Opportunity bias

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Abstract

This paper presents an alternative way of assessing equality of opportunity, called *opportunity bias*, and compares it with the standard approach. We consider that society is partitioned in a collection of types, consisting of population subgroups that gather people with similar circumstances. Opportunity bias refers to the differences between the outcomes distributions of the different types. We compare those distributions with a method that is valid for evaluation problems involving categorical variables (the *balanced worth*). An application to the analysis of equality of opportunity in educational achievements, according to PISA, is provided in order to illustrate this approach.

Key words: inequality of opportunity, opportunity bias, balanced worth, skills, PIAAC.

JEL classification numbers: D63, I24, B41

1 Introduction

1. 1. Equality of opportunity

The assessment of the degree of unfairness of an outcome distribution has evolved in two complementary directions since the early 70's of the XXth Century. On the one hand, analysing proper ways of measuring the observed inequality by means of sound indicators (the literature on inequality indices). On the other hand, discussing to what extent differences in the observed outcomes are unfair (the literature on "inequality of what"). In short, one strand cares for the measuring rod and the other for the measured variable.

Inequality indices can be regarded as dispersion measures that incorporate two particular value judgements: (a) Dispersion is bad, so that perfect equality is the optimum; and (b) The measure is more sensitive to changes in the lower than in the upper part of the distribution. Different sensitivities to those changes yield different inequality indices. We can say that there is already a well-established theory on the measurement of inequality (see for instance Chakravarty (2008), Villar (2017)).

Inequality indices measure observed outcomes independently of its origin. This can be sometimes too crude an approach as part of the observed outcome differences may be just the product of different people's choices and have nothing of unfair. Think for instance of the case of two people and suppose that one gets twice the income of the other because she works twice the number of hours. This outcome cannot be regarded as unfair, provided the two individuals have freely chosen the number of working hours.

Yet there are observed differences that cannot be considered as a mere product of "free choices". Take again the case of the distribution of labour income in a given society. The larger differences are mostly related to the workers' education levels. One can be tempted to conclude that those differences do not entail inequality but are rather the product of the differential retribution of skills (e.g. a surgeon gets better paid than a taxi driver because of the different investment in human capital involved). This would be fine as long as those people had the chance of choosing freely and knowingly the education obtained. Yet the data show that the family environment affects the likelihood of achieving high levels of education, which implies that part of the observed wage dispersion reflects differences in the family origin and are thus unfair.

The methodological approach to take into account the role of free choices and the role of circumstances (the "inequality of what" question) is referred to as *equality of opportunity*. This is nowadays one of the most relevant approaches to distributive justice, characterised by a wide spectrum of views with respect to what is required for equality of opportunity.[1] The bottom line behind the equality of opportunity principle is that external circumstances are to be taken into account when comparing outcomes, which is usually associated with the idea that people who are relatively disadvantaged deserve some kind of recognition or compensation. And, complementarily, that we should not be concerned for those

[1] The philosophical underpinnings of this approach are linked to the works of Rawls (1971), Dworkin (1981a, b), Sen (1985), Arnesson (1989), Cohen (1989), Van der gaer (1993), Roemer (1993). Among the many contributions developed by the economists, let us mention those of Fleurbaey (1995), Bossert (1995), Bossert & Fleurbaey (1996), Roemer (1996), (1998), Fleurbaey & Maniquet (2001), Peragine (2002), (2004), Ruiz Castillo (2003), Villar (2005), Moreno-Ternero & Roemer (2006), Bourguignon, Ferreira & Menéndez (2007), Fleurbaey (2008), Lefranc, Pistolesi & Trannoy (2008), (2009), Checchi & Peragine, (2010), Almas et al (2011), Calo-Blanco & García-Pérez, (2012), Fleurbaey & Peragine (2013). Romer & Trannoy (2013) provide a comprehensive survey of this literature.

outcome differences among people with the same circumstances, as long they derive from people's autonomous choices (e.g. differential effort).

Needless to say, the concepts of external circumstances and responsibility have fuzzy boundaries, vary from one to another problem, and open some difficult conceptual issues (e.g. to what extent are individuals responsible for their preferences, moral principles and believes? Is luck part of the circumstances? Is information gathering and processing type dependent?). In spite of those difficulties those concepts refer to relevant elements for the ethical evaluation of outcome distributions and have to be addressed one way or another.

1.2 The standard approach

One of the most successful approaches to inequality of opportunity among economists is that based on the work of Roemer (1998). According to his approach an outcome distribution can be regarded as the result of two different effects: *effort* and *opportunity*. Effort has to do with responsibility and involves people's autonomous choices on a common "playing field". Opportunity refers to the agents' external circumstances, which may include genes, race, gender, family socioeconomic and cultural background, and other aspects for which agents cannot be held responsible. A fair society should care for the agents' differences in opportunity but not for those differences derived from autonomous personal decisions.

Evaluating the degree of inequality of opportunity in a society from this perspective thus involves a double partition of their members. On the one hand, there are the ***types***, which gather agents who share the same circumstances. On the other hand, there are the ***effort groups***, which correspond to those population subgroups that exert a similar degree of effort. Within this framework, the outcome distribution of those agents of the same type can be regarded as determined by their effort decisions. In other words, agents of the same type have the same opportunity and all outcome differences within a type correspond to differences in people's effort decisions, which are ethically irrelevant. Since the relevant inequality refers to that between effort groups, we can measure inequality of opportunity by recurring to some inequality index applied to those effort groups (see Villar, 2005, 2017) for a model based on the decomposability properties of Theil's inequality indices).

There are different ways of operationalizing this approach, both in terms of measurement and in terms of policy implementation, and a large number of applications in different fields. Discussing those aspects is far beyond the scope of this work. The reader is referred to the thorough discussion in Roemer & Trannoy (2013), (2015) and the references provided there.

1.3 An alternative approach

We shall introduce here an alternative way of approaching equality of opportunity, under the heading of ***opportunity bias***. The idea is rather elementary, even if its implementation is less so. It boils down to estimating how different the outcome distributions of population subgroups that have diverse circumstances are. The differences between those distributions reflect the different opportunities faced by individuals depending on their non-chosen characteristics.

As in Roemer's framework, we consider that agents in society are divided into different *types*, depending on their circumstances. The outcome distribution of each type is interpreted as an estimate of its opportunity (i.e. the chances of success faced by a new agent entering this type). Consequently, there is equality of opportunity when the chances open to individuals do not depend on their external circumstances. That is to say, when all types exhibit similar outcome distributions.

We can think of the opportunity bias viewpoint as an application of a basic non-discrimination principle by which we try to ensure that any new member of society will have access to its average chances, no matter which social group (*type*) she ends up in. We can also see this principle as the application of a leximin social evaluation function on the space of opportunities. Needless to say, these principles are not new and have already appeared under different formats in the literature (Rawls (1971), Sen (1985) and particularly Roemer (1998) are obvious references). What is new is the way of transforming those ideas into a well-defined tool that measures equality of opportunity.

The rest of the paper is organised as follows. Section 2 describes the opportunity bias approach to equality of opportunity, assuming that the population is partitioned into different types according to people's differential circumstances. Then, the comparison of the types' outcome distributions, interpreted as an expression of their opportunities, is obtained by applying the methodology developed in Herrero & Villar (2013), (2017), which is summarised in this section. Section 3 applies this notion to the analysis of equality of opportunity in skills, focusing on the OECD countries, according to PIAAC 2012, 2015. The types are determined by gender (male and female), age (younger, middle and older), and the parents' education (primary, secondary and tertiary). The outcome variable corresponds to the average scores of the PIAAC tests, clustered into five levels of proficiency. A short discussion concludes.

2 The opportunity bias approach to equality of opportunity

This approach is based on a direct comparison of the outcome distributions between the different types. The observed distribution of a type is interpreted as a measure of its opportunity (the chances open to a newcomer to that type). There is no inequality of opportunity in society when the outcome distributions of all types are alike. We shall provide a measure of opportunity bias by recurring to the *balanced worth*, an evaluation method that provides a consistent cardinal way of comparing those distributions developed by Herrero & Villar (2013), (2017)).

Consider a society $M = \{1,2,\dots,n\}$ whose members' outcomes can be described in terms of L different ***levels of achievement***, ordered from best to worst. This society consists of τ ***types***, t = 1, 2, ..., τ, depending on their external circumstances. Each type, therefore, gathers individuals with similar circumstances. Our task consists of evaluating the outcome distributions of those τ different types, according to their levels of achievement. Let a_{tl}, t = 1, 2, …, τ, ℓ = 1, 2, …, L, denote the share of individuals of type t in level ℓ. The key principle for the comparison between types refers to the probability of getting better outcomes.

Let p_{tq} denote the probability that an individual of type t belongs to a higher level of outcome than an individual of type q. Such a probability can be calculated as follows:

$$p_{tq} = a_{t1}(a_{q2} + \cdots + a_{ql}) + a_{t2}(a_{q3} + \cdots + a_{ql}) + \cdots + a_{t(l-1)}a_{ql}$$

Similarly, we call $e_{tq} = e_{qt}$ as the probability that and individual of type t gets the same outcome level of an individual of type q. By definition, we have: $p_{tq} + p_{qt} + e_{tq} = 1$. Given the symmetry of the probability of a tie in binary comparisons, we shall split equally that probability between both types, so that we can rewrite the former equality as follows: $(p_{tq} + e_{tq}/2) + (p_{qt} + e_{qt}/2) = 1$.

We say that type t ***weakly dominates*** type q in a pairwise comparison when:

$$\frac{(p_{tq}+e_{tq}/2)}{(p_{qt}+e_{qt}/2} \geq 1 \qquad (1)$$

That is, when it is more likely that picking at random an individual from type t she belongs to a higher or equal level than that of another individual randomly chosen from type q, than the other way around. Note that equation (1) holds if and only if $p_{tq} \geq p_{qt}$. Yet this equation permits keeping track of the probability of ties, which are split evenly between the two types, and provides a cardinal measure of the size of the domination probabilities.

This is our basic comparison principle for two distributions: distribution t dominates distribution q when it is more likely that t produces better outcomes. How much better is measured by the ratio of the corresponding domination probabilities.

When there are more than two types involved this simple comparison is not enough, because of the indirect domination relationships that exist between these two types and the rest. Actually, the pairwise comparison given by domination probabilities is not transitive. That is, it may be that $p_{tq} \geq p_{qt}$ so that t is better than q, $p_{qs} \geq p_{sq}$ meaning that q is better than s, and yet $p_{st} \geq p_{ts}$ creating a cycle.

To get a consistent comparison in this general context we define the ***relative advantage of type t with respect to type q***, RA_{tq} as follows:

$$RA_{tq} = \frac{p_{tq}+(e_{tq}/2)}{\sum_{h\neq t}(p_{ht}+(e_{ht}/2)} \tag{2}$$

The relative advantage of type t with respect to type q is nothing else than the probability that an individual of type t belongs to a higher level of outcome than an individual of type q, divided by the sum of the probabilities that an individual of type t belongs to a lower level of outcome than an individual of some other type. Obviously (2) collapses to (1) when there are only two types. Note that $p_{tq} + (e_{tq}/2) = p_{qt} + (e_{qt}/2)$ does not ensure that $RA_{tq} = RA_{qt}$, because of the different domination probabilities of other types over t and q, respectively.

To obtain an overall evaluation of type t in society, we take a weighted average of its relative advantages with respect to all other types. That is, the ***relative advantage of type t*** is given by:

$$\boldsymbol{v_t} = \sum_{q\neq t} \lambda_q \, RA_{tq}$$

Here the weights λ_q reflect the relevance of the different types. How to choose those weights in a non-arbitrary way? The most natural option is to choose them by applying a consistency principle, i.e., by taking $\lambda_q = v_q$. In this way, each type enters the evaluation of the relative advantage of the others with the weight corresponding to its own relative advantage.[2] This implies that we have to find a vector $\boldsymbol{v}^* = (\boldsymbol{v}_1^*, \dots, \boldsymbol{v}_\tau^*) \in \boldsymbol{R}_+^\tau$ that solves the following simultaneous equation system:

$$\boldsymbol{v}_t^* = \sum_{q \neq t} \boldsymbol{v}_q^* \, RA_{tq} = \frac{\sum_{q \neq t}[p_{tq} + (e_{tq}/2)]}{\sum_{h \neq t}[p_{ht} + (e_{ht}/2)]}, \quad t = 1, \dots, \tau \qquad (3)$$

It can be shown (Herrero & Villar (2016)) that this vector $\boldsymbol{v}^*$ always exists, is strictly positive and unique (after normalisation, as by construction it has one degree of freedom).

Note that this evaluation protocol provides not only a consistent ranking (i.e. a transitive and complete criterion) but also a cardinal measure of the relative goodness of the distributions. It is easy to check that whenever the distributions of two different types, t, q, coincide, then $\boldsymbol{v}_t^* = \boldsymbol{v}_q^*$. Moreover, if $p_{tq} = 0$ for all $q \neq t$, then $\boldsymbol{v}_t^* = 0$ i.e. we give value zero to a type whose distribution is fully dominated. Finally, let us mention that this evaluation criterion satisfies *monotonicity* in the following sense: If the outcome distribution of a type improves whereas the rest remain the same, then the new evaluation will be higher for this type. As a consequence, this evaluation can be regarded as a transitive, complete and cardinal extension of the first order stochastic dominance criterion.

[2] Note the similarity with the notion of *page rank* that applies Google to order the web pages. See also a very close application of this principle to the evaluation of scientific influence in Palacios-Huerta & Volij (2004).

When we are evaluating a single society, the balanced worth vector of the corresponding types provides the required information on the opportunity bias. Note that this vector has a degree of freedom so that we can choose arbitrarily the units by means of a particular normalisation. Here we propose to normalise the values of the vector by taking the highest value equal to 1, so that all the remaining values are expressed as shares of the top value. This permits one to interpret the values of the balanced worth of the different types as the probabilities of reaching the best outcome distribution.

We might also be interested in getting an overall measure of inequality of opportunity based on the opportunity bias vector, especially if we are to compare different societies. We can think of this measure as an ***index of opportunity bias***. There is a number of ways of so doing depending on the aspects we are willing to emphasize (e.g. the difference or the ratio between the extreme values of the range, the coefficient of variation, or any conventional inequality measure). We shall take here the coefficient of variation as the relevant summary measure, because it is a familiar unit-free way of measuring the dispersion of a variable.

4 Opportunity bias in educational outcomes: the OECD according to PISA 2012

We shall now apply this way of measuring opportunity bias to educational outcomes, using PISA data corresponding to 2012.[3]

[3] Among the recent studies on equality of opportunity in education let us mention the works of Lefranc, Pistolesi & Trannoy (2009), Chechi & Peragine (2010), Ferreira & Gignoux (2011), Gamboa & Waltenberg (2012), Herrero, Méndez & Villar (2014), Carvalho, Gamboa & Waltenberg (2015), or Tansel (2015). Those works use average scores and different ways of defining social groups to analyse the dependence of the outcomes on those conditioning variables.

The OECD's *Programme for International Student Assessment* (PISA) provides the richest and most comprehensive database for the evaluation of the educational achievements of 15 year-old students in three different areas: mathematics, reading comprehension, and science. Sixty-five countries and large economies participated in the 2012 wave of PISA, focused on the field of mathematics, involving a sample of about half a million students (see OECD (2014a, b)).

Achievement in PISA is primarily measured in a 1000 points scale with a mean originally set a 500 and a standard deviation of 100. Besides, the Programme establishes six levels of proficiency that try to approximate the ability of the young to deal with different tasks. Each level corresponds to a different capacity or set of skills (see OECD 2014a, p. 61 for details). Even though the very notion of proficiency is intrinsically qualitative, those levels are parameterized in terms of intervals of the average scores of the tests that students realize in each subject. A simple inspection of the data regarding the distribution of the students among those levels of proficiency shows that there is a large diversity among countries, even between those with similar average scores. Those levels thus provide relevant information on the structural features of the different educational systems, which is not captured by the average scores of the test.

Table 1 identifies the cutting points that define those levels of proficiency (let us recall here that the average score of the OECD students is 494). It also contains information about the percentage of students in each level in the OECD countries.

Table 1: Levels of proficiency and thresholds of the test scores

Levels of proficiency	5 or above	4	3	2	1	< 1
Test scores for the levels	> 607	606 - 545	544 - 482	481 - 420	419 -358	< 358
OECD students per level	12,6%	18,2%	23,7%	22,5%	15%	8%

Another feature of the PISA that makes it an extraordinary database refers to the information collected on the students' family and school environment. In particular one can use data regarding socio-economic conditions to analyse the degree of equity of educational systems, in the understanding that more equity means less dependence of the results on the family environment. The OECD provides a summary variable that gathers the key information about the family environment: the index of Economic and Socio-Cultural Status (ESCS).[4]

The family background, as measured by this ESCS, is generally recognised as the most important external influence on the students' performance. In order to take into account the family environment, we divide the students of each country into four different *types*, according to the quartile distribution of the index of Economic and Socio-Cultural Status (ESCS). So we shall consider the population of each OECD country divided into four different *types*, which correspond to the quartiles of the ESCS *country-specific* distribution. Those types will be labelled Q1 (bottom 25%), Q2, Q3, and Q4 (top 25%).

We adopt a twofold approach to assessing the equity of educational systems.

(i) First, we evaluate within each country the relative performance of the

[4] The ESCS is a composite measure made of the following variables: the International Socio-Economic Index of Occupational Status (ISEI); the highest level of education of the student's parents, converted into years of schooling; the PISA index of family wealth; the PISA index of home educational resources; and the PISA index of possessions related to "classical" culture in the family home.

different social groups (the types) in terms of the distribution of the students by levels of proficiency. This evaluation provides, for each country, a vector of values that tells us how different are the outcomes by social groups. The coefficient of variation of those values can be regarded as a summary measure of the overall inequality of opportunity within countries.

(ii) Second, we compare the degree of fairness between the OECD countries in terms of those coefficients of variation of within-country opportunity bias, relative to the OECD value.[5]

Table 2 describes the distribution of the students by proficiency levels depending on their type, for the OECD as a whole. It also provides a measure of the opportunity bias so that we observe how far away are the students of types Q1, Q2, and Q3 from reaching the results of type Q4 students, in probability terms. The evaluation of each individual country is provided in Table 3 in the Appendix.

The data send a clear message: the family background is an important factor in the achievement of competences, especially for those at the top and the bottom ends of the distribution of the socioeconomic conditions. Students coming from families in the first quartile have about one third of the chances of getting the degree of success of those in the fourth quartile. Those data are slightly less than one half for students from families in the second quartile, and about two thirds for those in the third quartile. It is worth keeping in mind this rough division, 1/3, ½, 2/3, to help interpreting the data of individual countries (see Table 3 in the

[5] The OECD provides several measures of the degree of equity of educational systems, linking average scores with family characteristics. Yet the distribution of the students among the proficiency levels is not taken into account when analysing the equitability of educational systems.

Appendix). The last cell of the table gives us the coefficient of variation of the opportunity bias.

Table 2: Levels of proficiency in mathematics by quartiles of the ESCS in the OECD (PISA, 2012)

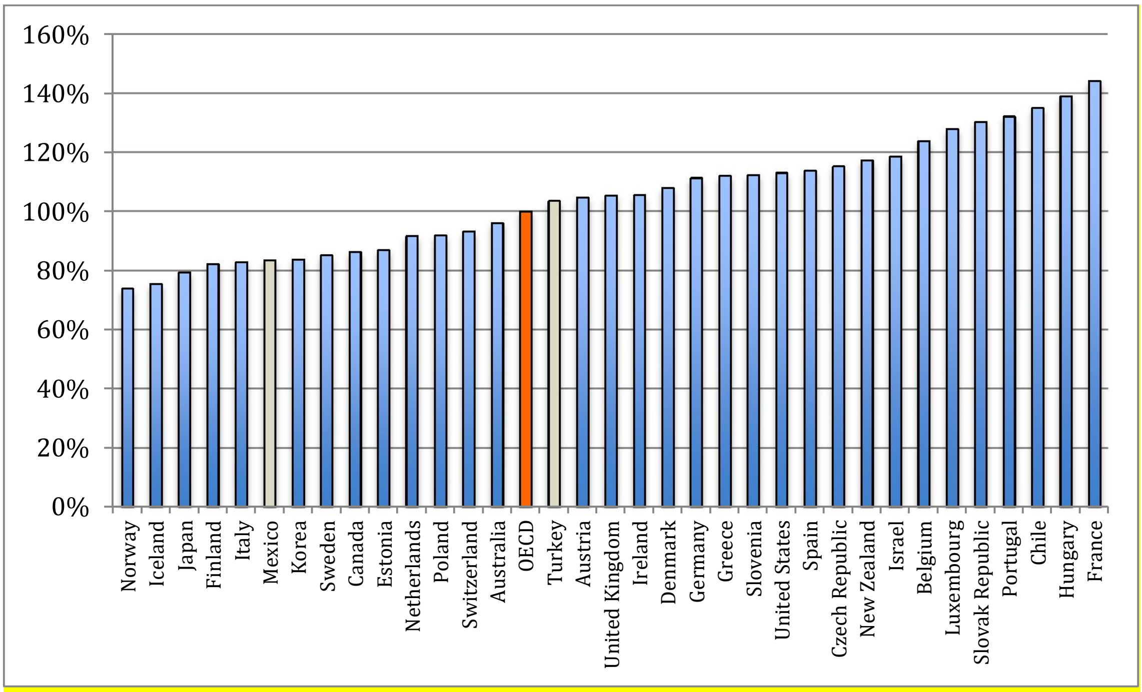
	Level 5*	Level 4	Level 3	Level 2	Level 1	Level < 1	Op. bias
Q1	4,63%	10,96%	20,62%	26,63%	22,46%	14,70%	0.331
Q2	8,43%	16,05%	24,76%	25,52%	17,10%	8,13%	0.475
Q3	13,85%	20,72%	25,77%	21,95%	12,49%	5,22%	0.644
Q4	24,68%	25,86%	24,41%	15,54%	6,95%	2,56%	1
Total	12,90%	18,40%	23,89%	22,41%	14,75%	7,66%	(0.408)

Regarding the comparison of the degree of equity between the OECD countries, we take the coefficients of variation of the countries' opportunity bias and make them relative to that of the OECD, to facilitate the comparison. This measure corresponds to the percentage of the OECD variability of opportunity bias by social groups.

There are two key messages that derive from this analysis. First, the degree of inequality of opportunity between social groups is extremely large (above 0.4 for the OECD as a whole). Second, there is also a large variability between OECD countries, ranging from 74% of the OECD value for the case Norway to 144% for the case of France. Figure 1 illustrates the variability of this equity measure (see also Table 4 in the Appendix, for details). We observe that Norway, Iceland, Japan, Finland, Italy, Korea and Sweden exhibit values that are more than 15 percentual points below that of the OECD. In the other extreme we find France, Hungary, Chile, Portugal, Slovak Republic, Luxembourg, Belgium, Israel and New Zealand, with values more than 15 percentual points above that of the OECD.[6]

It is worth noting that most of the countries with better values are also countries with better performance (higher mean scores in the tests). This illustrates, once more, that equity and efficiency are not alternative targets when dealing with education. The coefficient of correlation between the index of opportunity bias and the average scores is some -0.3.

Figure 1: Relative opportunity bias (% of the OECD) of the educational systems in the OECD countries (PISA 2012, mathematics).



6 There are some relevant differences among the OECD countries regarding the fraction of the 15-year old people that attend school (what is usually referred to as *access*), who are the only ones subject to evaluation. This is especially relevant in the case of Mexico and Turkey where more than one third of the 15-year old population has left the school (that's why they appear with a different colour in the figure). We shall not discuss here how to address this problem but simply alert on the non-comparability of the evaluation in those cases. For a discussion on how to combine data regarding access and achievement see Ferreira & Ginoux (2011), Ferreira, Ginoux & Aran (2011), Carvalho, Gamboa & Waltenberg (2012), Gamboa & Waltenberg (2015), and Tansel (2015).

As expected, all European Nordic countries (except Denmark) exhibit lower inequality of opportunity than the average. Somehow more surprisingly, all European Southern countries, with the notorious exception of Italy, are doing worse than the OECD average. This is true, in particular, in the cases of France, Portugal and Israel (but also for Greece and Spain where supposedly equity has been a major concern for their Governments).

Let us conclude this section by pointing out that the ranking of countries in Figure 1 does not differ much from that provided by the OCDE regarding the percentage of the variance explained by socioeconomic conditions (see in particular Figure II.2.2 in vol. II of OECD (2014)). Even though the Pearson coefficient is about 0.9, there are ten countries that move 4 or more positions in the ranking and only five countries whose ranking does not change (see Table 5 in the Appendix). The coefficient of correlation between the values of the explained variance and those of the opportunity bias index is some 0.95, which means that this rougher procedure captures practically all relevant information of the individual data (and also that the % of explained variance is a good measure of inequality of opportunity).

5 Discussion

We have presented here an alternative way of approaching equality of opportunity and provided an application to the case of educational outcomes in the OECD countries. We conclude this work by discussing the interest and applicability of this new approach.

Probably the basic question is the following: Do we need still another way of measuring equality of opportunity? Does the opportunity bias approach add something to this literature? One may reasonably think that the standard approach is always preferable to the one presented here because, among other things, it is an evaluation in which the size of the outcome differences is taken into account, whereas this is not the case in the opportunity bias approach. Indeed, the balanced worth only takes into account the distribution of the population of the different types, so that if we apply any monotone transformation to the outcome values, nothing changes. We shall see that this apparent weakness is, precisely, what makes it a relevant approach to equality of opportunity in some cases, because being a rougher measure is also a more robust one.

We consider here a series of scenarios in which the opportunity bias approach can be regarded as the right venue to analyse equality of opportunity.

Categorical variables

The first instance in which the measurement of equality of opportunity may be better served by this new approach is that in which the reference variables are categorical. In that case the standard approach is of no avail.

To illustrate this idea consider the following variant of the analysis presented above. Suppose we have to evaluate the equality of opportunity regarding university grades in a given state. Students' final grades are given in terms of a categorical scale consisting of five levels: A, B, C, D, and E, ordered from top to bottom. We want to analyse the relationship between students' achievements and their family origin. In a fair society, students' results should not depend on their family origin. So equality of opportunity here amounts to ensuring that the distribution of outcomes does not depend on the students' family characteristics.

Multilevel distributions

There are cases in which the reference variable is quantitative and yet we may consider that the range of variation of the outcome variable can be sensibly divided into a number of levels or categories, defined by a partition in terms of intervals, each of which may have a different meaning.

The main reason for that is that there are evaluation problems in which quantitative differences entail qualitative disparities. That is, different parts of the outcome distribution represent different aspects of that distribution from a descriptive or normative viewpoint. In the case of income distributions, for instance, values below 60% of the median identify the set of the poor, who are regarded as defining a particular category of agents. When measuring the scientific influence, publications in the top 1% or top 10% of the citation distribution represent contributions that are regarded as qualitatively different (*influential* contributions, we may say). The levels of proficiency in the PISA reports are actually conceived as qualitatively different, each one defining a particular set of competencies, even if they are operationalized by intervals of the test scores (see OECD (2014)).

Qualitative numbers

There are also instances in which the reference variable is quantitative but the numbers that measure outcomes are artefacts rather than genuine quantities. This happens when the outcome variable is essentially qualitative and the numbers are used to get summary measures, usually in the form of weighted sums. A case in point is that of the evaluation of self-perceived health states. People answer a questionnaire about their perceived health into five categories, from very good to very bad, say, and then those individual responses are aggregated for a given society attaching numbers from 5 to 1 to those categories. Something similar happens in the analysis of intellectual influence in terms of citation impact. It is common to find indicators that divide the range of citations into a set of categories, e.g. the percentiles of the citation distribution, and then using some average to evaluate institutions by weighting those categories attaching numbers 100, 99, …, and 1. Needless to say, those numbers are arbitrary and the classification we obtain is not robust to monotone transformations.

Another case in which numbers are not exactly what they appear to be is when there is no reason to assume that different agents have a common scale to evaluate performance and/or there are not good reasons to assume that the scale is linear. Think for instance the case in which one evaluates a new medical treatment against pain. Patients are asked to evaluate their situation before and after treatment in terms of a 0 – 10 pain scale. There is little ground to assume that all patients have the same scale or that this scale is linear for each single individual.

Since the balanced worth is invariant to any monotone transformation of the outcome variables, it provides an interesting evaluation method in all these cases.

The effort problem

The standard approach may be regarded as a richer model not only because it is sensitive to changes in the levels of the outcome variables but also because it introduces the idea of effort to explain and justify the outcome differences within types. Nothing of the sort appears in the opportunity bias approach.

Yet effort is a non-observable and type-dependent variable, which calls for the design of an index that permits comparing effort levels for agents of different types. The resulting measurement of inequality of opportunity is very sensitive to the way of defining the effort groups, always a challenging modelling choice.[7]

* * *

Let us conclude by stressing that the main message of this discussion is not that the opportunity bias is a *better* approach to measuring inequality of opportunity, but rather that it is one that can be more suitable in particular scenarios. Indeed, one can interpret our model as an extension of Roemer's one to the family of problems mentioned above (in particular to the case of categorical variables), by identifying our levels of performance with the effort levels. This is a possible interpretation even though, from a conceptual viewpoint, there is no effort in our model. The outcome distribution of a type is regarded as a sufficient

[7] To solve this problem Roemer (1998) assumes that effort is a single-valued variable that is positively correlated with outcome. So, even though the effort distribution is a characteristic of the type, we can take the quantiles of the effort distribution within types to compare the degrees of effort (i.e. two individuals of different types exert a comparable degree of effort if their outcomes belong to the same quantile of the outcome distribution of their corresponding types).

description of its chances, very much in the spirit of Sen's capabilities. Such distributions are partly related to the agents' effort decisions, but also to some other aspects that might be type-dependent, such as network structures (the extent and functionality of peer relationships), gathering and processing of information, or luck.

REFERENCES

Arnesson, R. (1989), Equality and equal opportunity for welfare, **Philosophical Studies**, 56 : 159–194.

Almas, I., Cappelen, A.,Lind, J.T., Sorensen, E. & Tungoden, B. (2011), Measuring unfair (in)equality, **Journal of Public Economics**, 95 : 488-499.

Bossert, W. (1995), Redistribution mechanisms based on individual characteristics, **Mathematical Social Sciences**, 29 : 1–17.

Bossert, W. & Fleuerbaey, M. (1996), Redistribution and compensation, **Social Choice and Welfare,** 13 : 343–355.

Bourguignon, F., Ferreira, F.H. & Menéndez, M. (2007), Inequality of opportunity in Brazil, **Review of Income and Wealth,** 53 : 585–618.

Calo-Blanco, A, & García-Pérez, J.I. (2012), On the welfare loss caused by inequality of opportunity, **The Journal of Economic Inequality**, DOI 10.1007/s10888-013-9247-6.

Carvalho, M., Gamboa, L.F. & Waltenberg, F.D. (2015), Equality of educational opportunity: Taking both achievement and access into account, **Ecineq working paper** 2012-277.

Checchi, D. & Peragine, V. (2010). Inequality of opportunity in Italy, **Journal of Economic Inequality**, 8 : 429-450.

Cohen, G.A. (1989), On the currency of egalitarian justice, **Ethics**, 99 : 906–944.

Dworkin, R. (1981a), What is equality? Part 1: Equality of welfare, **Philosophy and Public Affairs**, 10 : 185–246.

Dworkin, R. (1981b). What is equality? Part 2: Equality of resources, **Philosophy and Public Affairs**, 10 : 283–345.

Ferreira, F. and Gignoux, J. (2011), The measurement of educational inequality: Achievement and opportunity, **Ecineq working paper** nº 240.

Ferreira, F. H. G, Gignoux J. and Aran, M. (2011), Measuring inequality of opportunity with imperfect data: The case of Turkey, **Journal of Economic Inequality**, 9 : 651-680.

Fleuerbaey, M. (1995), Equality and responsibility, **European Economic Review**, 39 : 683–689.

Fleuerbaey, M. (2008), **Fairness, Responsibility, and Welfare**. Oxford: Oxford University Press.

Fleuerbaey, M. & Maniquet, F. (2001), Compensation and Responsibility. In K. Arrow, A. Sen & K. Suzumura (eds.), **Handbook of Social Choice and Welfare**, Vol.- 2, Amsterdam: North-Holland, 2001.

Fleurbaey, M. & Peragine, V. (2013), Ex ante and ex post equality of opportunity, **Economica**, 80 : 118-130.

Gamboa, L. F. and F.D. Waltenberg (2012), Inequality of opportunity in educational achievement in Latin America: Evidence from PISA 2006-2009, **Economics of Education Review**, 31: 694-708.

Herrero, C., Méndez, I. & Villar, A. (2014), Analysis of Group Performance with Categorical Data when Agents are Heterogeneous: The Evaluation of Scholastic Performance in the OECD through PISA, **Economics of Education Review**, 40 : 140-151.

Herrero, C. & Villar, A. (2013), On the Comparison of Group Performance with Categorical Data, **PLoS ONE**, Vol. 8, No. 12.

Herrero, C. & Villar, A. (2017), The Balanced Worth: a Procedure to evaluate performance in terms of ordered attributes, **Social Indicators Research.** DOI: 10.1007/s11205-017-1818-7

Lefranc, A., Pistolesi, N. & Trannoy, A. (2008), Inequality of opportunities vs. inequality of outcomes: Are western societies all alike?, **Review of Income and Wealth**, Vol. 54, pp. 513-546.

Moreno-Ternero, J. & Roemer, J. (2006), Impartiality, priority and solidarity in the theory of justice, **Econometrica,** Vol 74(5), pp. 1419-1427

Lefranc, A., Pistolesi, N. & Trannoy, A. (2009). "Equality of opportunity and luck: definitions and testable conditions, with an application to outcome in France", *Journal of Public Economics*, Vol. 93, Issue 11-12, pp. 1189-1207.

OECD (2014A), **PISA 2012 Results: What Students Know and Can do: Student Performance in Mathematics, Reading and Science** (vol. I). Paris: OECD Publishing.

OECD (2014b). **PISA 2012 Results: Excellence through Equity** (vol. II). París: OECD Publishing.

Palacios-Huerta, I. & Volij, O (2004), The Measurement of Intellectual Influence, **Econometrica**, 72 : 963-977.

Peragine, V. (2002), Opportunity egalitarianism and outcome inequality: the rank-dependent approach, **Mathematical Social Sciences**, 44, 45–64.

Peragine, V. (2004), Measuring and implementing equality of opportunity for outcome, **Social Choice and Welfare**, 22, 1–24.

Rawls, J. (1971), **A Theory of Justice,** Cambridge, Mass., Harvard U. Press

Roemer, J. (1996), **Theories of Distributive Justice**, Cambridge: Massachusetts Harvard University Press.

Roemer, J. (1998), **Equality of Opportunity**, Cambridge: Massachusetts Harvard University Press.

Ruiz-Castillo, J. (2003), The Measurement of Inequality of Opportunities, **Research in Economic Inequality**, 9 : 1-34.

Sen, A. (1985), **Commodities and Capabilities,** Amsterdam: North Holland.

Tansel, A. (2015), Inequality of opportunities in educational achievements in Turkey over time, **IZA working paper** DP 9005.

Van de gaer, D. (1993), **Equality of opportunity and investment in human capital**, PhD Dissertation, Catholic University of Leuven.

Villar, A. (2005), On the welfare evaluation of income and opportunity, **Contributions to Theoretical Economics** 5, 1, (2005b): article 3.

[http://www.bepress.com/bejte/contributions/vol5/iss1/art3]

Villar, A. (2017), **Lectures on Inequality, Poverty and Welfare**, Springer-Verlag, New York.

APPENDIX

Table 3: Distribution of population by quartiles of the ESC and levels of proficiency, and evaluation of opportunity bias in OECD countries.

		Level 5+6	Level 4	Level 3	Level 2	Level 1	Level < 1	Balanced worth
Australia	Q1	5,90%	12,10%	22,03%	27,04%	21,89%	11,04%	0,340
	Q2	10,15%	16,75%	26,85%	24,78%	15,50%	5,98%	0,488
	Q3	18,03%	22,23%	26,16%	20,57%	9,58%	3,43%	0,711
	Q4	26,59%	26,49%	24,46%	14,76%	5,85%	1,87%	1,000
Austria	Q1	4,60%	12,12%	20,83%	28,52%	22,29%	11,64%	0,297
	Q2	11,60%	17,99%	25,10%	24,54%	14,70%	6,07%	0,479
	Q3	15,03%	25,16%	26,21%	20,98%	9,55%	3,06%	0,652
	Q4	26,34%	29,22%	24,60%	13,37%	4,88%	1,59%	1,000
Belgium	Q1	6,02%	12,20%	22,34%	25,45%	20,06%	13,93%	0,248
	Q2	12,20%	19,11%	24,63%	22,75%	14,35%	6,97%	0,387
	Q3	23,21%	24,62%	24,83%	15,69%	8,30%	3,35%	0,621
	Q4	38,34%	27,73%	18,93%	9,52%	3,86%	1,62%	1,000
Canada	Q1	7,75%	16,91%	26,83%	26,80%	15,23%	6,48%	0,390
	Q2	12,97%	20,62%	27,26%	24,82%	11,31%	3,02%	0,521
	Q3	17,81%	24,88%	28,29%	18,86%	8,15%	2,01%	0,676
	Q4	28,72%	28,99%	24,20%	12,89%	4,13%	1,06%	1,000
Chile	Q1	0,08%	0,86%	5,40%	18,71%	35,42%	39,53%	0,231
	Q2	0,68%	2,64%	12,52%	25,72%	33,88%	24,55%	0,372
	Q3	0,68%	5,18%	16,87%	30,50%	31,09%	15,68%	0,503
	Q4	4,98%	16,26%	27,37%	26,54%	17,38%	7,47%	1,000
Czech Republic	Q1	3,71%	11,30%	21,35%	26,14%	23,15%	14,35%	0,278
	Q2	7,84%	17,34%	27,75%	25,60%	15,02%	6,44%	0,438
	Q3	12,23%	22,12%	27,79%	21,86%	12,33%	3,68%	0,568
	Q4	28,25%	28,22%	22,20%	13,09%	5,99%	2,24%	1,000
Denmark	Q1	2,41%	11,96%	23,63%	31,94%	21,44%	8,62%	0,298
	Q2	5,08%	17,82%	31,68%	27,57%	13,92%	3,92%	0,451
	Q3	11,36%	21,60%	33,36%	23,00%	8,61%	2,08%	0,632
	Q4	21,83%	29,06%	28,62%	14,80%	4,56%	1,13%	1,000
Estonia	Q1	7,62%	17,73%	31,36%	27,44%	12,65%	3,21%	0,417
	Q2	10,10%	21,98%	30,03%	24,92%	10,71%	2,26%	0,496

	Q3	14,24%	25,06%	30,26%	21,50%	7,43%	1,50%	0,616
	Q4	27,22%	29,78%	26,00%	13,74%	2,86%	0,39%	1,000
Finland	Q1	8,07%	16,67%	28,78%	26,36%	13,58%	6,53%	0,410
	Q2	10,86%	21,50%	30,65%	23,88%	10,60%	2,51%	0,530
	Q3	15,83%	27,57%	30,04%	18,15%	6,44%	1,97%	0,712
	Q4	27,09%	28,16%	26,87%	13,27%	3,88%	0,73%	1,000
France	Q1	3,34%	8,45%	20,27%	27,64%	22,55%	17,74%	0,209
	Q2	7,20%	15,28%	24,60%	26,58%	16,45%	9,89%	0,327
	Q3	13,39%	22,39%	27,74%	22,55%	9,60%	4,33%	0,515
	Q4	29,04%	31,01%	23,98%	11,31%	3,36%	1,29%	1,000
Germany	Q1	6,61%	14,08%	21,91%	26,29%	20,33%	10,78%	0,280
	Q2	12,84%	19,46%	27,62%	21,59%	13,03%	5,46%	0,434
	Q3	22,01%	28,84%	24,14%	15,58%	7,32%	2,10%	0,690
	Q4	35,97%	28,21%	19,81%	10,07%	3,95%	1,98%	1,000
Greece	Q1	0,35%	4,36%	14,56%	27,42%	28,36%	24,94%	0,302
	Q2	1,80%	7,92%	20,80%	28,48%	24,67%	16,33%	0,431
	Q3	3,08%	12,12%	23,78%	30,30%	19,80%	10,92%	0,567
	Q4	10,54%	20,63%	29,49%	22,66%	11,91%	4,77%	1,000
Hungary	Q1	1,53%	5,77%	15,33%	26,82%	29,39%	21,16%	0,211
	Q2	4,76%	12,56%	22,83%	29,96%	20,28%	9,61%	0,374
	Q3	8,63%	15,15%	26,55%	27,84%	16,10%	5,73%	0,494
	Q4	22,60%	24,55%	27,90%	16,87%	6,37%	1,71%	1,000
Iceland	Q1	5,55%	13,11%	22,84%	27,20%	19,84%	11,46%	0,447
	Q2	7,23%	15,19%	26,93%	27,32%	15,88%	7,45%	0,549
	Q3	14,70%	20,07%	25,48%	22,79%	11,96%	5,01%	0,763
	Q4	18,18%	25,15%	28,36%	17,22%	7,57%	3,52%	1,000
Ireland	Q1	3,43%	11,29%	25,07%	30,50%	19,82%	9,90%	0,311
	Q2	7,25%	16,64%	29,67%	26,83%	14,40%	5,21%	0,453
	Q3	11,55%	23,16%	31,23%	21,98%	9,17%	2,92%	0,635
	Q4	20,91%	30,61%	27,60%	16,12%	4,17%	0,59%	1,000
Israel	Q1	1,60%	5,53%	14,23%	22,87%	26,70%	29,08%	0,250
	Q2	5,75%	10,85%	20,33%	26,79%	20,43%	15,85%	0,420
	Q3	11,58%	18,55%	24,54%	21,76%	13,99%	9,59%	0,658
	Q4	19,91%	24,56%	25,58%	15,52%	9,26%	5,16%	1,000

Italy	Q1	3,46%	9,83%	20,27%	28,08%	23,58%	14,78%	0,388
	Q2	7,57%	14,82%	24,40%	25,77%	18,11%	9,33%	0,557
	Q3	11,73%	19,16%	26,75%	22,95%	13,47%	5,94%	0,743
	Q4	17,18%	23,64%	27,35%	19,38%	8,93%	3,52%	1,000
Japan	Q1	11,81%	20,47%	25,85%	22,82%	13,04%	6,01%	0,405
	Q2	18,82%	24,50%	27,17%	18,92%	8,24%	2,35%	0,571
	Q3	27,80%	24,94%	25,86%	13,89%	5,44%	2,08%	0,747
	Q4	38,12%	26,17%	20,20%	10,98%	3,67%	0,86%	1,000
Korea	Q1	16,34%	21,88%	26,76%	21,02%	9,34%	4,65%	0,403
	Q2	24,31%	24,09%	23,79%	16,80%	8,07%	2,95%	0,523
	Q3	35,08%	25,49%	19,60%	13,01%	4,84%	1,98%	0,717
	Q4	48,15%	24,21%	15,56%	7,59%	3,21%	1,28%	1,000
Luxembourg	Q1	2,29%	8,77%	18,63%	27,80%	25,15%	17,36%	0,242
	Q2	6,10%	13,20%	24,94%	27,09%	19,55%	9,11%	0,368
	Q3	12,98%	21,87%	27,85%	21,15%	11,24%	4,91%	0,603
	Q4	23,81%	30,98%	23,58%	13,62%	5,41%	2,60%	1,000
Mexico	Q1	0,12%	1,07%	6,57%	21,52%	35,68%	35,04%	0,395
	Q2	0,20%	2,48%	11,56%	27,60%	34,05%	24,11%	0,558
	Q3	0,43%	3,43%	13,57%	30,03%	32,40%	20,14%	0,645
	Q4	1,78%	7,98%	21,11%	32,51%	25,38%	11,24%	1,000
Netherlands	Q1	8,42%	16,55%	25,60%	24,56%	18,22%	6,65%	0,355
	Q2	13,66%	23,52%	27,17%	20,27%	12,12%	3,26%	0,511
	Q3	22,29%	28,63%	23,62%	14,83%	7,66%	2,98%	0,715
	Q4	34,25%	27,94%	20,77%	11,07%	4,42%	1,56%	1,000
New Zealand	Q1	4,30%	8,75%	18,00%	27,96%	25,37%	15,61%	0,246
	Q2	10,53%	18,11%	25,48%	24,19%	15,42%	6,26%	0,458
	Q3	14,75%	22,37%	27,42%	20,31%	11,64%	3,50%	0,597
	Q4	32,34%	24,65%	21,23%	13,82%	6,26%	1,70%	1,000
Norway	Q1	4,68%	12,41%	21,39%	27,98%	22,36%	11,19%	0,443
	Q2	6,45%	16,30%	26,26%	26,71%	16,59%	7,69%	0,576
	Q3	11,70%	20,44%	27,64%	22,93%	12,71%	4,58%	0,776
	Q4	15,72%	25,26%	28,32%	19,00%	7,95%	3,76%	1,000
Poland	Q1	5,77%	13,27%	25,06%	29,40%	19,35%	7,15%	0,280
	Q2	10,14%	20,95%	26,32%	26,20%	13,22%	3,16%	0,411
	Q3	17,50%	23,08%	27,51%	21,24%	8,46%	2,21%	0,557

	Q4	33,99%	28,24%	22,51%	11,46%	3,16%	0,64%	1,000
Portugal	Q1	2,91%	8,73%	19,59%	26,57%	25,42%	16,78%	0,245
	Q2	6,20%	14,93%	24,52%	27,33%	19,07%	7,94%	0,379
	Q3	9,67%	19,02%	27,66%	24,16%	13,47%	6,02%	0,497
	Q4	24,55%	29,12%	25,72%	13,54%	4,95%	2,12%	1,000
Slovak Republic	Q1	2,37%	6,90%	14,88%	24,14%	25,12%	26,61%	0,213
	Q2	5,98%	13,64%	25,32%	28,57%	18,90%	7,58%	0,413
	Q3	11,13%	19,23%	25,46%	24,01%	13,63%	6,52%	0,553
	Q4	25,15%	26,63%	23,44%	15,34%	7,38%	2,06%	1,000
Slovenia	Q1	4,36%	9,85%	22,07%	30,30%	24,59%	8,83%	0,293
	Q2	9,45%	16,47%	23,98%	27,01%	17,32%	5,77%	0,433
	Q3	13,98%	22,13%	25,59%	22,27%	12,32%	3,72%	0,587
	Q4	27,64%	26,86%	24,43%	14,27%	5,44%	1,36%	1,000
Spain	Q1	2,50%	8,42%	19,88%	29,54%	24,58%	15,09%	0,287
	Q2	4,59%	14,02%	26,43%	28,09%	19,01%	7,85%	0,426
	Q3	8,02%	19,94%	28,92%	25,52%	12,44%	5,16%	0,591
	Q4	17,28%	28,55%	29,38%	16,52%	6,55%	1,72%	1,000
Sweden	Q1	2,48%	9,06%	19,67%	28,72%	24,86%	15,22%	0,379
	Q2	5,21%	15,51%	23,66%	26,87%	18,63%	10,12%	0,545
	Q3	9,40%	19,14%	27,11%	24,34%	14,69%	5,33%	0,738
	Q4	15,81%	23,21%	27,17%	20,03%	10,73%	3,05%	1,000
Switzerland	Q1	9,18%	18,21%	25,61%	24,17%	15,18%	7,65%	0,348
	Q2	15,60%	24,85%	26,26%	19,89%	9,72%	3,68%	0,511
	Q3	23,50%	27,14%	25,28%	15,88%	6,40%	1,80%	0,684
	Q4	37,95%	25,89%	21,02%	10,49%	3,80%	0,85%	1,000
Turkey	Q1	1,35%	4,34%	12,14%	25,29%	32,53%	24,35%	0,345
	Q2	2,81%	7,69%	16,58%	27,16%	29,38%	16,37%	0,469
	Q3	5,37%	8,92%	16,50%	27,56%	27,93%	13,72%	0,535
	Q4	14,16%	19,55%	21,11%	22,46%	15,76%	6,97%	1,000
United Kingdom	Q1	3,98%	11,62%	23,60%	28,76%	20,04%	11,99%	0,335
	Q2	6,97%	14,74%	26,18%	26,57%	16,82%	8,72%	0,424
	Q3	12,57%	22,30%	26,52%	22,71%	11,12%	4,78%	0,623
	Q4	25,20%	26,65%	24,43%	15,30%	6,40%	2,02%	1,000
United States	Q1	2,50%	8,16%	18,68%	29,64%	26,48%	14,54%	0,303

	Q2	3,84%	12,28%	22,61%	30,57%	21,97%	8,73%	0,403
	Q3	9,66%	18,54%	26,01%	25,62%	15,31%	4,86%	0,610
	Q4	19,66%	25,02%	26,57%	19,28%	6,76%	2,72%	1,000
OECD	Q1	4,63%	10,96%	20,62%	26,63%	22,46%	14,70%	0,331
	Q2	8,43%	16,05%	24,76%	25,52%	17,10%	8,13%	0,475
	Q3	13,85%	20,72%	25,77%	21,95%	12,49%	5,22%	0,644
	Q4	24,68%	25,86%	24,41%	15,54%	6,95%	2,56%	1,000

Table 4: Coefficients of variation of the balanced worth (absolute and relative)

Countries	CV	% OECD
Australia	0,392	96%
Austria	0,427	105%
Belgium	0,505	124%
Canada	0,352	86%
Chile	0,550	135%
Czech Republic	0,470	115%
Denmark	0,440	108%
Estonia	0,354	87%
Finland	0,335	82%
France	0,588	144%
Germany	0,454	111%
Greece	0,457	112%
Hungary	0,567	139%
Iceland	0,308	75%
Ireland	0,430	106%
Israel	0,484	119%
Italy	0,338	83%
Japan	0,324	79%
Korea	0,342	84%
Luxembourg	0,522	128%
Mexico	0,341	84%
Netherlands	0,374	92%
New Zealand	0,478	117%
Norway	0,301	74%
Poland	0,375	92%
Portugal	0,539	132%
Slovak Republic	0,531	130%
Slovenia	0,458	112%
Spain	0,464	114%
Sweden	0,347	85%
Switzerland	0,380	93%
Turkey	0,422	104%
United Kingdom	0,430	105%
United States	0,461	113%
OECD	0,408	100%

Table 5: Ranking by explained variance* and opportunity bias

Countries	% explained variance	Ranking by explained variance	Ranking by Op. Bias	Difference
Australia	12,3	12	14	-2
Austria	15,8	22	17	5
Belgium	19,6	30	29	1
Canada	9,4	5	9	-4
Chile	23,1	34	33	1
Czech Republic	16,2	23	26	-3
Denmark	16,5	24	20	4
Estonia	8,6	3	10	-7
Finland	9,4	4	4	0
France	22,5	32	35	-3
Germany	16,9	26	21	5
Greece	15,5	19	22	-3
Hungary	23,1	33	34	-1
Iceland	7,7	2	2	0
Ireland	14,6	16	18	-2
Israel	17,2	27	28	-1
Italy	10,1	8	5	3
Japan	9,8	6	3	3
Korea	10,1	7	7	0
Luxembourg	18,3	28	30	-2
Mexico	10,4	9	6	3
Netherlands	11,5	11	11	0
New Zealand	18,4	29	27	2
Norway	7,4	1	1	0
Poland	16,6	25	12	13
Portugal	19,6	31	32	-1
Slovak Republic	24,6	35	31	4
Slovenia	15,6	20	23	-3
Spain	15,8	21	25	-4
Sweden	10,6	10	8	2
Switzerland	12,8	14	13	1
Turkey	14,5	15	16	-1
United Kingdom	12,5	13	19	-6
United States	14,8	18	24	-6

(*) Explained variance refers to the coefficient of determination of the relationship between test scores and socio-economic conditions calculated at individual level for each country.