Inheritance taxation in a model with intergenerational time transfers[*]

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Abstract

We consider a two-period overlapping generation model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. Starting from a steady state where public spendings are financed through taxation on saving income and labor, we analyze a tax reform that consists in a shift of the tax burden from saving income tax towards inheritance tax, leaving the capital-labor ratio unchanged. In the standard Barro model with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. We assume the young have elastic labor supply and can receive time transfers from their parents. Then inheritance tax modifies the trade-off parents make between both kinds of private transfers, and can be Pareto-improving. The Pareto improvement strongly depends on the strength of the positive effect of time transfers on the young's labor supply and on the strength of the effect of higher labor supply on the production of market goods.

Keywords: family transfers, altruism, bequests, time transfers, inheritance tax.
JEL Classifications: H22, H24, J22.

1 Introduction

Inheritance taxation is one of the most controversial subjects in the public policy debate and among economists. Currently, an increasing number of countries are without inheritance tax or have significantly reduced it such as United States or United kingdom. For the opponents, the inheritance tax discourages capital accumulation and the incentive to work, and it is an immoral tax which increases the pain suffered by mourning families. They claim that tax on bequest involves "double taxation" of saving incomes which have been already taxed. A second line of argument suggests that if people have a long enough horizon (through altruistic behavior), inheritance taxation that impacts distant consumption is inefficient. This point has been shown by Chamley (1986) in a model of individuals with rational altruism *à la* Barro (1974). The long run optimal tax on inheritance converges towards zero even if saving income taxation is different to a tax on bequests.[1]

Over the past few years, an extensive literature has shown that we can overturn the Chamley-Judd result of zero capital income (inheritance) taxation by relaxing some of their hypotheses.[2] However, the previous theoretical literature about inheritance taxation has essentially focused on financial bequests as the single source of intergenerational transfers within family. Nevertheless, a number of empirical studies suggest that time transfers from parents to their children are substantial and on average almost as important as monetary transfers (see for example Cardia and Ng (2003) and Schoeni et al. (1997)).[3] Some studies based on the SHARE survey[4], such as Attias-Donfut et al. (2005) or also Albertini et al. (2007), show that parent's time transfers to children consists mainly in childcare. According to Wolff and Attias-Donfut (2007), two-fifth of grandparents keep their grandchildren every week. A common finding is that grandparents still support parents' home production with household tasks for instance.

Thanks to the intergenerational transfers of time in the form of grandparenting, parents free up more time for working and taking care of their children. Labor supply of the heirs as well as life cycle resources are affected differently by time transfers compared to inheritances as shown by Cardia and Michel (2004) or Belan et al. (2010). Taking into account time transfers allows to deal with the trade-off between both types of transfers. Time transfers have some macroeconomic implications through labor supply of the next generation, while bequests enhance its private wealth (Cardia and Ng, 2003).

Despite their importance and macroeconomic implications, the theoretical literature about fiscal

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[1]See Chamley (1986, p. 613).

[2]A non-zero bequest tax result is potentially achieved by assuming other household's bequest motives (Cremer and Pestieau, 2011) or for example, focusing on a model with heterogeneous random tastes for bequest and for wealth per se (Piketty and Saez, 2013) or, lastly, considering imperfect competition on capital market (Farhi and Werning, 2010).

[3]For example Cardia and Ng (2003, Table 1) uses the Health and Retirement Study of 1992 and report that the mean time transfer for total sample (7547 households) of 325 hours has a value of $1950 (using a time cost of $6 per hour), which is similar of the sample mean of $1868 for monetary transfers.

[4]The Survey of Health, Ageing and Retirement in Europe is conducted since 2004 in ten Western European countries.

incidence of inheritance tax has not devoted attention to time transfers. Whether or not time transfers are introduced, inheritance taxation reduces the incentive to leave resources to the next generation. Taking account of time transfers adds a substitution effect since inheritance tax now affects the trade-off between monetary and time transfers, making time transfers more attractive. From this point of view, taxing bequests may enhance the young's labor supply, giving room to an increase in resources disposable for consumption. Nevertheless, the positive effect on labor supply has to be balanced with the potential reduction in private wealth that may be detrimental for capital accumulation. At least, the resulting capital-labor ratio could be lower, moving the economy away from the Golden-rule of capital accumulation.

In this paper, considering time transfers in a second-best world, we analyze whether shifting from savings income tax towards inheritance tax may be a welfare-improving tax reform. The fall in saving taxation may compensate the negative effects of inheritance tax on savings, capital stock and capital-labor ratio. But simultaneously, the reform may increase stead-state resources since the inheritance tax has a positive effect on labor supply. To analyze and disentangle the above effects, we consider a two-period overlapping generation model with rational altruism taking into account both types of family transfers (inheritance and time transfers) from grandparents to parents. Individuals work when young (i.e. parent) and then retire in their second period of life (i.e. when they are grandparents). In each period, every household consumes a composite good that aggregates market good and home production. Parent's labor supply decision depends on the trade-off between formal work and home production. Then, grandparents contribute to home production of the parents through both family transfers. Furthermore, the government finances public spending using taxation on inheritance, saving income and labor income.

As in the standard Barro model with rational altruism, inheritance tax decreases the accumulated capital stock and thus, reduces the capital-labor ratio at steady state. But the fall in saving income tax allows to neutralize this steady-state effect. Assuming that the tax reform is designed in order to leave the steady-state capital-labor ratio constant, we identify situations where life-cycle utility increases. First, steady-state utility is likely to increase when the substitution effect between consumption of market good and time devoted to home production is strong. Indeed, in this case, inheritance tax makes time transfers more attractive, that is, the grandparents prefer to leave higher time transfers and lower bequests. The higher the substitution effect, the higher the increase in labor supply of the parents. Secondly, even if the substitution effect is not too strong, the tax reform may have a positive effect on steady-state utility through the size of the additional production of market goods generated by the increase in labor supply. We show that the strength of the latter effect depends crucially on the gaps between the marginal rates of transformation and the marginal rates of substitution between consumption in market goods and time devoted to home production. The reform is likely to increase utility if lower time devoted to home production allows the production sector to generate more market goods than necessary for leaving individual with the same level of utility. However, keeping the steady capital-labor ratio constant shifts the burden of the initial public debt towards the first generations and introduces some intergenerational redistributions.

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Using a numerical example, we illustrate that the effect of the tax reform on household's welfare of each generation can be positive along the transitional dynamics.

The paper is organized as follow. In Section 2, the model is presented. Section 3 analyzes the steady-state equilibrium with operative bequest and positive time transfers. Then, in Section 4, we present the tax reform and study its effects on households' utility without time transfers or when both types of transfers are positive. In Section 5, we conduct numerical illustrations in order to study the impact of the tax reform on the whole dynamics. The final section concludes.

2   The model

We consider a two-period overlapping generation model. Time is discrete. Population is constant and normalized to unity. It consists in one dynasty where the representative agent of generation t has one child, born in $t + 1$. We consider dynastic altruism *à la* Barro (1974) from parents to children.

2.1   Households

The representative household of generation t works during his first period of life (i.e. when young, or equivalently parent) and then retires (i.e. when old, or equivalently grand parent). Labor supply when young is elastic and depends on the allocation of a one-unit time endowment between formal work and home production. In both periods, the household consumes a composite good that aggregates market good and home production. Life-cycle utility writes

$$u\left(f^y\left(c_t, T_t^y\right)\right) + v\left(f^o\left(d_{t+1}, T_{t+1}^o\right)\right)$$

where u and v are increasing and strictly concave. Function $f^y\left(c_t, T_t^y\right)$, respectively $f^o\left(d_{t+1}, T_{t+1}^o\right)$, is the quantity of composite good when young, resp. when old. The former is obtained with market good expenditures c_t and time devoted to home production T_t^y. In the latter, d_{t+1} represents market good expenditures when old, while T_{t+1}^o is time spent in home production. Home production functions f^y et f^o are assumed to be linear homogenous and concave. Marginal products are strictly positive and strictly decreasing.

Let ℓ_t denotes labor supply of the young in the formal sector. Household's labor supply relies on the trade-off between formal work and home production work. Time devoted to home production when young, T_t^y, aggregates time for his home production $1 - \ell_t$ and time transfer from his parent (denoted by λ_t):

$$T_t^y = 1 - \ell_t + \mu\lambda_t \tag{1}$$

where $\mu > 0$ represents the relative efficiency of time transfer of the parent. Since the parent is retired, time spent in home production when old is the fraction of the one-unit endowment that is

not transferred to his child

$$T_t^o = 1 - \lambda_t \tag{2}$$

In the following, τ_t^w, τ_t^x and τ_t^R are the respective period-t tax rates on wages, bequests and saving income. R_t and w_t denote the gross interest rate and the wage rate. When young, a household born in t receives after-tax wage income $(1 - \tau_t^w) w_t \ell_t$ and after-tax bequest $(1 - \tau_t^x) x_t$. Theses resources are allocated between consumption spendings c_t and saving s_t:

$$c_t + s_t = (1 - \tau_t^w) w_t \ell_t + (1 - \tau_t^x) x_t \tag{3}$$

When old, this household allocates after-tax saving income $(1 - \tau_{t+1}^R) R_{t+1} s_t$ between consumption spendings d_{t+1} and bequest x_{t+1}:

$$d_{t+1} + x_{t+1} = (1 - \tau_{t+1}^R) R_{t+1} s_t \tag{4}$$

Following Barro (1974), households are altruistic in the sense that they enjoy utility of their children. Utility of the household born in t, U_t, depends on his consumptions in composite goods in both periods and utility of his child U_{t+1}:

$$U_t = u \left(f^y (c_t, T_t^y) \right) + v \left(f^o \left(d_{t+1}, T_{t+1}^o \right) \right) + \beta U_{t+1} \tag{5}$$

where β denotes the degree of altruism, $0 < \beta < 1$.

Using equations (1)-(4), both consumptions in market goods rewrite

$$c_t = (1 - \tau_t^w) w_t \left[1 - T_t^y + \mu (1 - T_t^o) \right] + (1 - \tau_t^x) x_t - s_t \tag{6}$$

$$d_{t+1} = (1 - \tau_{t+1}^R) R_{t+1} s_t - x_{t+1} \tag{7}$$

Plugging (6)-(7) into U_t gives household's utility as a function of s_t, x_{t+1}, T_t^y and T_{t+1}^o. The representative household maximizes U_t with respect to these four variables. For an interior solution, this leads to the following first-order conditions:

- with respect to s_t
$$-u_t' f_{c_t}^y + (1 - \tau_{t+1}^R) R_{t+1} v_{t+1}' f_{d_{t+1}}^o = 0 \tag{8}$$

where u_t', $f_{c_t}^y$, v_{t+1}' and $f_{d_{t+1}}^o$ respectively stand for $\frac{\partial u_t}{\partial f_t^y}$, $\frac{\partial f_t^y}{\partial c_t}$, $\frac{\partial v_{t+1}}{\partial f_{t+1}^o}$ and $\frac{\partial f_{t+1}^o}{\partial d_{t+1}}$.

- with respect to T_t^y

$$- (1 - \tau_t^w) w_t f_{c_t}^y + f_{T_t^y}^y = 0, \text{ if } 0 < T_t^y < 1 + \mu (1 - T_t^v) \tag{9}$$

where $f_{T_t^y}^y$ stands for $\frac{\partial f_t^y}{\partial T_t^y}$.

- with respect to x_{t+1}

$$-v'_{t+1} f^o_{d_{t+1}} + \beta \left(1 - \tau^x_{t+1}\right) u'_{t+1} f^y_{c_{t+1}} = 0, \text{ if } x_{t+1} > 0 \tag{10}$$

- with respect to T^o_{t+1}

$$v'_{t+1} f^o_{T^o_{t+1}} - \beta\mu \left(1 - \tau^w_{t+1}\right) w_{t+1} u'_{t+1} f^y_{c_{t+1}} = 0, \text{ if } 0 < T^o_{t+1} < 1 \tag{11}$$

where $f^o_{T^o_{t+1}}$ stands for $\frac{\partial f^o_{t+1}}{\partial T^o_{t+1}}$.

All constraints for an interior solution are not necessarily satisfied at equilibrium. The less critical one is the constraint $T^y_t < 1 + \mu\left(1 - T^o_t\right)$, which is equivalent to $\ell_t > 0$. Assuming that it is satisfied remains to consider equilibria where the production sector uses labor. Two other constraints should be satisfied with small additional assumptions: $T^y_t \geq 0$ and $T^o_{t+1} \geq 0$. Time spent in home production remains positive if substitutability with market goods is not too strong.

Finally, non-negativity constraints on bequests and time transfers deserve some discussion. It depends on the utility gains that parents may expect with both kinds of transfers. As shown by Weil (1987), in the standard Barro framework without time transfers, positive bequests are obtained at steady state if the steady-state capital-labor ratio in the corresponding Diamond economy is below the modified Golden-rule. With time transfers, assuming logarithmic utility and Cobb-Douglas technology, Cardia and Michel (2004) have given conditions for the existence of intertemporal equilibria where both transfers are positive. They also state conditions for the case with zero bequests and positive time transfers. In the Section 3, since our concern is inheritance tax, we focus on a steady state where both transfers are positive.

2.2 Equilibrium

The production sector consists in a representative firm that behaves competitively, and produces output with labor and capital. The production function $F(k, \ell)$ is linear homogenous and concave, and includes capital depreciation. Marginal products are strictly positive and strictly decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$w_t = F_L\left(k_t, \ell_t\right) \tag{12}$$
$$R_t = F_K\left(k_t, \ell_t\right) \tag{13}$$

where k_t is quantity of capital. F_L and F_K stand for the partial derivative of F with respect to labor and capital.

At equilibrium, household savings s_t splits into private capital that will be used in $t+1$ and public

debt Δ_t

$$k_{t+1} + \Delta_t = s_t$$

In each period, government spendings amounts to a fraction Γ of total production. Government resources come from taxation on labor income, saving income and bequests. Then, public debt accumulates according to the following equation:

$$\Delta_t = R_t \Delta_{t-1} + \Gamma F(k_t, \ell_t) - \tau_t^w \ell_t w_t - \tau_t^R R_t s_{t-1} - \tau_t^x x_t$$

where the initial public debt and the inital capital stock are given: $\Delta_{-1} = \bar{\Delta}_{-1}$, $k_0 = \bar{k}_0$.

The assumption that government spendings is proportional to production leads to some externality created by the level of production. Indeed, consider the resource constraint in period t

$$c_t + d_t + k_{t+1} = (1 - \Gamma) F_K(k_t, \ell_t) \tag{14}$$

By increasing capital (resp. labor) used in production, the social marginal product for consumption and investment is $(1 - \Gamma) F_K(k_t, \ell_t)$ (resp. $(1 - \Gamma) F_L(k_t, \ell_t)$), while the private marginal product are higher, equal to $F_K(k_t, \ell_t)$ (resp. $F_L(k_t, \ell_t)$) as stated by the first-order condition of the representative firm (12) and (13). Such externalities are internalized by the private sector if the government sets the tax rates on saving and labor incomes to $\tau_t^R = \tau_t^w = \Gamma$.

3 Steady state with positive transfers

We consider steady states with operative bequests and positive time transfers. Tax rates and public debt are assumed to be constant over time. From the marginal conditions (8) and (10), the gross interest rate satisfies the modified Golden-rule, and is equal to R_M defined as

$$\beta(1 - \tau^x)(1 - \tau^R) R_M = 1 \tag{15}$$

which characterizes the capital-labor ratio $k/\ell = z_M$ and the wage rate $w_M = F_L(z_M, 1)$.

From equations (8),(9) and (11), the other marginal conditions of the household problem can be rewritten as

$$\frac{v' f_d^o}{u' f_c^y} = \beta(1 - \tau^x) \equiv P^R \tag{16}$$

$$\frac{f_{T^y}^y}{f_c^y} = (1 - \tau^w) w_M \equiv P^y \tag{17}$$

$$\frac{f_{T^o}^o}{f_d^o} = \mu \frac{(1 - \tau^w) w_M}{1 - \tau^x} \equiv P^o \tag{18}$$

where P^R is the relative price of market good consumed when old d in terms of market good

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consumed when young c, P^y (resp. P^o) is the relative price of time devoted to home production in terms of market good when young (resp. when old).

Time constraint when young (1) gives the household's labor supply

$$\ell = 1 - T^y + \mu (1 - T^o). \tag{19}$$

Then, the resource constraint (14) can be rewritten as

$$c + d = C_M \left[1 - T^y + \mu (1 - T^o) \right] \tag{20}$$

where C_M denotes aggregate consumption per labor unit

$$C_M \equiv (1 - \Gamma) F (z_M, 1) - z_M \tag{21}$$

Consequently, for given tax rates $\left(\tau^w, \tau^R, \tau^x \right)$, equations (16)-(18) and the resource constraint (20) characterize household's choice at steady-state equilibrium in terms of consumption in market goods, c and d, and time devoted to home production, T^y and T^o.

The household's intertemporal budget constraint (obtained by eliminating s_t from (6) and (7)) allows to compute steady-state bequest. Indeed, using the time constraint (19) and the relation between relative prices $P^R P^o = \beta \mu P^y$ given by (16)-(18), one gets

$$c + P^y T^y + P^R \left(d + \beta^{-1} P^o T^o \right) = P^y (1 + \mu) + (1 - \tau^x) (1 - \beta) x \tag{22}$$

Bequests are positive if the present value of market goods spendings $c + P^R d$ is higher than net wage income $(1 - \tau^w) w_M \ell$.

Finally, public debt is deduced from the budget constraint of the government

$$\Delta = \left(\left(1 - \tau^R \right) R_M - 1 \right)^{-1} \left(\tau^x x + \left[\tau^w w_M + \tau^R R_M z_M - \Gamma F (z_M, 1) \right] \ell \right) \tag{23}$$

For instance, if all tax rates are zero, steady-state public debt is negative, equal to $- (R_M - 1)^{-1} \Gamma F (z_M, 1) \ell$. This means that, at each period, the government uses interests on public capital to finance public spendings $\Gamma F (z_M, 1) \ell$. Of course, this is possible either if there is initial public capital that finances the whole sequence of public spendings, or if the government has taxed households in order to accumulate some public capital amount to this end.

As stated before, the case where saving and labor incomes are taxed at the same rate $\tau^R = \tau^w = \Gamma$ allows to eliminate the externality created by public spendings. Since F is linear homogenous, these taxes would then finance the whole current public spendings. Additionally, if the initial debt $\bar{\Delta}_{-1} = 0$, a zero inheritance tax would allow to reach a first-best optimum.

In the following, we consider an initial public debt $\bar{\Delta}_{-1} > 0$. The debt burden is allocated among

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generations by an additional taxation on savings incomes, that is the tax rate on saving income exceeds Γ. Thus, government finances $\bar{\Delta}_{-1}$ using $\bar{\tau}^R > \Gamma$ over time with $\bar{\tau}^w = \Gamma$ and $\bar{\tau}^x = 0$. This higher saving-income tax rate creates distortion in household's decisions, leading to a lower capital-labor ratio than the one obtained without initial debt at a first-best optimum.

4   Fiscal reform

The issue we address is whether a tax shift from savings income tax towards inheritance tax would be welfare enhancing. Therefore, the tax reform consists to set up a positive inheritance tax rate $\tau^x > 0$ and reduces the saving-income tax τ^R in order to make it closer to Γ.

In overlapping-generation models with rational altruism, saving income is divided between second-period consumption and inheritance (see equation (4)). This implies that inheritance is lower than saving income. Therefore, if the government tries to keep the primary surplus (fiscal receipts minus public spendings) constant, it needs to put inheritance tax rate larger than the fall in the saving-income tax rate. This means that the product $(1 - \tau^x)(1 - \tau^R)$ decreases and becomes lower than $(1 - \bar{\tau}^R)$, leading to a fall in the capital-labor ratio (see equation (15)) and the real wage rate. This reduces the resource available for market-good consumption.

In the following, we conduct the analysis by first assuming that the fiscal reform is designed in order to keep the capital-labor ratio constant. Therefore, the shift from saving income to inheritance tax is such that

$$(1 - \tau^x)(1 - \tau^R) = 1 - \bar{\tau}^R$$

Since this implies proportional changes in both tax rates, the fiscal reform decreases the steady-state primary surplus. Therefore, the steady-state public debt is lower (see equation (23)). The tax reform shift the burden of the initial debt towards the first-generations. Thus, it introduces an intergenerational redistribution of resources from the first generations towards the ones far in the future.

At this stage we will focus on the effect of the reform on steady-state life-cycle utility:

$$V = u\left(f^y\left(c, T^y\right)\right) + v\left(f^o\left(d, T^o\right)\right) \tag{24}$$

and postpone the issue of intergenerational redistribution to the next section, through a numerical illustration.

The rest of the section decomposes the marginal effect of the tax reform on steady-state household's life cycle utility in different settings. We start from the Barro model, assuming inelastic labor supply and no time transfer. We then extend the discussion to elastic labor supply, still without time transfer. Finally, we consider the complete framework with elastic labor supply and where both transfers are positive.

4.1 Tax reform without time transfer at the steady state

To decompose the different effects of a tax reform, we first analyze a shift from saving income taxation toward inheritance taxation (leaving constant the capital-labor ratio) in an economy where time transfers are inoperative. We thus leave aside the fact that inheritance taxation modifies the trade-off between both parental transfers. At a steady-state equilibrium with positive bequests and zero time transfer, i.e. $x > 0$ and $T^o = 1$, the capital-labor ratio, the gross interest rate and the wage rate are at their modified Golden-rule levels. Market goods consumptions (c and d) and time spent to home production when young T^y are characterized by equations (16), (17) and (20) and may change with the tax reform implemented. Thus, the marginal changes in τ^x reduces the relative price P^R and then modifies the household's intertemporal allocation of resources between consumptions in market good when young and old. The magnitude of the effect crucially depends on the elasticity of substitution between the composite goods f^y and f^o. Let us denote by σ^u, the absolute value of this intertemporal elasticity of substitution. Then

$$\frac{df^y}{f^y} - \frac{df^o}{f^o} = \sigma^u \frac{d\left(\frac{f_c^y P^R}{f_d^o}\right)}{\frac{f_c^y P^R}{f_d^o}} = \sigma^u \left(\frac{df_c^y}{f_c^y} - \frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R}\right) \tag{25}$$

4.1.1 Tax reform in a standard Barro model

We first show that the tax reform in the standard Barro (1974) model with inelastic labor supply ($T^y = 1$) has a negative effect on household's welfare.

Proposition 1. *At a steady-state equilibrium with no time transfer and inelastic labor supply, consider a switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first-period consumption in the market good c decreases, while the second-period consumption d increases. Moreover, steady-state life-cycle utility (24) decreases.*

Proof. Differentiating steady-state life-cycle utility $V = u\left(f^y\left(c, 1\right)\right) + v\left(f^o\left(d, 1\right)\right)$, and using marginal condition (16), dV has the same sign as

$$dc + P^R dd$$

Moreover, differentiating the resource constraint (20), one gets

$$c\frac{dc}{c} + d\frac{dd}{d} = 0 \tag{26}$$

Thus dV has the same sign as

$$\left(P^R - 1\right)\frac{dd}{d}$$

We now need to state the sign of dd. Let us define the shares of market good cost in the total cost of production of the composite good for the young $\alpha^y \equiv f_c^y c / f^y$ and the old $\alpha^o \equiv f_d^o d / f^o$.

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Equation (25) then rewrites as

$$\alpha^y \frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(\frac{f_{cc}^y(c,1) \, c}{f_c^y(c,1)} \frac{dc}{c} - \frac{f_{dd}^o(d,1) \, d}{f_d^o(d,1)} \frac{dd}{d} + \frac{dP^R}{P^R} \right)$$

since

$$\frac{df_c^y}{f_c^y} = \frac{f_{cc}^y(c,1) \, c}{f_c^y(c,1)} \frac{dc}{c} \text{ and } \frac{df_d^o}{f_d^o} = \frac{f_{dd}^o(d,1) \, d}{f_d^o(d,1)} \frac{dd}{d}$$

and

$$\frac{df^y}{f^y} = \alpha^y \frac{dc}{c} \text{ and } \frac{df^o}{f^o} = \alpha^o \frac{dd}{d}$$

Then, using (26), one easily checks that dd has an opposite sign to dP^R. Since the tax reform considered implies a fall in $P^R = \beta(1 - \tau^x)$, one gets $dV < 0$, which concludes the proof. ■

The fall in the relative price between both intertemporal market goods consumptions P^R increases the market good consumed when old d and pushes down the market good consumed when young c. These opposite effects are stronger when the substitutability between composite goods consumed in both periods is important (i.e. high σ^u). In addition, from (26), the marginal rate of transformation between d and c ($MRT_{d/c}$) is equal to one. As the marginal rate of substitution between d and c ($MRS_{d/c} \equiv P^R$) is lower than the $MRT_{d/c}$ and declines with the tax reform, household's welfare is negatively affected by the reform.

4.1.2 Tax reform with elastic labor supply

Extending the model to elastic labor supply when young ($T^y \leq 1$) modifies the effect of the tax reform, introducing labor supply effects. From equation (17), since home production functions are linear homogeneous, one deduces that the ratio c/T^y can be written as a function of P^y: $c/T^y = \phi^y(P^y)$, where ϕ^y is increasing. Since the tax reform does not modify the relative price P^y, market good consumption c varies in the same proportion as time devoted to home production T^y. Then any reallocation of resources from c to d will be associated with a reduction in T^y by the same percentage as the reduction in c. One gets the following result.

Proposition 2. *At a steady-state equilibrium with no time transfer, let us consider a switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first-period consumption in the market good c and time spent in home production T^y decrease, while the second-period consumption d increases. Moreover, steady-state utility increases iff*

$$C_M - \frac{P^y}{P^R} > \phi^y(P^y) \left(\frac{1}{P^R} - 1 \right). \tag{27}$$

Proof. Since the home production function when young f^y is linear homogenous and $dP^y = 0$, we deduce from (17) that

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

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Then, equation (25) rewrites as

$$\frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(-\frac{f_{dd}^o(d,1)\,d}{f_d^o(d,1)} \frac{dd}{d} + \frac{dP^R}{P^R} \right)$$

Differentiating the resource constraint (20), one gets

$$(c + C_M T^y) \frac{dc}{c} = -d \frac{dd}{d} \qquad (28)$$

Thus, straightforward computations lead to

$$\left[\frac{-f_{dd}^o(d,1)\,d}{f_d^o(d,1)} \sigma^u + \frac{d}{c + C_M T^y} + \alpha^o \right] \frac{dd}{d} = -\sigma^u \frac{dP^R}{P^R}$$

which shows that the sign of dd is opposite to dP^R, while dc and dT^y have the same sign as dP^R. Moreover, the sign of dV is the same as

$$dc + P^y dT^y + P^R dd = (c + P^y T^y) \frac{dc}{c} + P^R d \frac{dd}{d}$$

Using (28), $dV > 0$ is equivalent to condition (27), since the tax reform considered implies a fall in $P^R = \beta (1 - \tau^x)$. ∎

To interpret results in Proposition 2, recall that the tax reform consists in a fall in second-period consumption price P^R that increases d and reduces c and T^y. The fall in T^y improves total resources for market good consumption $C_M (1 - T^y)$ through the increase of the labor supply. The positive effect of the tax reform on labor supply attenuates or reverses the Barro effect on utility stressed in Proposition 1. Notice that the increase in labor supply should be stronger when the substitutability between both periods is important (i.e. high σ^u).

Since the capital-labor ratio is kept constant, the increase in labor supply is associated with an increase in the capital stock, and thus in savings. Households when young work more, consume less and then save more for their second period of life.

The consumption per additional labor unit C_M corresponds to the marginal rate of transformation between T^y and d, while P^y / P^R corresponds to the marginal rate of substitution between both variables. Then, $C_M > P^y / P^R$ means that the fall in T^y allows to produce more market goods for second-period consumption than the amount necessary to preserve the same welfare.

The condition $C_M > P^y / P^R$ is sufficient to guarentee welfare improvement if $P^R > 1$. But, with the initial values of the instruments that we consider ($\tau^w = \Gamma$, $\tau^x = 0$ and $\bar{\tau}^R > \Gamma$), the relative price P^R is equal to β, and is lower than 1. In this case, the condition $C_M > P^y / P^R$ is no longer sufficient: welfare increases if the ratio ϕ^y is small enough. Indeed, a low ϕ^y corresponds to a situation where the first-period market good consumption c is relatively small to T^y. Thus, the proportional reduction of c and T^y leads to small reduction in c (small negative effect on welfare)

and a sharp increase in labor supply.

In a country where people consume a large (resp. small) amount of market goods, the ratio ϕ^y would be high (resp. low) and then the tax reform would be detrimental for welfare (resp. welfare enhancing). The situation where consumption relies essentially on market goods can be associated with a developped country. By contrast, in a developing country, time devoted to home production becomes more important and consumption in market goods lower, leading to a small ratio ϕ^y. Following this interpretation, under the condition $C_M > P^y/P^R$, the tax reform is likely to be welfare enhancing in developing rather than developped countries.

4.2 Tax reform when both transfers are positive

Let us now introduce time transfers by considering the tax reform at steady state where both private transfers are positive: $x > 0$ and $T^o < 1$. Compared with the preceding section without time transfers, the marginal shift from saving income tax towards inheritance tax also modifies the parent's trade-off between bequests and time transfers. As we shall see, this adds new positive or negative effects on the young's labor supply.

The steady state is characterized by equations (16)-(18) and (20). In these equations, the tax reform not only decreases the relative price P^R between both market good consumptions, but also increases P^o, the relative price between market good and time used in home production when old. In the following, consequences of the fall in P^R will be named *interperiod* effects, while those resulting from higher P^o will be named *intraperiod* effects.

We first detail the interperiod effects. The fall in P^R has similar consequences on labor supply than those stressed in the preceding subsection (4.1.2), but also introduces an additional effect through changes in the time transfer. Indeed, lower P^R involves a negative effect on c and T^y and a positive effect on d and T^o. The elasticity of substitution σ^u between both composite goods may amplify these effects. The resulting impact on the young's labor supply is ambiguous: the negative effect on T^y affects positively the labor supply whereas the positive effect on T^o leads to a negative impact on time transfers, hence on the young's labor supply.

We now turn to the intraperiod effects, that come from the increase in P^o. The equality between marginal rate of substitution and relative price, $MRS_{T^o/d} = P^o$, implies that the marginal rate of substitution between d and T^o increases with the tax reform. This has a positive impact on d and a negative effect on T^o. The negative effect on T^o affects positively the labor supply. The magnitude of the intraperiod effect on T^o depends crucially on the elasticity of substitution between T^o and d. Let us denote by σ^o, the absolute value of this elasticity of substitution associated with home production technology f^o. By definition:

$$\frac{dd}{d} - \frac{dT^o}{T^o} = \sigma^o \frac{dP^o}{P^o} = -\sigma^o \frac{dP^R}{P^R} \tag{29}$$

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The following lemma signs the marginal effect on the second-period consumption d.

Lemma 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. Then, marginal effect on second-period consumption d is positive and given by*

$$\frac{dd}{d} = -\left[\left(1 - \frac{d}{(1+\mu)\,C_M}\right)\sigma^o + \frac{c + C_M T^y}{(1+\mu)\,C_M}\alpha^o\left(\sigma^u - \sigma^o\right)\right]\frac{dP^R}{P^R} > 0 \qquad (30)$$

where $\alpha^o \equiv f_d^o d/f^o$.

Proof. As the home production function when old f^o is linear homogenous,

$$\frac{df^o}{f^o} = \alpha^o \frac{dd}{d} + (1 - \alpha^o)\frac{dT^o}{T^o} = \frac{dd}{d} + (1 - \alpha^o)\sigma^o\frac{dP^R}{P^R}$$

where the second equality is obtained with (29). Since $dP^y = 0$ and the home production function when young f^y is linear homogenous, one deduces

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

Then, equation (25) rewrites as

$$\frac{dc}{c} - \frac{dd}{d} - (1 - \alpha^o)\sigma^o\frac{dP^R}{P^R} = \sigma^u\left(-\frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R}\right)$$

Linear homogeneity of f^o implies $T^o f_{dT^o}^o(d, T^o) = -df_{dd}^o(d, T^o)$ and $\frac{-f_{dd}^o d}{f_d^o}\sigma^o = 1 - \alpha^o$. Then, one gets

$$\frac{df_d^o}{f_d^o} = \frac{f_{dd}^o dd + f_{dT^o}^o dT^o}{f_d^o} = \frac{-f_{dd}^o d}{f_d^o}\sigma^o\frac{dP^R}{P^R} = (1 - \alpha^o)\frac{dP^R}{P^R}$$

Consequently, the preceding relation between $\frac{dc}{c}$ and $\frac{dd}{d}$ becomes

$$\frac{dc}{c} - \frac{dd}{d} = [\sigma^u\alpha^o + (1 - \alpha^o)\sigma^o]\frac{dP^R}{P^R} \qquad (31)$$

Differentiation of the resource constraint (20) yields

$$c\frac{dc}{c} + d\frac{dd}{d} + C_M\left(T^y\frac{dT^y}{T^y} + \mu T^o\frac{dT^o}{T^o}\right) = 0$$

and, combining with (31), allows to compute dd/d:

$$\frac{dd}{d} = -\frac{(c + C_M T^y)[\sigma^u\alpha^o + (1 - \alpha^o)\sigma^o] + \mu C_M T^o\sigma^o}{c + d + C_M(T^y + \mu T^o)}\frac{dP^R}{P^R} > 0$$

which is equivalent to (30).■

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Lemma 1 shows that tax reform results in an increase in d whatever the initial values of the instruments, as soon as they allow for positive bequests and positive time transfers. We now turn to the variations of c, T^y and T^o that depend crucially on both elasticities of substitution σ^u and σ^o, that respectively drives up the size of the interperiod and intraperiod effects.

Lemma 2. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. Then, one gets the following sufficient conditions:*

(i) *If $\sigma^o \geq \sigma^u$, the marginal effect on time devoted to home production when old T^o is negative.*

(ii) *If $\sigma^u \geq \sigma^o$, the marginal effects on first-period consumption in market good c and time devoted to home production T^y are negative.*

(iii) *If σ^o/σ^u is close to zero, c and T^y decrease, while T^o increases.*

(iv) *If σ^o/σ^u is close to unity, then c, T^y and T^o decrease.*

(v) *If σ^o/σ^u tends to infinity, c and T^y increase, while T^o decreases.*

Proof. Marginal effects on c, T^y and T^o can be computed from expressions (29), (30) and (31):

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \sigma^o \frac{d + \mu C_M T^o}{(1+\mu) C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) + \frac{d}{d + \mu C_M T^o} \right] \frac{dP^R}{P^R}$$

$$\frac{dT^o}{T^o} = - \sigma^o \frac{c + C_M T^y}{(1+\mu) C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) - \frac{d}{c + C_M T^y} \right] \frac{dP^R}{P^R}$$

This proves results (i)-(iv). Let us show result (v). Assuming that σ^o/σ^u tends to infinity, one gets that dc and dT^y are positive iff

$$\alpha^o > \frac{d}{d + \mu C_M T^o}$$

which is equivalent to $\mu C_M > P^o$, since $\alpha^o = d/(d + P^o T^o)$. The proof is complete.∎

Notice that the assumption $\mu C_M > P^o$ is satisfied at the initial steady state, that is, with $\tau^x = 0$, $\tau^w = \Gamma$ and $\bar{\tau}^R > \Gamma$. Indeed, since $P^o = \beta \mu P^y/P^R$, straightforward calculations using linear homogeneity of the technology F show that the inequality $\mu C_M > P^o$ is always true.[5] At equilibrium, the relative price P^o is equal to the marginal rate of substitution between T^o and d ($MRS_{T^o/d}$). Moreover, from the resource contraint, the marginal rate of transformation between T^o and d is: $MRT_{T^o/d} = \mu C_M$. Thus, the assumption $\mu C_M > P^o$ means that the MRT between

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[5]With $\tau^x = 0$, inequality $\mu C_M > P^o$ is equivalent to $C_M > P^y$. Using the linear homogeneity of F, one gets

$$C_M = (1 - \Gamma) F_L + [(1 - \Gamma) F_K - 1] z_M > P^y$$

where the last inequality is obtained using $\tau^w = \Gamma$ and $(1 - \Gamma) F_K > \left(1 - \bar{\tau}^R\right) F_K = 1/\beta > 1$.

T^o and d is higher than the MRS, that is, for given (c, T^y), any fall in T^o increases labor supply, and then leaves enough additional resources for second-period consumption, to increase utility.

From the proof of the preceding Lemma, one may notice that increases in both consumptions c and d, and both home production times T^y and T^o cannot arise simultaneously, since $dc > 0$ requires $\sigma^u < \sigma^o$, which implies $dT^o < 0$. Therefore, only three cases can arise:

- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o > 0$. This case happens when σ^o/σ^u is close to zero. Intergenerational time transfers have been reduced by the increase in the inheritance tax.

- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u is close to one, as with logarithmic utility.[6] It induces a rise in intergenerational time transfers.

- $dc > 0$, $dT^y > 0$, $dd > 0$ and $dT^o < 0$. This case happens when σ^o/σ^u tends to infinity. Intergenerational time transfers increase with the inheritance tax.

We now analyze the marginal effect of the tax reform on the household life-cycle utility in each of these three cases. In the following Proposition, we establish the condition for the tax reform to be welfare improving.

Proposition 3. *At the steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. The marginal effect on utility dV has the same sign as*

$$\left[P^R - \Theta\right] d - \alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) \left[(c + P^y T^y) - \Theta \left(c + C_M T^y\right)\right] \tag{32}$$

where

$$\Theta \equiv \frac{c + P^y T^y + P^R d + \beta \mu P^y T^o}{(1 + \mu) C_M} \tag{33}$$

Proof. Using the marginal conditions of the household problem (16)-(18), dV has the same sign as

$$dc + P^y dT^y + P^R dd + \beta \mu P^y dT^o \tag{34}$$

Since $dP^y = 0$, relative changes dc/c and dT^y/T^y are equal. Consequently, replacing (29) and (31) in (34) and using (30) in Lemma 1, one obtains that dV has the same sign as

$$-\sigma^o \left[P^R - \Theta\right] d \frac{dP^R}{P^R} + \alpha^o \left(\sigma^u - \sigma^o\right) \left[(c + P^y T^y) - \Theta \left(c + C_M T^y\right)\right] \frac{dP^R}{P^R}$$

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[6]This is the case, for instance, if the life-cycle utility function is:

$$\alpha^y \ln c + (1 - \alpha^y) \ln (1 - T^y) + \gamma \left[\alpha^o \ln d + (1 - \alpha^o) \ln (1 - T^o)\right]$$

where a^y, α^o and γ are positive parameters, $\alpha^y < 1$ and $\alpha^o < 1$.

which concludes the proof.■

To interpret condition (32), we distinguish the above three cases according as the value of the elasticity ratio σ^o/σ^u.

4.2.1 Tax reform with $\sigma^u = \sigma^o$

In this situation, that encompasses the case of a logarithmic utility function, the second-period consumption d increases thanks to lower c, T^y and T^o. From expression (32), welfare increases if only if $P^R > \Theta$, which can be rewritten as:

$$dV > 0 \Leftrightarrow C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y + (\mu C_M - P^o)\frac{T^o}{T^y} > 0 \tag{35}$$

In the latter inequality, we observe the same term as in (27): $C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y$. The tax reform increases welfare in the model with elastic labor supply and no time transfer iff this term is positive. This leads to the same kind of interpretation: the fall in the second-period consumption price P^R reduces c and T^y and increases d. Then, the reduction in T^y increases the young's labor supply involving a positive effect on resources in market goods.

Moreover, the positive effect on labor supply is reinforced by the increase in time transfers since T^o decreases with the reform. This positive effect on welfare appears in the second-term in (35). As stated before, the substitution from T^o to d is welfare enhancing since the initial equilibrium satisfies $\mu C_M > P^o$, that is, $MRT_{T^o/d} > MRS_{T^o/d}$.

Therefore, taking the Barro model with elastic labor supply as a benchmark, introducing intergenerational time transfers creates an additional positive effect on steady-state welfare. Moreover, as soon as condition (27) is satisfied, the tax reform improves steady-state welfare. The falls in T^y and T^o involve a rise in labor supply. Simultaneously, reducing c and increasing d imply higher savings, and lead to higher capital stock. All these additional inputs allow to produce more market goods, that will be consumed in second-period of life.

4.2.2 Tax reform with $\sigma^u \gg \sigma^o$

In this case, interperiod effects (from the decrease in P^R) dominate intraperiod effects (from the increase in P^o). This arises with a high elasticity of substitution between both composite goods σ^u, or with a low elasticity of substitution σ^o.

A high σ^u involves a significant shift in resources from the first to the second period of life. Thus, the market good consumption d and the time devoted to home production when old T^o strongly increase thanks to lower c and T^y.

For a low elasticity of substitution σ^o, the tax reform has a negative effect on time transfers. Indeed,

the increase in d associated with strong complementarity between d and T^o results in an increase in T^o as $\phi^o = d/T^o$ remains constant.

In both cases, the effect on labor supply is ambiguous as the labor supply is positively affected by the reduction in T^y and negatively by the increase in T^o.

The marginal effect on household life cycle utility may be worse off than with a logarithmic utility as the effect on labor supply is attenuated or reversed. From expression (32), the welfare is improved iff:

$$dV > 0 \Leftrightarrow - \left[(c + P^y T^y) - \Theta \left(c + C_M T^y \right) \right] > 0$$

Using expression (33), one gets

$$C_M - \frac{P^y}{P^R} > \left(\frac{\phi^o + \mu C_M}{\phi^o + P^o} \frac{1}{P^R} - 1 \right) \phi^y \tag{36}$$

With the initial values of the instruments: $\mu C_M > P^o$ and $P^R < 1$. Therefore, the difference $C_M - \frac{P^y}{P^R}$ has to be positive for the tax reform to improve welfare. Comparing (27) and (36), the right-hand side in (36) is higher. Consequently, situations where the tax reform has a positive effect on welfare are less likely to happen with operative time transfers than in the Barro model with elastic labor supply. Increase in T^o reduces time transfer to the young and then affects negatively their labor supply.

The ratio ϕ^y has still to be low in order to get a positive effect of the tax reform. As in preceding sections, low ϕ^y means a sharp decrease in T^y, and thus a important increase in labor supply. With time transfers, the ratio ϕ^o has also an impact. Indeed, since the tax reform increases P^o, the ratio ϕ^o also increases. Thus, if ϕ^o is initially high, the rise in T^o will be small and has also a small negative effect on labor supply.

4.2.3 Tax reform with $\sigma^u << \sigma^o$

Here, intraperiods effects (through higher P^o) dominate interperiod effects (through lower P^R). This case arises if σ^o is high, or if σ^u is small.

On the one hand, for a high elasticity of substitution σ^o, increasing relative price P^o involves higher second-period consumption of market good d, lower time devoted to home production T^o, and so, higher time transfer to the young. The young enjoy more resources, and then consume more composite good, increasing both c and T^y.

On the other hand, a low elasticity of substitution between both periods σ^u means that both composite goods are complements. This involves a small effect of P^R and a small shift of resources from the first to the second period. But, higher P^o increases the ratio d/T^o, leading to a fall in T^o. The latter involves a positive effect on labor supply of the young.

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Corollary 1. *At the steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from saving income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. If the ratio σ^o/σ^u tends to infinity, the marginal effect of the tax reform on utility is positive.*

Proof. Putting σ^u/σ^o at zero in (32), one gets that dV is positive iff

$$\left[P^R - \Theta\right] d + \alpha^o \left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] > 0$$

Then, plugging Θ, from (33) into the preceding inequality yields

$$(1 + \mu) C_M > c + C_M T^y + d + P^o T^o$$

which is true if $\mu C_M > P^o$. ∎

In the case with σ^o/σ^u closed to unity, both T^y and T^o were reduced by the tax reform. With higher σ^o/σ^u, the negative effect of the tax reform on time devoted to home production by the old, T^o, is strengthened. This increases first-period resources, and allows a rise in time devoted to home production by the young T^y. This shows that the effect on welfare is likely to be positive if the increase in labor supply only comes from a rise in time transfers from the grandparents.

5 Numerical illustration when $\sigma^u << \sigma^o$

As stated before, our aim is to identify situations where a tax shift from saving income tax towards inheritance tax would be Pareto-improving. In this section, we use numerical examples to analyze the impact of the tax reform on welfare along the transitional dynamics. Welfare of any generation t corresponds to the infinite sum

$$W_t = \sum_{i=t}^{+\infty} \beta^{i-t} V_i$$

where V_i is life-cycle utility of generation $i \geq t$. Then a Pareto-improvement is achieved if the tax reform does not reduce W_t, for any generation $t \geq -1$, and increases W_t for at least one generation.

We start from the same values of the instruments as those considered in the steady-state analysis, for any $t \geq 0$: $\tau_t^R = \bar{\tau}^R > \Gamma$, $\tau_t^w = \Gamma$, $\tau_t^x = 0$. We first focus on the same kind of fiscal reform as in Section 4, that is a fiscal reform that keeps the capital-labor ratio constant in the long-run. However, this tax reform reduces fiscal receipts at steady state, thus shifting the burden of the initial public debt to the first generations. It involves some intergenerational redistribution towards generations living in the long run.

Secondly, in order to attenuate the intergenerational redistribution involved by the assumption of constant capital-labor ratio, we consider a tax reform example that allows for a lower capital-labor ratio in the long-run.

Furthermore, we concentrate on situations where the tax reform increases c, T^y and the labor supply through the positive effect on time transfer (i.e. $\sigma^u << \sigma^o$). In this case, the fiscal reform implemented in Section 4, involves an increase of the steady state households' life-cycle utility. This is likely the most favorable situation to improve the household welfare. For this purpose, we assume that d and T^o are substitutes and that the elasticity of substitution between both periods σ^u is low. Table 1 presents values of all parameters.

As noticed before, without assuming $z = z_M$, a shift from saving income tax towards inheritance reduces the capital-labor ratio and the real wage rate and thus the resource available for market good consumption. In order to attenuate this negative effect on the whole dynamics and in the long run, we consider that production factors are complements as well as c and T^y .

Table 1: Base-case parameter value

Parameter		Value
Government		
Initial public debt	$\bar{\Delta}_{-1}$	0.1
Fraction of production devoted to public sector	Γ	0.1
Production function		
Technological parameter	A	20
Share parameter of physical capital	a	0.4
Share parameter of labor supply	b	1
Elasticity of substitution between production factors	σ^F	0.5
Representative household		
Home production function when young f^y		
Share parameter of market good c	a^y	0.1
Elasticity of substitution between c and T^y	σ^y	0.5
Home production function when old f^o		
Share parameter of market good d	a^o	0.1
Elasticity of substitution between d and T^o	σ^o	10
Taste		
Degree of altruism	β	0.7
Efficiency of time transfer	μ	0.7
Elasticity of substitution between f^y and f^o	σ^u	0.2
Time preference	γ	0.5

5.1 Tax reform with constant steady state capital-labor ratio

This numerical example is based on the tax reform analyzed in Section 4. Thus, a shift from saving income tax towards inheritance tax is implemented, leaving the steady state capital-labor ratio constant, such as $\tilde{\tau}_t^R = \frac{\bar{\tau}^R - \tau^x}{1 - \tau^x}$ for any generation $t \geq 1$, with adjustment of τ_0^R in order to satisfy the intertemporal budget constraint.

Figure 1 describes the effect of the tax reform on households' welfare when $\sigma^u << \sigma^o$. In this case, the steady state household's life-cycle utility increases through the positive effect of the tax reform on the young's labor supply. The tax reform involves an increase of the first generation's saving income tax τ_0^R since the burden of the public debt is shifted towards them. This results in a negative impact on the life-cycle utility of the first generation (decreasing from -1.7103 to -1.7109) while the effect on life-cycle utility along the transitional dynamics is positive. The tax reform involves efficiency gains and intergenerational redistribution that leads to higher welfare for each generation thanks to the households' altruistic behavior (see Figure 1(b)).

Figure 1: Tax reform with constant steady state capital-labor ratio
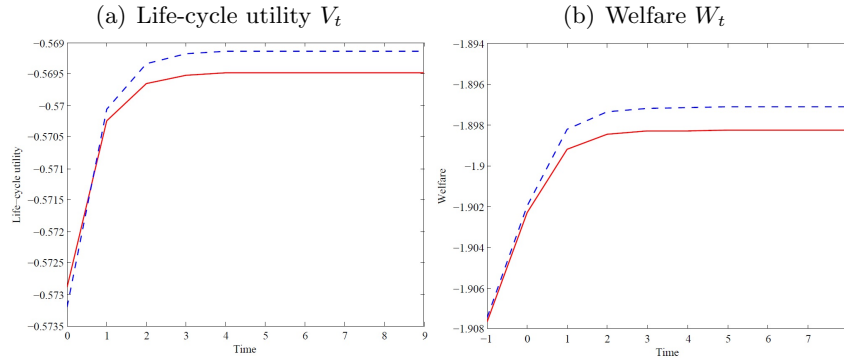
(a) Life-cycle utility V_t (b) Welfare W_t

Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. Setting up a positive inheritance tax of $\tau_t^x = 0.03$ for any $t \geq 0$, the tax reform involves $\tau_0^R \simeq 0.2895$ and $\tilde{\tau}_t^R \simeq 0.2603$ for any $t \geq 1$.

However, the adjustment of the first generation's saving income tax τ_0^R allows to shift part of the burden of the initial debt towards a lump-sum tax. This feature of the tax reform influences the result obtained. For this reason, we thereafter focus on another fiscal reform that keeps the lump-sum tax τ_0^R constant.

5.2 Tax reform with constant tax rate on saving income from period 1
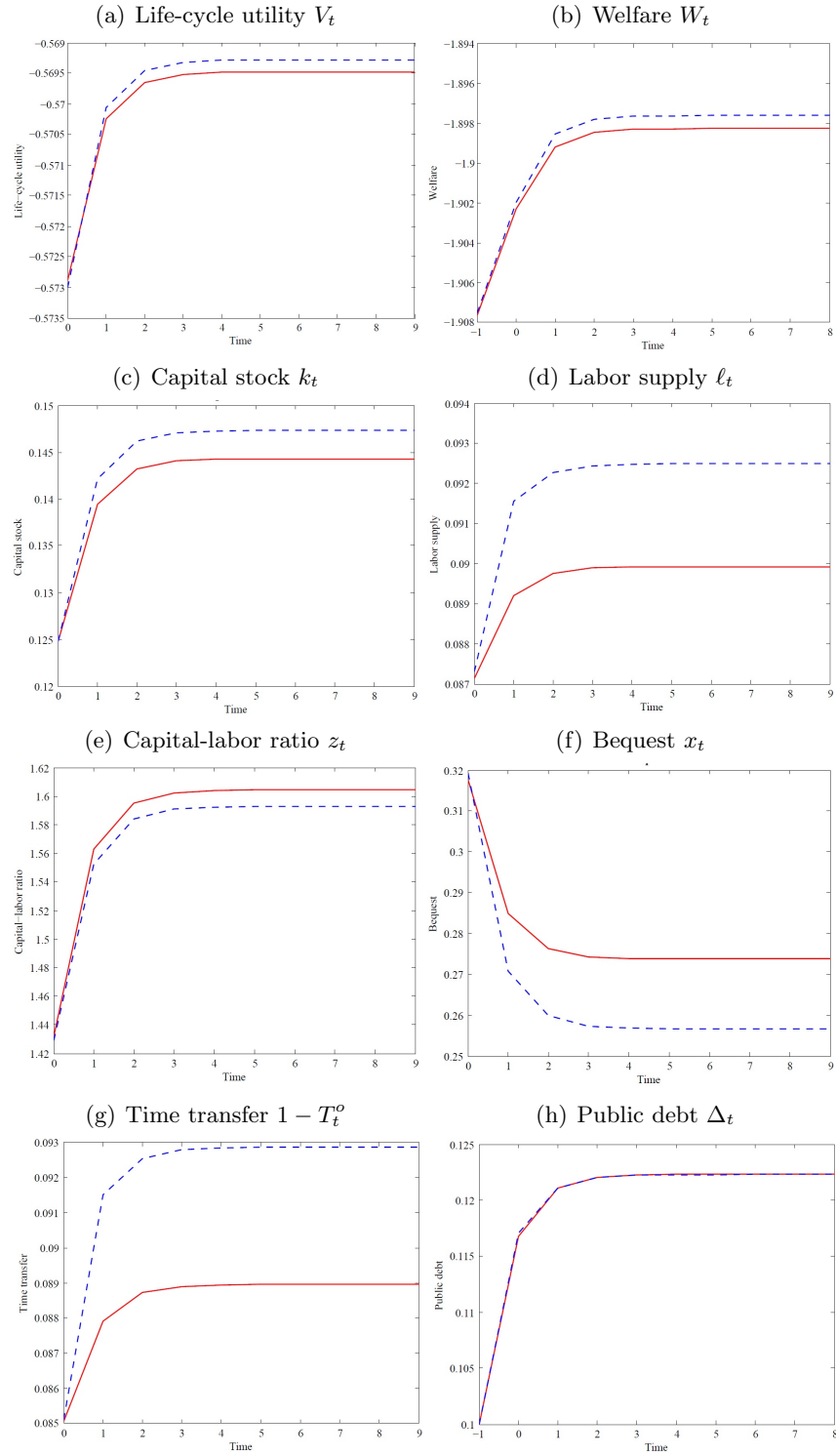
The tax reform now consists to set up a positive inheritance tax rate $\tau_t^x > 0$ and decrease the saving income tax $\tau_t^R = \hat{\tau}^R$ for any period $t \geq 1$ such as $\hat{\tau}^R$ balances the intertemporal budget constraint. The initial saving tax rate is kept constant: $\tau_0^R = \bar{\tau}^R$.

The results are reported in Figure 2. In this numerical example, we get similar effects on utility and welfare to the preceding illustration while the burden of the public debt is smoothed between generations starting from period 1. The tax reform implemented illustrates the trade-off for the government between keeping the steady-state primary surplus constant (constant steady-state public debt) and leaving the capital-labor ratio constant (lower steady-state public debt). The tax reform considered involves a decrease of the steady-state capital-labor ratio and an increase of the steady-state public debt compared to Subsection 5.1. Indeed, the increase of the capital stock (thanks to the fall in the relative price between both intertemporal market goods consumptions P^R) is low in relation to the significant rise of the labor supply after the reform. This positive effect on the labor supply relies on the rise of time transfers (through the increase of P^o). In addition, setting up positive inheritance tax rate reduces bequests (since households have more incentive transfer time).

Notice that the bequests decrease before and after the reform given that the labor supply increases in both case along the transitional dynamics which involves a reduction of the unfunded market goods component by labor supply.

As in the previous subsection, the households' life cycle utility is improved for each generation excepting the first old (which decreases from -1.7103 to -1.7113) and the household's welfare increases with the tax reform (even if the effect is lower than in the previous subsection).

Figure 2: The tax reform effect with constant tax rate on saving income

(a) Life-cycle utility V_t

(b) Welfare W_t

(c) Capital stock k_t

(d) Labor supply ℓ_t

(e) Capital-labor ratio z_t

(f) Bequest x_t

(g) Time transfer $1 - T_t^o$

(h) Public debt Δ_t

Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. Setting up a positive inheritance tax of $\tau_t^x = 0.03$ for any $t \geq 0$, the tax reform involves $\tau_0^R = \bar{\tau}^R$ and $\tilde{\tau}_t^R \simeq 0.2689$ for any $t \geq 1$.

6 Conclusion

Introducing time transfers in a model with rational altruism *à la* Barro, a shift from saving income tax towards inheritance tax can be Pareto-improving. The Pareto improvement strongly depends on the strength of the positive effect of time transfers on the young's labor supply and on the strength of the effect of higher labor supply on the production of market goods.

Our results have some implications for the debate on inheritance tax. Indeed, the optimal fiscal policy may differ from the standard Chamley result taking into account time transfers. Furthermore, some empirical studies suggest that the distributions of time transfers are less skewed than for inheritance. Hence, possible extension would focus on intragenerational heterogeneity, in particular assuming that households differ in altruism and do not transfer the same resources to their offspring.

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