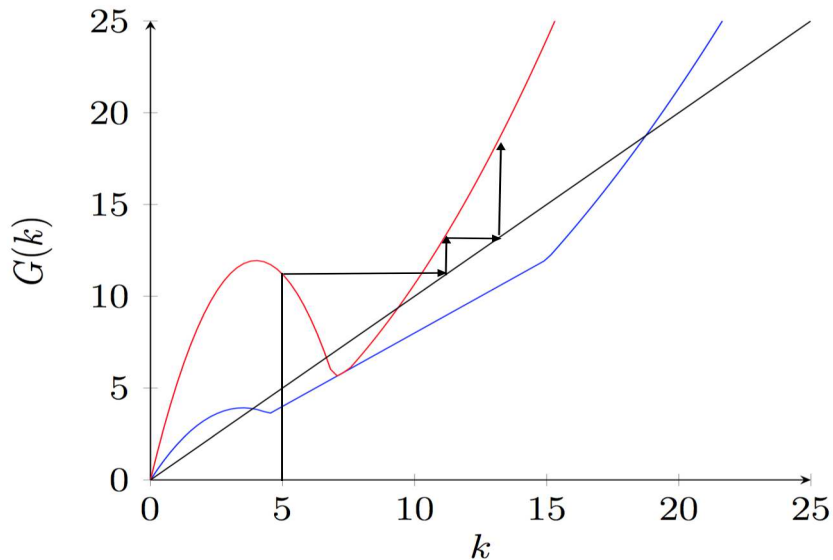
Figure 5: *Lucky growth vs. middle-income trap.* Low curve corresponds to $G(x_3) < G(k^{**})$, with parameters $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.4, \sigma = 1, \alpha_i = 0.7; \bar{a} = 12, b = 3, \phi = 2$. High curve corresponds to $G(x_3) > G(k^{**})$, with $\bar{a} = 14$, $\alpha_i = 0.8, \sigma = 2$, other parameters unchanged.

The first point in Proposition 7 indicates that when the dynamics of capital are weak, then the economy never surpasses the middle-income trap.

Point 2 of Proposition 7 suggests that a poor country, receiving development aid and having strong dynamics of capital, may surpass the middle-income in a finite period and achieve growth in the long run. To illustrate this point, let us consider $k_0 \in U_0(k^{**})$. When the dynamics of capital are strong $(G(x_3) > k^{**})$, the stock of capital at the next period will be high (thanks to development aid) and surpass the middle-income trap k^{**} (i.e., $k_1 = G(k_0) > k^{**}$), and then the recipient economy may reach growth. However, in some cases, the economy needs more than one period to surpass the middle-income trap (for example, when $k_0 \in U_T(k^{**}$, the economy only surpasses k^{**} after T periods).

Figure 5 indicates that if $k_0 = 5$, then the economy grows in the long run. However, if $k_0 = k^* > 5$, we have $k_t = k^*$ for any t. So, a higher initial capital does not necessarily help the economy to have more growth. Having growth without bounds, k_0 must belong to $U(k^{**})$. For that reason, we use the term "lucky growth": with the same rule of aid, a poorer country may have growth but a richer country may not.

4.4 Discussion

We have seen in previous sections that the same rule of aid $(\bar{a}, \phi)$ may generate very different effects in the recipient country, following its initial circumstances. Focusing on autonomous technology, government effort, efficiency in the use of aid, efficiency in public investment and its fixed cost, we can distinguish 4 levels of circumstances, ranked from low to high quality: low circumstances, intermediate circumstances with government effort focus, intermediate circumstances with foreign aid focus, and high circumstances. If the recipient has a relatively high quality of circumstances, the development aid may help it to reach economic growth whatever the initial capital. Consequently, there will exist

a period when this economy no longer needs international aid to stimulate its economic development. In the opposite circumstances with low quality circumstances, our analysis shows that the recipient country would obtain an economic take-off only if aid flows are sufficiently high. This result might justify a scaling-up of aid for countries suffering initial disadvantages which are not in favor of generating economic growth.

Concerning two intermediate circumstances, as we have shown in Section 4.2 and Section 4.3, their equilibrium outcomes are very different and hard to compare. On the one hand, under the intermediate circumstances 1 with domestic investment focus, k^* is the only steady state and can be viewed as a poverty trap of the economy. The economy will collapse in the long run if and only if the initial capital of the country is lower than this trap. In this case, development aid may promote growth in the recipient country, but under the condition that the use of aid is efficient enough. On the other hand, under the intermediate circumstances 2 with foreign aid focus, there exists two steady states. The lower one k^* can be interpreted as a poverty trap while the higher one k^{**} can be interpreted as a middle income trap. In this case, with foreign aid, the economy never collapses, even if its initial capital is very low. However it does not necessarily mean that the economy will grow in the long run. Instead, the outcomes are fragile. Indeed, it may converge to the poverty trap level or fluctuate around it. With some luck (strong dynamics of capital), the economy may benefit development aid to improve its public investment (including R&D) and thanks to this, it can surpass the middle income trap and get grow after a finite period.

It should be noticed that if we consider a general setup of endogenous productivity and aid flows: $A_t = \mathcal{A}(B_t)$ and $a_t = \Phi(k_t)$, the dynamics of capital stocks become:

$$k_{t+1} = \beta \frac{1 - \delta + \mathcal{A}(\tau k_t + \alpha_i \Phi(k_t))]}{1 + \tau} k_t \tag{36}$$

Since $\mathcal{A}$ is an increasing function while Φ is a decreasing function, the general form (36) may yield similar characteristics of the capital path (e.g., convergence, non-monotonicity, two-period cycles, multiple steady states, etc.) as in our above analyses. The advantage of our framework rests on its tractability and it can also cover all possible scenarios.

Let us consider a particular case with a full depreciation of capital ($\delta = 1$), there is no fixed cost ($b = 0$), and no capital tax ($\tau = 0$). If we consider a simple case $\mathcal{A}(B_t) = \sigma B_t$, and set the rule of aid flows as $a_t = \theta k_t^\lambda$ where $\theta > 0, \lambda < 0$ as in Dalgaard (2008), we have:

$$k_{t+1} = \beta A \sigma \alpha_i \theta k_t^{\lambda+1} \tag{37}$$

Then, we recover a dynamic system similar to that in Dalgaard (2008). The transitional dynamics of capital stock in (37) are much simpler than (11) in our framework. In Dalgaard (2008) or in (37), the characteristics of the transitional path are determined by λ (the degree of inequality aversion on the part of the donor) while in our model they depend on all parameters. In particular, the model in Dalgaard (2008) has at most one steady state while ours may have two. Our framework allows us to study whether a poor country can surpass not only the low steady state but also the high one, then obtain growth in the long run.

5 Concluding remarks

Our paper presents a simple and tractable model to investigate the effectiveness of foreign aid given the donors' rules. It contributes to the debate regarding the effectiveness of aid in terms of economic growth, comprising numerous empirical investigations. We have fully characterized the transitional dynamics of capital in all scenarios. The effectiveness of foreign aid depends strongly on the manners in which aid is used in recipient countries and on the absorptive capacity of these countries as well as the initial development level of the recipient countries.

Some countries with high circumstances do not need aid to grow. Some others with intermediate circumstances need aid for the first stages of their development process. Foreign aid may help a poor country to avoid collapse, to converge towards its low steady state, or to get an economic take-off. But focusing on foreign aid may make the country dependent on aid and hence economic fluctuations may arise. Our analyses show that the recipient's TFP, the efficiency of public investment and in the use of aid play major roles in the country?s development.

In our framework, the recipient country receives foreign aid with exogenous rules $(\bar{a}, \phi)$ (although the aid flow $(\bar{a} - \phi k_t)^+$ is endogenous). For future research, it would be interesting to endogenize the aid rules as well as the efficiency in the use of aid. By doing this, we can investigate the optimal design of development aid and the reaction of the recipient country's government (specially when corruption may happen).

A Appendix

A.1 The solution of the consumer's problem in Section 2

Lemma 5. *Consider the optimal growth problem*

$$\max_{(c_t, s_t)_t} \sum_{t=0}^{\infty} \beta^t \ln(c_t) \tag{38}$$

$$c_t + s_{t+1} \leq A_t s_t, \quad c_t, s_t \geq 0. \tag{39}$$

The unique solution of this problem is given by $s_{t+1} = A_t s_t$ for any $t \geq 0$.

Proof. Indeed, the Euler condition $c_{t+1} = \beta A_{t+1} c_t$ jointly with the budget constraint becomes $s_{t+2} - \beta A_{t+1} s_{t+1} = A_{t+1}(s_{t+1} - \beta A_t s_t)$. Thus, a solution is given by $s_{t+1} = \beta A_t s_t$. It is easy to check the transversality condition $\lim_{t \to \infty} \beta^t u'(c_t) s_{t+1} = 0$.

By the concavity of the utility function, the solution is unique.

□

A.2 Properties of function f and G

To prove Lemma 1, we need the following results.

Claim 1 (Properties of f)**.**

    1. The function $f_1(k) \equiv (k - a)^+$ is increasing in k.

2. *The function $f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+$ is increasing on $[0, \infty]$ if $\tau \geq \alpha_i \phi$. When $\tau < \alpha_i \phi$, the function f_2 is decreasing on $[0, \bar{a}/\phi]$ and increasing on $[\bar{a}/\phi, \infty]$.*

3. $f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+ \geq \bar{a} \min(\alpha_i, \tau/\phi)$.

4. $f(k_t) \geq \dfrac{\beta}{1 + \tau}\Big[1 - \delta + A\Big(1 + \big(\sigma\bar{a}\min(\alpha_i, \tau/\phi) - b\big)^+\Big)\Big]$.

Proof. The three first points are obvious. Let us prove the last point. We consider 2 cases. If $k \geq \bar{a}/\phi$, it is easy to see that $f_2(k) \geq \tau k \geq \tau\bar{a}/\phi \geq \bar{a}\min(\alpha_i, \tau/\phi)$.

If $k \leq \bar{a}/\phi$, then $f_2(k) = \alpha_i\bar{a} + (\tau - \alpha_i\phi)k$.

When $\tau - \alpha_i\phi \geq 0$, we have $f_2(k) \geq \alpha_i\bar{a}$.

When $\tau - \alpha_i\phi \leq 0$, we have $f_2(k) \geq \alpha_i\bar{a} + (\tau - \alpha_i\phi)\bar{a}/\phi = \alpha_i\bar{a}/\phi$. $\qquad\square$

Claim 2.   *1. G is increasing on $[x_1, \infty)$.*

2. *Assume that $x_2 > 0$. We have G increasing on $[x_2, \infty)$.*
Consequently, G is increasing on $[\min(x_1, x_2), \infty)$.

Proof of Claim 2.   1. G is increasing on $[x_1, \infty)$ because when $x \geq x_1$, we have

$$G(x) = \beta\frac{1 - \delta + A\Big(1 + (\sigma\tau x - b)^+\Big)}{1 + \tau}x.$$

2. If $x_1 < x_2$, it is trivial that G is increasing on $[x_2, \infty)$ because it is increasing on $[x_1, \infty)$.

   We now consider the case where $x_1 > x_2$. Let x and y such that $x \geq y \geq x_2$. We have to prove that $G(x) \geq G(y)$. It is easy to see that $G(x) \geq G(y)$ when $x, y \in [x_2, x_1]$ or $x, y \in [x_1, \infty)$. We now assume that $x \geq x_1 \geq y$. In this case, we have

$$G(x) = \beta\frac{1 - \delta + A\Big(1 + (\sigma\tau x - b)^+\Big)}{1 + \tau}x \geq \beta\frac{1 - \delta + A}{1 + \tau}x$$

$$G(y) = \beta\frac{1 - \delta + A\Big(1 + (\sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi - \tau)y)^+\Big)}{1 + \tau}y = \beta\frac{1 - \delta + A}{1 + \tau}y$$

   where the last equality is from the fact that $y \geq x_2 \equiv \dfrac{\sigma\alpha_i\bar{a} - b}{\sigma(\alpha_i\phi - \tau)}$. So, it is clear that $G(x) \geq G(y)$.

$\qquad\square$

Proof of Lemma 1.   1. When $\tau \geq \alpha_i\phi$, by using point 3 of Lemma 1, we get that G is increasing on $[0, \infty)$.

2. When $\tau < \alpha_i\phi$ and $x_2 < 0$. We consider two cases.
   If $x \leq \bar{a}/\phi$, then $\big(\sigma(\tau x + \alpha_i(\bar{a} - \phi x)^+) - b\big)^+ = (\sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi - \tau)x)^+ = 0$
   (because $\sigma\alpha_i\bar{a} - b < 0$). So, in this case, we have $G(x) = \beta\dfrac{1 - \delta + A}{1 + \tau}x$.

If $x \geq \bar{a}/\phi$, we have

$$G(x) = \beta \frac{1 - \delta + A\left(1 + (\sigma\tau x - b)^+\right)}{1 + \tau}x.$$

It is easy to see that G is increasing on $[0, \infty)$.

3. We now consider the last case where $\tau < \alpha_i\phi$ and $x_2 > 0$, and $x_3 > \min(x_1, x_2)$.

First, according to Lemma 2, we observe that G is increasing on $[\min(x_1, x_2), \infty)$.

Second, we also see that G is increasing on $(0, x_3)$. Since $x_3 > \min(x_1, x_2)$, we obtain that G is increasing on $[0, \infty)$.

$\square$

Proof of Lemma 2. According to Lemma 2, we have that G is increasing on $[\min(x_1, x_2), \infty)$. We now consider G on $[0, \min(x_1, x_2)]$. Let $x \in [0, \min(x_1, x_2)]$. We have

$$G(x) = f_3(x) = \beta \frac{1 - \delta + A\left(1 + \sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi - \tau)x\right)}{1 + \tau}. \tag{40}$$

By definition of x_3, we have $f_3'(x_3) \geq 0$ if and only if $x \leq x_3$. Therefore, G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$.

$\square$

A.3 Proof Proposition 4

When x is low enough, we have $G(x) = f_3(x)$. So, we can easily compute $G'(x)$ for any x low enough. Under condition $D + b < \sigma\alpha_i\bar{a}$, we can verify that $G'(0) > 1$. Hence, $G'(x) > 1$ for any x low enough. By consequence, $k_1 = G(k_0) > k_0$ for any k_0 low enough.

A.4 Proof of Proposition 5

Point 1. First, we need the following result.

Claim 3. *Assume that $\bar{a}\alpha_i > \frac{D+b}{\sigma} > \bar{a}\tau/\phi$ and $x_3 < x_2$.*
If $x_3 > k^*$, then $G(x_3) < x_3$. And therefore, $G(x_3) < x_3 < x_2 < k^* = G(k^*)$. In this case, we have $G(x) < k^*$ for any $x < k^*$.

Proof of Claim 3. It is easy to see that if $x_3 > k^*$, then $G(x_3) < x_3 < x_2 < k^* = G(k^*)$.
If $x < k^*$, then we have $G(x) \leq \max_{x \leq k^*} G(x) \leq G(x_3) < x_3 \leq k^*$.
$\square$

We now come back to the proof of Proposition 5.
If $k_0 < k^*$, according to Claim 3, we have $k_1 = G(k_0) < k^*$. By induction, we have $k_t < k^*$ for any t.
We now prove that $\lim_{t \to \infty} k_t = k^*$ for any $k_0 \in (0, k^*)$.
Case 1: $k_0 \in (0, x_3]$. Since G is increasing on $[0, x_3]$, we have $\lim_{t \to \infty} k_t = k^{**}$ for any $k_0 \in (0, x_3]$.
Case 2: $k_0 \in (x_3, x_2]$. We see that $k_1 = G(k_0) \leq \max_{x \in [0, x_2]} G(x) = G(x_3) < x_3$. Therefore $k_1 < x_3$, and so $\lim_{t \to \infty} k_t = k^{**}$.

Case 3: $k_0 \in [x_2, \bar{a}/\phi]$, we have $k_1 = G(k_0) = \frac{\beta(1-\delta+A)}{1+\tau}k_0$. Since $\frac{\beta(1-\delta+A)}{1+\tau} < 1$, there exists t_0 such that $k_{t_0} < x_2$. Thus $\lim_{t\to\infty} k_t = k^{**}$.

Case 4: $k_0 \in [\bar{a}/\phi, k^*]$, we have $G(k_0) < k_0$ which means that $f(k_0) < 1$. Combining with $k_1 = f(k_0)k_0$, there exists t_1 such that $k_1 < \bar{a}/\phi$. This implies that $\lim_{t\to\infty} k_t = k^{**}$.

Point 2. Recall that

$$G(k) = f_3(k) \equiv \frac{\beta}{1+\tau}\Big[1 - \delta + A\Big(1 + \sigma\alpha_i\bar{a} - \sigma(\alpha_i\phi - \tau)k - b\Big)\Big]k \tag{41}$$

$$= \frac{\beta}{1+\tau}\Big[1 - \delta + A\big(1 + \sigma\alpha_i\bar{a} - b\big) - A\sigma(\alpha_i\phi - \tau)k\Big]k \tag{42}$$

$$G'(k) = f_3'(k) = \frac{\beta}{1+\tau}\Big[1 - \delta + A\big(1 + \sigma\alpha_i\bar{a} - b\big) - 2A\sigma(\alpha_i\phi - \tau)k\Big]. \tag{43}$$

It is easy to compute that

$$G'(k^*) = \frac{\beta}{1+\tau}\Big[1 - \delta + A(1 + b + 2B - \sigma\bar{a}\alpha_i)\Big]. \tag{44}$$

According to Bosi and Ragot (2011), k^* is locally stable if and only if $\|G'(k^*)\| < 1$. Since $x_3 < k^*$, have have $G'(k) < 0$. So, k^* is locally stable if and only if $G'(k) > -1$ which is equivalent to

$$3\frac{1+\tau}{\beta} - (1-\delta) + A(b - 1 - \sigma\alpha_i\bar{a}) > 0. \tag{45}$$

A.5   Proof of Lemma 4

We will find $y_1, y_2 > 0$ such that (32). Let us denote $n = 1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)$ and $m = A\sigma(\alpha_i\phi - \tau)$. y_1, y_2 must satisfy

$$\frac{\beta}{1+\tau}(n - my_1)y_1 = y_2, \quad \frac{\beta}{1+\tau}(n - my_2)y_2 = y_1. \tag{46}$$

Since $y_1 \neq y_2$, we have $\frac{\beta}{1+\tau}(n - m(y_1 + y_2)) = -1$. So, we obtain

$$H(y_1) \equiv \frac{\beta}{1+\tau}(n - my_1)y_1 + y_1 - \frac{1}{m}\Big(n + \frac{1+\tau}{\beta}\Big) = 0 \tag{47}$$

We have $H(y_1) < 0$. We also see that $H(k^*) > 0$ if condition (31) is satisfied.

Under condition (31), there exists y_1 such that $H(y_1) = 0$. Therefore, y_1 and $y_2 = f_3(y_1)$ satisfy (32).

A.6   Proof of Proposition 7

Point (1). Since conditions in Assumption 3 hold, Lemma 2 implies that G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$. So, $\max_{x \leq k^{**}} G(x) \leq Max(G(x_3), G(k^{**})) \leq k^{**}$. Therefore $k_t = G(k_{t-1}) \leq k^{**}$ for any $k_0 \leq k^{**}$.

Point (2). If $G(x_3) > k^{**}$, then $x_3 \in U_0(k^{**}) \subset U(k^{**})$. So, $U(k^{**}) \neq \emptyset$. Now, let $k_0 \in U(k^{**})$, then there exists t_0 such that $G^{t_0}(k_0) > k^{**}$, where $G^1 \equiv G$ and $G^{s+1} \equiv G(G^s)$ for any $s \geq 1$. So, $k_{t_0} = G^t(k_0) > k^{**}$. This implies that $(k_t)_{t \geq t_0}$ is an increasing sequence and $\lim_{t\to\infty} k_t = \infty$.

References

Alesina, A., & Dollar, D. (2002). Who Gives Foreign Aid to Whom and Why?. *Journal of Economic Growth* 5(1), 33-63.

Azariadis, C., & Drazen A. (1990). Threshold externalities in economic development. *Quarterly Journal of Economics*, 105, 501-526.

Azariadis, C., & Stachurski, J. (2005). Poverty traps. In Aghion F. and Durlauf (Ed.), *Handbook of Economic Growth*, Vol. 1.1. Elsevier B.V.

Barro, R. (1990). Government Spending in a Simple Model of Endogeneous Growth. *Journal of Political Economy*, 98, S103-S125.

Berthelemy, J.-C., & Tichit, A. (2000). Bilateral donors' aid allocation decisions? a three-dimensional panel analysis. *International Review of Economics & Finance* 13(3), 253-274.

Bosi, S., & Ragot L. (2011). Discrete time dynamics: an introduction. *Bologna: CLUEB.*

Bruno O., Le Van, C., & Masquin, B. (2009). When does a developing country use new technologies?. *Economic Theory*, 40, 275-300.

Burnside, C., & Dollar, D. (2000). Aid, policies and growth. *American Economic Review* 90(4), 409-435.

Carter, P. (2014). Aid Allocation Rules. *European Economic Reviews*, 71, 132-151.

Carter, P., Postel-Vinay, F., & Temple, J.R.W. (2015). Dynamic aid allocation. *Journal of International Economics*, 95(2), 291-304.

Chauvet, L., & Guillaumont, P. (2003). Aid and Growth Revisited: Policy, Economic Vulnerability and Political Instability. In Tungodden B., Stern N. and Kolstad (Ed.), *Toward Pro-Poor Policies-Aid, Institutions and Globalization*, Oxford University Press, New York.

Chauvet, L., & Guillaumont, P. (2009). Aid, Volatility and Growth Again: When Aid Volatility Matters and When it Does Not. *Review of Development Economics,* 13(3): 452-463.

Charterjee, S., & Sakoulis, G., & Tursnovky, S. (2003). Unilateral capital transfers, public investment, and economic growth. *European Economics Review*, 47, 1077-1103.

Charteerjee, S., & Tursnovky, S. (2007). Foreign aid and economic growth: the role of flexible labor supply. *Journal of Development Economics*, 84, 507-533.

Clemens, A. M., Radelet, S., Bhavnani, R .R., & Bazzi, S. (2012). Counting chickens when they hatch: timing and the effects of aid aid on growth. *Economic Journal*, 122(561), 590-617.

Collier, P., & Dollar, D. (2001). Can the world cut poverty in half? How policy reform and effective aid can meet intrenational development goals. *World Development* 29(11), 1787-1802.

Collier, P., & Dollar, D. (2002). Aid allocation and poverty reduction. *European Economic Review* 46, 1475-1500.

Dalgaard, C.-J. (2008). Donor Policy Rules and Aid Effectiveness. *Journal of Economic Dynamics & Control*, 32, 1895-1920.

Easterly, W. (2003). Can foreign aid buy growth. *Journal of Economic Perspectives*, 17 (3), 23-48.

Easterly, W., Levine, R., & Roodman, D. (2004). Aid, policies and growth: comment. *American Economic Review,* 94 (3), 774-780.

Feeny, S., & McGillivray, M. (2010). Aid and public sector fiscal behavior in failing states. *Economic Modelling,* 27, 1006-1016.

Franco-Rodrigez, S., Morrissey, O., & McGillivray, M. (1998). Recent Developments in Fiscal Response with an Application to Costa Rica. *World Development* 26(7), 1241-1250.

Guillaumont, P., & Chauvet, L. (2001). Aid and performance: A reassessment. *Journal of Development Studies*, 37(6), 6-92.

Guillaumont, P., & Guillaumont Jeanneney, S. (2010). Big Push versus absorptive capacity: How to reconciliate the two approaches?. In G. Mavrotax (Ed.), *Foreign aid and development issues, challenges, and the New Agenda (297-322). Oxford: Oxford University Press.*

Guillaumont, P., & Wagner, L. (2013). Aid Effectiveness for Poverty Reduction: Lessons from Cross?country Analyses, with a Special Focus on Vulnerable Countries. *Revue d'Économie du Dveloppement*, vol. 22, 217-261.

Guillaumont, P., McGillivray, M., & Wagner, L. (2017). Performance assessment, vulnerability, human capital and the allocation of aid among developing countries. *World Development*, 90, 17-26.

Hansen, H., & Tarp, F. (2001). Aid and growth regressions. *Journal of Development Economics,* 64, 547-570.

Islam, M. N. (2005). Regime changes, economic policies and the effect of aid on growth. *Journal of Development Studies*, 41:8, 1467-1492, DOI:10.1080/00220380500187828.

Jones, L., & Manuelli, R. (1990). A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy*, 98(5), 1008-1038.

Kamihigashi, T., & Roy, S. (2007). A nonsmooth, nonconvex model of optimal growth. *Journal of Economic Theory*, 132, 435-460.

Khan, H., & Hoshino E. (1992). Impact of foreign aid on the fiscal behavior of LDC governments. *World Development*, 20(10), 1481-1488.

Kraay, A., & Raddatz, C. (2007). Poverty trap, aid, and growth. *Journal of Development Economics*, 82, 315-347.

Marx, A., & Soares, J. (2013). South Korea's transition from recipient to DAC donor: assessing Korea's development cooperation policy. *International Development Policy*, 4.2, 107-142.

McGillivray, M., & Pham, T.K.C. (2017). Reforming performance based aid allocation practice. *World Development*, 90, 1-5.

Ouattara, B. (2006). Aid, debt and fiscal policies in Senegal. *Journal of International Development*, 18(8), 1105-1122.

Roodman, D. (2007). The anarchy of numbers, development, and cross-country empirics. *World Bank Economic Review*, 21(2), 255-277.

Sachs, J. (2005). *The end of poverty. How we can make it happen in our lifetime.* London: Penguin Book.

Wagner, L. (2014). Identifying thresholds in aid effectiveness. *Review of World Economics*, 150(3), 619-638.