Collateral and Development

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Abstract

This paper presents a model economy with endogenous credit constraints and endogenous growth, in which agents face a trade-off between investing resources to improve the pledgeability of collateral assets and the accumulation of human capital. The model generates both growth miracles and stagnant economies.
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"One may say that lending upon security has for long ages been the normal form of lending",
John Hicks (1969), A Theory of Economic History

1 Introduction

There are some well established facts in the literature on economic development over the post-war period.[1] First, US income per capita has been growing at a fairly steady average rate of about 2% per year; second, income per capita has been catching up with US levels in several OECD coutries, at faster rates between 1950 and 1980 (the so-called *Glorious Thirties*) than between 1980 and 2010; third, with the exception of China and a few other Asian countries, that have started to grow over the last thirty years, exhibiting persistently high investment rates and high returns to capital, the rest of the world has seen no sign of an overall tendency of convergence of living standards toward the frontier, with income per capita remaining stagnant in several African countries over the entire period.[2]

This paper proposes a simple environment that fits these observations, combining two well-known macroeconomic frameworks: the Uzawa-Lucas endogenous growth model and the Kiyotaki-Moore imperfect credit model. In particular, we consider an economy with human capital accumulation in which agents are unable to commit to future actions. This gives rise to credit imperfections, that are remedied using the undepreciated physical capital stock available at any point in time as collateral. Crucially, the agents may decide to devote resources to reduce the depreciation of capital, thus, increasing its redeployability and pledgeability, and relaxing the collateral constraint, but at the expense of human capital accumulation, thus, potentially, hindering economic growth. This generates a trade-off between improving credit conditions and fostering human capital. The tightness of collateral constraints and

[1] See Jones (2016) for a recent survey.

[2] Some countries seem to be converging to their own steady state rather than to the frontier, as argued by Mankiw, Romer and Weil (1992).

investment in human capital are both endogenously determined by the model.

An economy starting with a large initial stock of capital has enough collateral to overcome its credit constraint and remain indefinitely on its undistorted growth path. This captures the behavior at the frontier. Economies with smaller stocks of capital, but where human capital is relatively easy to accumulate and the agents are patient, undergo an initial phase of rapid growth with tight financial markets and distorted allocations, relative to first best, eventually ending up onto their long-run growth path, with lower growth rates but unhindered credit markets and undistorted allocations. These are the economies that converge to the frontier. During the phase of rapid growth with constrained financial markets, the return on physical capital exceeds its marginal return in production since capital carries a liquidity premium due to its collateral role. This leads to over-investment in physical capital relative to the efficient benchmark. Since more new capital is accumulated, there are lower incentives to reduce the depreciation of old capital, thus freeing resources to be used for human capital accumulation. This is the reason why, in our model, these economies grow faster in the initial phase, with both high investment rates and high returns to capital. Depending on the agents' impatience and the difficulty of accumulating human capital, other cases are also possible: some economies remain stagnant, while others enter a period of "happy de-growth" after an initial growth phase. This captures the non-convergence cases.

In the second part of the paper, we extend the model to include, among other aspects, an agricultural sector, and the possibility of policy intervention. Interestingly, the model suggests that developing economies with larger agricultural sectors in the early stages of development will tend to experience higher growth, relative to developing economies with smaller agricultural sectors. Valuable land may play a key role, early on in the development process, in relaxing collateral constraints, thus, reducing the need to invest in capital redeployability and freeing resources for human capital accumulation. Since allocations are distorted away from first best efficiency in economies that operate with binding collateral constraints, policy intervention may - in these circumstances - be beneficial, provided the public authorities have more commitment ability than the agents themselves. For instance, issuing public debt that can be used as collateral or financing a well functioning legal system that helps improve the enforcement of contracts, the Government may be able to reduce the need for investing private resources to enhance pledgeability, thus, favoring human capital accumulation.

The model combines aspects of the endogenous growth model of Lucas (1988) and the imperfect credit model of Kiyotaki and Moore (1997). The result is a model that has both endogenous growth and endogenous collateral constraints, which is able to generate miracle economies that experience over-investment with returns above fundamental value in the rapid growth phase and efficient investment with fundamental returns once the frontier has been reached. It is well known that neoclassical growth theory can generate convergence to the frontier but has a hard time generating excess investment during the transition, while endogenous growth theory has a hard time generating convergence. Our paper shows that endogenous growth with collateral constraints allows to reconcile convergence with excess returns on capital. Other models with different types of borrowing constraints,[3] based on imperfect monitoring technologies, rather than collateral, would have a hard time generating the endogenous slacking of such constraints. In Aghion, Howitt and Mayer-Foulkes (2005), who examine the effect of financial development on convergence in a Schumpeterian growth model, the exogenous level of financial development constitutes merely an obstacle to potential growth, while in the present context a country may be able to grow out of financial underdevelopment, through a process of capital deepening that accompanies the growth process and helps relax financial constraints. Consistently with the evidence on China, Song, Storesletten and Zilibotti (2011) generate high growth rates

[3] Azariadis and Kaas (2008), Lochner and Monge-Naranjo (2011) and Piazza (2014) have growth model with different borrowing imperfections.

accompanied by high returns on capital, in a multi-sector model in which resources are misallocated across sectors due to credit market imperfections. Our paper generates persistently high growth rates and high returns on capital in a one-sector model. The collateral role of land in developing economies has been pointed out by De Soto (2000) and further examined by Besley and Ghatak (2009). The declining value of land during the transition from stagnant to growing living standards is at the heart of Hansen and Prescott (2002). Government debt as a tool to relax credit constraints have been studied by Woodford (1990) and Holmstrom and Tirole (1998) in models without endogenous growth. The model points out one particular mechanism through which finance and growth interact. A variety of other channels of interaction could be included to enrich the framework, including the link between finance and human capital accumulation, pointed out by De Gregorio and Kim (2000).[4]

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the competitive equilibrium. Section 4 examines efficiency. Section 5 sums up our findings and relates them to the evidence. Section 6 presents the extensions. Section 7 considers policy intervention. Section 8 concludes. The proofs, tables and figures are in the Appendix.

2 The Model

Fundamentals Time is discrete and continues forever, indexed by t. There is a single consumption/investment good and two agents, an entrepreneur and a worker. At any point in time, the entrepreneur produces output, y_t, with a technology that combines physical capital, k_t, labor, l_t, and human capital, h_t, represented by a Cobb-Douglas production function,

$$y_t = k_t^{\alpha} \left(l_t h_t\right)^{1-\alpha}. \tag{1}$$

[4]Thorough surveys on the relationship between finance and growth can be found in Levine (2005) and Beck (2012).

Every period, the entrepreneur has the option to exert effort, u_t, up to a total of 1 unit, that can be allocated to enhance either the redeployability of physical capital or the accumulation of human capital, the following period. Physical capital is available initially in amount k_0 and depreciates after being used in production at a rate $1-u_{t-1}$, whereby the effort exerted by the entrepreneur in one period reduces depreciation the following period, leaving an amount of undepreciated capital after production equal to

$$u_{t-1}k_t. \tag{2}$$

At the initial date, depreciation is nil, since capital has not been used in production yet. Human capital, available initially in amount h_0, is accumulated at a per period net rate $\delta(1-u_{t-1})$, whereby the effort devoted in one period by the entrepreneur to human capital enhancement, instead of capital redeployability, increases human capital the following period by a positive factor δ, giving rise to the human capital accumulation rule

$$h_t = h_{t-1}\left[1+\delta(1-u_{t-1})\right]. \tag{3}$$

The entrepreneur derives spot utility $U(\cdot)$ from consumption of the good, c_t, with standard properties, and discounts the future at a positive rate $\beta < 1$, giving rise to life-time utility,

$$\sum_{t=0}^{\infty}\beta^t U(c_t). \tag{4}$$

The worker provides labor at a competitive wage w_t in units of the good, obtaining a spot payoff $V(\cdot)$ from consumption and labor with the partial derivative wrt labor $V_l < 0$ to reflect its disutility and other standard properties.

Debt Labor is acquired by the entrepreneurs issuing debt. Agents are unable to commit themselves to future actions. Due to the agents' inability to commit to repay, debts need to be collateralized. A debtor agrees to repay the amount borrowed by the end of the same period. Capital can be pledged as collateral up to its undepreciated end of period value, given by (2). If debts are not honored, the creditor has the right to seize the amount of the good pledged as collateral and consume it. Human capital is non-contractible, hence, it is neither traded nor pledged as collateral. No contract can be credibly written on final output.

Assumption In order to have an economy that has the potential to grow, we assume that human capital is sufficiently relevant as an input in production,

$$1 - \alpha > \frac{\beta}{\delta}, \tag{5}$$

which, in turn, requires $\delta > \beta$, to be feasible.

3 Collateral and Human Capital

In this model economy, traders are unable to commit to repayment. Hence, credit is collateralized by physical assets, specifically, the undepreciated capital available after production. The point of view of this paper is that more innovative production processes mould capital in new ways making it more specific and, hence, more difficult to redeploy and pledge with outsiders. Activities that help reduce capital depreciation and increase its redeployability, thus, also enlarging the collateral base, may be brought into play, but subtracting resources from the accumulation of human capital, which may hinder the process of growth. The resolution of the trade-off between enlarging the collateral base and enhancing growth is the subject of this section, where we construct an equilibrium with collateralized credit.

3.1 Trade

The sequence of trades within a period is as follows. The entrepreneur acquires labor from the worker on credit, then, produces the final good and pays off his debts. Next, the entrepreneur allocates time between the activity that improves the accumulation of human capital and the one that reduces the depreciation of physical capital, and accumulates assets for the following period. Finally, consumption occurs. We first describe the decision problem of the two agents taking the wage as given, since the labor market is competitive. At any point in time, the entrepreneur chooses consumption c_t, labor demand l_t, effort u_t, and capital holdings for the future k_{t+1}, to maximize life-time utility (4), subject to a budget and collateral constraint. The budget constraint, whose non-negative, discounted multiplier appears in square brackets, is,

$$c_t + k_{t+1} + w_t l_t = y_t + u_{t-1} k_t, \qquad \left[\beta^t \lambda_t\right] \tag{6}$$

which states that production, given by (1), and undepreciated capital can be used to acquire consumption, capital and labor. Effort reduces next period depreciation of capital, (2), but is exerted at the expense of the enhancement of next period human capital, through (3), potentially with negative effects on productivity. In turn, this trade-off has a knock-on effect on credit markets, since debts are collateralized with undepreciated capital as reflected in the collateral constraint, whose non-negative, discounted multiplier appears in square brackets,

$$w_t l_t \leq u_{t-1} k_t, \qquad \left[\beta^t \zeta_t\right] \tag{7}$$

where the acquisition of labor is limited by the amount of available, undepreciated capital, (2), used as collateral to secure repayment. This is reminiscent of the collateral constraint appearing in Kiyotaki and Moore (1997), with the crucial difference that the effort exerted the previous period by the entrepreneur to salvage capital, appears on the RHS, since it increases the stock of collateralizable activities. The worker cannot use capital for productive purposes, hence, has no reason to accumulate it, therefore, he just chooses statically consumption, $\widehat{c}_t$, and labor supply, $\widehat{l}_t$, each period, to maximize $V\left(\widehat{c}_t, \widehat{l}_t\right)$ subject to $\widehat{c}_t = w_t \widehat{l}_t$.

3.2 Equilibrium Conditions

First, we derive the optimality conditions for the agents, then, we consider market clearing. The entrepreneur determines consumption to satisfy at any point in time the first order condition,

$$U'(c_t) = \lambda_t, \tag{8}$$

which equates the marginal utility benefit of extra consumption to the marginal cost of tightening the budget constraint. The optimal decision for the labor demand at any point in time leads to the determination of the non-negative shadow value of the collateral constraint,

$$\lambda_t + \zeta_t = \lambda_t \frac{(1-\alpha)\, y_t}{w_t l_t}, \tag{9}$$

which reflects the potential divergence of the wage from the marginal product of labor and allows to write the complementary slackness condition for the collateral constraint, (7), as

$$[(1-\alpha)\, y_t - w_t l_t]\,(u_{t-1} k_t - w_t l_t) = 0, \tag{10}$$

where the two expressions in parenthesis are constrained to be non-negative, the first corresponding to the shadow value of the collateral constraint, (9), and the second to the constraint itself, (7). A divergence between wage and labor productivity is a symptom of a constrained situation. Equation (10) states precisely that a positive spread between labor productivity and the wage and a slack collateral constraint are incompatible with the entrepreneur's optimization. A distinctive element of this model is the crucial role played, next to more traditional equilibrium conditions, by the complementary slackness condition (10). Through such an equation, the equilibrium determines endogenously whether the economy is in a regime with a binding collateral constraint or not. The choice of effort satisfies at any point in time the first order condition,

$$\lambda_{t+1} + \zeta_{t+1} = \lambda_{t+1} \frac{(1-\alpha)\, y_{t+1}}{k_{t+1}} \frac{\delta h_t}{h_{t+1}}, \tag{11}$$

which reflects the alternative allocation of effort to either improve the future redeployability of capital, which in turn enhances the ability to borrow against it in the future, or to accumulate human capital, according to (3), which improves the future productivity. The effect of the effort on the borrowing ability of the entrepreneur is represented, in equation (3), by the multiplier of the collateral constraint, ζ_{t+1}, which may be positive or nil depending whether the collateral constraint is tight or not, as determined by the complementary slackness condition (10) at time $t+1$. In Lucas (1988), human capital accumulation boosts productivity but subtracts time and effort to regular labor activities, while here it boosts productivity but subtracts time and effort to activities that help salvage capital after it has been used in production, which, in turn, helps enlarge the stock of collateralizable assets. At any point in time, capital is accumulated according to the Euler equation

$$\lambda_t = \beta \left[\lambda_{t+1} \frac{\alpha y_{t+1}}{k_{t+1}} + (\lambda_{t+1} + \zeta_{t+1}) u_t \right], \tag{12}$$

which weighs the cost of accumulating an extra unit of capital, against its future benefits, accruing both from increased output and a larger asset base to be used also as collateral. Hence, capital is an asset that has both a fundamental value, being an input in production, and a liquidity value, being used as collateral to obtain loans. The liquidity value of capital is represented, in equation (12), by the multiplier of the collateral constraint, ζ_{t+1}, which may be positive or nil depending whether the collateral constraint binds or not, as determined by (10) at time $t+1$. The effort, u_t, appears in the Euler equation, since it helps enlarge the future asset base. Finally, there is the budget constraint, (6), and the transverality condition,

$$\lim_{t \to \infty} \beta^t \lambda_t k_t = 0, \tag{13}$$

ruling out overaccumulation of capital at infinity, which is incompatible with the entrepreneur's optimization. These conditions, together with the initial conditions k_0 and h_0, determine effort, capital, consumption and labor demand taking the wage as given. The optimality condition for the worker's labor supply is $w_t V_c \left(\widehat{c}_t, \widehat{l}_t \right) +$

$V_l\left(\widehat{c}_t, \widehat{l}_t\right) = 0$, at all t. The market clearing conditions for labor and the good hold at any point in time.

3.3 Collateralized Credit Equilibrium

Before deriving the equilibrium conditions, we point out an interesting relationship between the real interest rate and the tightness of the credit market. The marginal product of capital including its undepreciated fraction in this economy is $\frac{\alpha y_t}{k_t} + u_{t-1}$, since depreciation is endogenously determined by effort. The real interest rate is $\frac{\lambda_{t-1}}{\lambda_t}\frac{1}{\beta}$ as usual. In this economy capital plays both a productive role and a collateral role, enhancing the functioning of the credit market, i.e. making it more liquid. Define the liquidity premium on capital as $\Lambda_t \equiv \frac{\lambda_{t-1}}{\lambda_t}\frac{1}{\beta} - \alpha\frac{y_t}{k_t} - u_{t-1}$, namely the excess of the real return over the marginal product of undepreciated capital. The next Lemma shows that there is a positive premium on capital if and only if the multiplier of the collateral constraint is strictly positive.

Lemma 1 $\Lambda_t > 0$ *iff* $\zeta_t > 0$.

When $\Lambda_t > 0$, capital commands a positive liquidity premium arising from its role as collateral. Since the multiplier of the collateral constraint is strictly positive only if the credit constraint is binding, by (10), necessary condition for capital to have a liquidity premium is that the economy is credit constrained. In other words, when the credit constraint is not binding, capital cannot possibly carry any extra return for relaxing it. This result was derived using the optimality conditions for the entrepreneur only. We will show later that the economy being credit constrained is also sufficient condition to have a liquidity premium on capital, using the other equilibrium conditions. Next, we move to the full equilibrium system. Combine (9) delayed one period and (12), obtaining

$$\frac{k_{t+1}}{w_{t+1}l_{t+1}} = \delta\frac{h_t}{h_{t+1}}. \tag{14}$$

Combine, also, (11) and (12), obtaining

$$\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta} = \frac{y_{t+1}}{k_{t+1}}\left[\alpha + (1-\alpha)\,\delta u_t \frac{h_t}{h_{t+1}}\right]. \tag{15}$$

The complementary slackness condition for the collateral constraint, the budget constraint, and the equations (14) and (15) together constitute the optimality conditions for the entrepreneur. The worker fulfills his budget constraint $\widehat{c}_t = w_t \widehat{l}_t$ and his labor supply condition. Labor market clearing $l_t = \widehat{l}_t$ holds at any point in time. The market clearing condition for the good is redundant by Walras Law. Combining these equilibrium conditions together, we can obtain a more manageable equilibrium system. The complementary slackness condition, (10), becomes

$$[(1-\alpha)\,y_t - \widehat{c}_t]\,(u_{t-1}k_t - \widehat{c}_t) = 0. \tag{16}$$

Use (3) into (14), obtaining

$$k_{t+1} = \frac{\delta \widehat{c}_{t+1}}{1+\delta-\delta u_t}. \tag{17}$$

Use (8) , (3) and (17) into (15), rearranging obtain

$$y_{t+1} = \frac{\frac{\delta}{\beta}\frac{U'(c_t)}{U'(c_{t+1})}\widehat{c}_{t+1}}{\alpha\,(1+\delta) + (1-2\alpha)\,\delta u_t}. \tag{18}$$

The budget constraint, (6), becomes

$$k_{t+1} = y_t + u_{t-1}k_t - \widehat{c}_t - c_t, \tag{19}$$

which coincides with the market clearing condition for the good. Equations (16) to (19), together with the initial conditions and the transversality condition, (13), hold at all times and determine the equilibrium path of effort, physical capital, and consumption for the two agents. Human capital is determined through (3), once effort is pinned down. Output is determined through (1), once human and physical capital are pinned down. The wage is determined implicitly by $w_t V_c\,(\widehat{c}_t, \widehat{c}_t/w_t) + V_l\,(\widehat{c}_t, \widehat{c}_t/w_t) = 0$ once the worker's consumption has been pinned down. Since the wage is determined residually, henceforth, we ignore it. Next, we define a collateralized credit equilibrium.

Definition 1 *A collateralized credit equilibrium (CCE) is a four-tuple* $(u_t,\, k_t,\, c_t,\, \widehat{c}_t)$ *satisfying (16) to (19) at every point in time.*

The value of effort needs to be strictly positive to guarantee that there is some pledgeable collateral. Instead, $u = 1$, which leads to a stagnant economy, is compatible with equilibrium. The economy may be in a regime with a binding collateral constraint - a constrained CCE regime- or not - an unconstrained CCE regime. Among CCE, there are equilibria with both constant and non-constant growth rates of human and physical capital over time. We postpone the analysis of equilibria with non-constant growth rates to a later section, and consider first equilibria with constant growth rates of human and physical capital, output and consumption, namely balanced growth equilibria.

3.4 Balanced Equilibria

Under our assumptions, the growth rates of the endogenous variables across constrained and unconstrained regimes are bound to differ. The closest we can get to a balanced equilibrium is an equilibrium in which the growth rate is constant within a regime. We will define this as a Balanced CCE.

Definition 2 *A Balanced CCE (BCCE) is a CCE with constant within-regime growth rates of* h, k, y, c *and* $\widehat{c}$.

At a BCCE, a change in growth rate is admissible only if accompanied by a switch from a constrained to unconstrained regime or viceversa.

3.4.1 Characterization

For simplicity, we let the utility function of the entreprenur be logarithmic.[5] By (3), for the growth rate of human capital to be constant within a regime, effort

[5] This parametrization simplifies the computations without affecting the gist of the argument, which would go through with, for instance, a constant relative risk aversion function.

must be time independent within the regime, $u_t = u$. By the equilibrium system, it immediately follows that, at a BCCE, all the equilibrium variables grow at the same rate at which human capital grows, $1 + \delta - \delta u$. The time sub-scripts can be dropped, and the equilibrium system, (16)-(19), can be rewritten as

$$[(1-\alpha) y - \widehat{c}] (uk - \widehat{c}) = 0; \tag{20}$$

$$k = \frac{\delta \widehat{c}}{1 + \delta - \delta u}; \tag{21}$$

$$y = \frac{\delta (1 + \delta - \delta u) \widehat{c}}{\beta [\alpha (1 + \delta) + (1 - 2\alpha) \delta u]}; \tag{22}$$

$$c = y - \widehat{c} - (1 + \delta)(1 - u) k. \tag{23}$$

Moreover, there is the complementary slackness condition at the initial date,

$$[(1-\alpha) y_0 - \widehat{c}_0] (k_0 - \widehat{c}_0) = 0. \tag{24}$$

Notice that the system (20)-(23) is recursive. Substituting (21) and (22) into (20), one obtains a single equation in the effort alone. The solution for the effort, if it exists, can then be substituted back into the other equilibrium conditions, (21), (22) and (23), to determine the rest of the equilibrium values, namely capital and consumption for the two agents. Then, human capital and output are derived, through (3) and (1). The wage is determined implicitly by $w v_c (\widehat{c}, \widehat{c}/w) + v_l (\widehat{c}, \widehat{c}/w) = 0$ once the worker's consumption is pinned down by the BCCE. The first Lemma establishes that the case in which the economy is constrained and the spread between the marginal product of labor and the wage is zero cannot emerge at a BCCE. This result allows us to limit attention to just two BCCE situations.

Lemma 2 *At a BCCE,* $(1-\alpha) y \neq uk$.

Hence, by (20), the two terms in the complementary slackness condition cannot be simultaneously equal to zero at equilibrium. Since we have shown above (Lemma 1) that the liquidity premium on capital is positive if and only if the multiplier of the collateral constraint is strictly positive, it follows that the economy is constrained if and only if there is a positive liquidity premium on capital at a BCCE. The next Proposition establishes the existence of a BCCE.

Proposition 1 *A BCCE exists and is unique.*

The equilibrium works as follows. Every period, the entrepreneur acquires labor on credit against collateral from the worker. Capital and labor enter as inputs in the production process, generating output. Debts are settled, consumption and the accumulation of assets for the future take place at the end of every period. The entrepreneur spends effort to salvage capital and possibly to enhance human capital for next period. A situation in which the entrepreneur devotes all his efforts to the activities that salvage capital, neglecting completely human capital accumulation, thus, compromising growth, is an admissible BCCE, in which there is no investment in human capital at any time, and the economy remains stagnant. The collateral constraint may or may not bind at equilibrium. A BCCE is constrained if the collateral constraint binds at equilibrium at some date. The following Proposition determines when the BCCE is constrained at the initial date.

Proposition 2 *A BCCE is initially constrained, iff* $k_0 \leq h_0 (1-\alpha)^{\frac{1}{1-\alpha}}$.

Hence, the initial capital stock relative to the initial amount of human capital determines whether its financial markets are hindered or not, at least temporarily. A small initial stock of capital constrains the amount entrepreneurs can borrow from the outset, while a large initial stock endows the economy with enough collateral to overcome its borrowing limits. When the collateral constraint is binding, from (20) and (21), effort in redeploying capital is

$$u = \frac{1}{2}\frac{1+\delta}{\delta}, \tag{25}$$

which may or may not be smaller than 1, i.e. give rise to growth, depending on the value of δ. The next Proposition establishes under which conditions a BCCE -constrained or otherwise- exhibits growth.

Proposition 3 *a. A constrained BCCE exhibits growth iff* $\delta > 1$. *b. An unconstrained BCCE exhibits growth iff* $\beta > \frac{(1-\alpha)\delta}{(1-\alpha)\delta+\alpha}$.

Both constrained and unconstrained BCCE may exhibit growth, in different circumstances. The net growth rate of human and physical capital and output at a constrained CCE with growth is

$$\gamma_c = \frac{\delta - 1}{2}, \tag{26}$$

which reflects the technological rate at which human capital is accumulated. For a constrained equilibrium to exhibit growth, the entrepreneur needs to devote at least some effort to human capital accumulation. To ensure this, human capital should be sufficiently easy to accumulate. The net growth rate of human and physical capital and output at an unconstrained BCCE with growth is

$$\gamma_u = \frac{\beta\left[(1-\alpha)\,\delta + \alpha\right] - (1-\alpha)\,\delta}{(1-\alpha)\,(\beta+\delta) - \beta\alpha}, \tag{27}$$

which reflects both technological and preference parameters. For an unconstrained equilibrium to exhibit growth, the agents should be sufficiently patient. Whenever there is growth at a BCCE, all the variables grow at the same rate at which human capital grows. The next Proposition shows that an initially constrained economy converges to an unconstrained situation.

Proposition 4 *An initially constrained BCCE with growth becomes unconstrained at some finite date.*

A constrained growing economy, even if not gifted with a large initial capital stock, will be able to progressively relax its borrowing constraint, investing effort in human capital accumulation. The lower collateralizability per unit of capital is more than compensated by the increase in the capital stock over time, which enlarges the overall collateral base. Hence, the collateral constraint tends to be relaxed over time, reducing the spread between the marginal product of labor and the wage, which remains positive throughout the period with a tight collateral constraint, until eventually, the collateral base becomes sufficiently large to make the collateral constraint not binding. Once the collateral constraint is slack, the wage is pinned down by the marginal productivity of labor. At a BCCE, (26) is larger than (27), hence, developing economies that take off, initially grow sufficiently fast to catch-up with mature economies. This happens because an economy accumulates more capital in constrained periods than unconstrained ones, due to the presence of a positive liquidity premium for its collateral role in the former but not in the latter case. This extra incentive to accumulate new capital reduces the need to exert effort to salvage old capital, thus, freeing resources for human capital accumulation, which, in turn, fosters growth. Finally, the next Proposition examines stagnant economies. A BCCE is stagnant if its growth rate is nil.

Proposition 5 *A stagnant BCCE remains indefinitely either constrained or unconstrained.*

In stagnant economies, the stock of capital remains for ever the same. A stagnant economy with a relatively small amount of physical capital, has a small collateral base, making its borrowing ability rather limited. On the other hand, a stagnant economy with a sufficiently large initial endowment of physical capital, can be borrowing unconstrained, despite its inability to grow. Hence, even economies that are initially poor may start to grow, provided their citizens are enticed to invest their time and efforts in human capital. Conversely, economies that are rich of natural resources may remain stagnant, although with unhindered financial markets.

3.5 Dynamics

We have so far confined attention to balanced equilibria. Alongside balanced equilibria, there may also be dynamic ones, in which the growth rate is non-constant over time even within a regime. The first Proposition shows that, when the economy is constrained, the balanced equilibrium derived above is the only equilibrium.

Proposition 6 *In the constrained regime, there is only the BCCE.*

Looking at the equilibrium conditions, in particular (16) and (17), when the collateral constraint is binding, the effort is determined by a time independent condition, which leads to the result. When the collateral constraint is not binding, instead, there are other non-balanced equilibria alongside the balanced one. At an unconstrained equilibrium, $(1-\alpha)\,y_t = \widehat{c}_t$ holds. Defining $z \equiv \frac{k}{c}$, we can reduce the system (17), (18) and (19) to two first order difference equations in z and u,

$$z_{t+1} = F\left(z_t, u_{t-1}\right) \equiv \beta^{-1} \frac{\left[\frac{\alpha(1+\delta)}{(1-\alpha)\delta} + \frac{1-2\alpha}{1-\alpha} u_{t-1}\right] z_t - 1}{\frac{1+\delta}{\delta} - \frac{1-2\alpha}{(1-\alpha)\delta}\left[\frac{1+z_t(1+\delta-\delta u_{t-1})^{\frac{1}{1-\alpha}}}{\frac{\alpha(1+\delta)}{(1-\alpha)\delta}+\frac{1-2\alpha}{1-\alpha}u_{t-1}}\right]^{\frac{1-\alpha}{\alpha}}}, \tag{28}$$

$$u_t = G\left(z_t, u_{t-1}\right) \equiv \frac{1+\delta}{\delta} - \frac{1}{\delta}\left[\frac{1+z_t\left(1+\delta-\delta u_{t-1}\right)^{\frac{1}{1-\alpha}}}{\frac{\alpha(1+\delta)}{(1-\alpha)\delta}+\frac{1-2\alpha}{1-\alpha}u_{t-1}}\right]^{\frac{1-\alpha}{\alpha}}. \tag{29}$$

The following Proposition studies the dynamics of the system (28)-(29) relative to the balanced equilibrium obtained above.

Proposition 7 *In the unconstrained regime, the BCCE is a saddle point iff $\beta > \frac{1}{2}$.*

When the agents are less patient, the balanced equilibrium is a source. Hence, under the appropriate conditions, the path leading to development is essentially unique, going through an initial phase with constrained financial markets but high growth rates, to jump, after a quick transition, onto the long-run balanced equilibrium with lower growth but unconstrained financial markets.

4 Efficiency

We have, so far, devoted attention to competitive equilibrium allocations. We now turn to efficient allocations. To characterize first best Pareto efficient allocations, we set up the Negishi problem for this model economy. We look for amounts of effort, labor, capital and consumption for the two agents that maximize

$$\mu \sum_{t=0}^{\infty} \beta^t U(c_t) + (1-\mu) \sum_{t=0}^{\infty} \beta^t V(\widehat{c}_t, l_t), \tag{30}$$

with a positive Pareto weight $\mu < 1$, subject to the feasibility constraint, $c_t + \widehat{c}_t + k_{t+1} = y_t + u_{t-1} k_t$, at every t. The first order conditions for a maximum are the following,

$$(1-\alpha) y_t = \frac{-V_l(\widehat{c}_t, l_t) l_t}{V_c(\widehat{c}_t, l_t)}, \tag{31}$$

$$k_{t+1} = \frac{\delta (1-\alpha) y_{t+1}}{1 + \delta - \delta u_t}, \tag{32}$$

$$\frac{U'(c_t)}{U'(c_{t+1})} = \frac{\beta [\alpha (1+\delta) + (1-2\alpha) \delta u_t]}{\delta (1-\alpha)}, \tag{33}$$

$$k_{t+1} = y_t + u_{t-1} k_t - \widehat{c}_t - c_t, \tag{34}$$

$$\mu U'(c_t) = (1-\mu) V_c(\widehat{c}_t, l_t). \tag{35}$$

Since (31)-(35) are necessary conditions for an allocation to be efficient, any allocation that violates them cannot be efficient. Since (30) is a convex combination of strictly concave functions, (31)-(35) are also sufficient for efficiency, hence, any allocation that satisfies them for at least some μ is efficient.The next Proposition shows when the competitive equilibrium is efficient.

Proposition 8 *In the constrained regime, the CCE is inefficient; in the unconstrained regime, the CCE is efficient.*

In economies in which the collateral constraint is binding, the return to capital and its marginal productivity diverge, a liquidity premium being present, which results in allocations that are distorted relative to the first best frontier, with over-accumulation of capital relative to first-best, while in economies in which the collateral constraint is slack, the return and the marginal productivity of capital coincide, resulting in allocations that are on the first best frontier. No further distortion is present, instead, relative to the second best frontier, hence the CCE is always constrained efficient.

5 Discussion

This section summarizes our findings and uses the available evidence to test the key prediction of the model. The model predicts a higher growth rate of GDP in constrained periods than unconstrained ones, due to the liquidity premium for the collateral role of capital, which makes it more attractive to accumulate capital afresh rather than exert effort to salvage it. This, in turn, frees resources for human capital accumulation, which fosters growth.

Summary of theoretical findings The exogenous factor affecting the accumulation of human capital, δ, the share of capital in production, α, and the rate of impatience, β, determine whether an economy grows or not, while the initial endowment of physical capital, k_0, relative to initial human capital, h_0, determines whether the economy has tight credit markets or not, at least for some time. Consider first the case of a small initial capital stock relative to human capital, $k_0 \leq h_0 (1-\alpha)^{\frac{1}{1-\alpha}}$. Among these economies, those in which the accumulation of human capital is easier, i.e. characterized by $\delta > 1$, begin to grow but facing two different destinies, depending on the rate of impatience. If the agents are sufficiently patient, $\beta > \frac{(1-\alpha)\delta}{(1-\alpha)\delta+\alpha}$,[6] the economy initially grows at high rates with constrained financial markets and distorted allocations, to eventually converge onto a growth path with lower growth rates but unconstrained markets and undistorted allocations. This type of economy, that catches up with the developed ones, accomplishes what might be termed a "growth miracle". If the agents are impatient, $\beta \leq \frac{(1-\alpha)\delta}{(1-\alpha)\delta+\alpha}$, instead, the economy, after a similar fast growth phase with constrained financial markets and distorted allocations, converges eventually to a growthless path with unhindered financial markets and undistorted allocations. This is the case of an initially fast growing economy, that enters into an indefinite phase of "happy de-growth", in which financial mar-

[6] For $\alpha < 1/2$, as it is typically the case in real world economies, and $\beta > 1/2$, which guarantees that the equilibrium is a saddle, this is always satisfied.

kets work smoothly but the economy grinds to a halt. The economies in which the accumulation of human capital is more difficult, i.e. with $\delta \leq 1$, remain stagnant, although some of them, those with patient agents, have growth potential. Development strategies aimed at increasing an economy asset base may play a role in this last case, but not in the others. With a sufficiently large initial capital stock relative to human capital, $k_0 > h_0 (1-\alpha)^{\frac{1}{1-\alpha}}$, once the economy has reached its unconstrained, undistorted path, with growth if the agents are sufficiently patient, without growth otherwise, remains indefinitely there. These are mature economies that remain on their long run equilibrium path. Figure 1 summarizes graphically the main patterns.

Alternative modelling assumptions The model has two distinguishing features, relative to an endogenous growth model, namely endogenous capital depreciation and the borrowing constraint secured by collateral, (7). To see that both of these assumptions are crucial for the key results, let us drop them, in turn. The same model with constant, exogenous depreciation of capital would exhibit either constrained or unconstrained equilibria, but not both for the same parameters values, and the same growth rate of GDP across constrained and unconstrained regions, being, therefore, unable to generate convergence. In particular, it is necessary to have effort determine endogenously the depreciation rate, in order to obtain two regions, with binding and non-binding collateral constraint. If the effort consisted only in a financial activity, helpful to relax the credit constraint but not to redeploy capital physically, the LHS of equation (11) would feature only the multiplier of the collateral constraint, which would, therefore, have to be always strictly positive. As regards the borrowing constraint, without it the model would be a version of Lucas (1988) with endogenous depreciation, which would generate only the unconstrained path. Consider now a different type of credit, enforced through a monitoring technology, rather than collateral, as follows. Suppose the agents can exert effort u each period to enhance the monitoring technology the following period, linearly and one for one, as well as to reduce capital depreciation. If an agent fails to repay his debt, with probability u is caught and left without consumption, while with the complementary probability he is not caught and can enjoy the extra consumption that arises from not repaying the debt. For simplicity, the agents enter the following period with a clean sheet, although more severe punishments could be considered. This monitoring technology gives rise to the borrowing constraint, $u_{t-1}U(0) + (1 - u_{t-1}) U(c_t + w_t l_t) \leq U(c_t)$, which replaces (7). The rest remains the same. This model can generate two separate equilibria, with a slack and tight borrowing constraint, but not an equilibrium with capital/financial deepening and endogenous slacking of the constraint. Intuitively, this happens because the state variable does not enter directly the borrowing constraint, as in (7). Hence, both endogenous capital depreciation and collateral constraints are crucial to obtain convergence. As for the interaction of the two ingredients, effort u and the stock of capital k should not be perfect complements in constraint (7), while any other form of even mild substitution would do, since our result requires only that extra capital accumulation substitutes for some effort to enhance redeployability. Clearly, endogenous growth through human capital accumulation is necessary. Without it there would be no sustained growth and no trade-off in the effort dimension.

Evidence Using publicly available data from the Penn World Tables 9.0 (PWT) and the World Bank,[7] we performed some checks on the evidence that constitutes the background for our theory of development. First, we confirmed that for all OECD countries, over the period 1950-2010, the pattern of (log) real GDP over time is concave, except for the US, where the pattern is essentially linear. For OECD countries, we found evidence of the presence of a structural break in the slope of the trend with higher growth rates before the break.[8] Second, we regressed the capital output-ratio

[7]Table 1 in the Appendix contains the summary statistics and variables descriptions. Details of our (fairly standard) empirical checks have been omitted but are available upon request.

[8]The presence of a single break in the trend function is statistically significant for Australia, Canada, Germany, Mexico, New Zealand, Poland, Slovenia, Switzerland and UK, using the Zivot-

against 10-years growth rates by country and found positive and significant coefficients for all OECD countries (see Table 2). Third, using data on credit provided by the World Bank, we found that the average real GDP growth rates and average credit to private sector over GDP ratios, for each country in the sample, are negatively related.[9] These observations together point in the direction of a pattern of catch-up for several countries that suggests the possibility of an initial phase with high growth rates of GDP, high investment in physical capital and some credit distortion, and a second phase with lower GDP growth, lower capital investment and smaller credit distortions.

Next, we put the key predictions of the model to the test. From the analysis above, an economy is constrained if and only if the liquidity premium on capital Λ_t is strictly positive. By equations (3) and (12), the effort in human capital accumulation is given by

$$1 - u_t = \frac{\frac{h_{t+1}-h_t}{h_t}\left(1-\alpha\right)\frac{y_{t+1}}{k_{t+1}}}{\frac{h_{t+1}}{h_t}\left(\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta} - \alpha\frac{y_{t+1}}{k_{t+1}}\right) + \frac{h_{t+1}-h_t}{h_t}(1-\alpha)\frac{y_{t+1}}{k_{t+1}}}, \tag{36}$$

which can be inserted into Λ_t obtaining a liquidity premium on capital which is positive if and only if

$$\frac{h_{t+1}}{h_t}\left[\left(\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta} - 1\right) - \alpha\frac{y_{t+1}}{k_{t+1}}\right] + \left(\frac{h_{t+1}-h_t}{h_t}\right)\left(1-\alpha\right)\frac{y_{t+1}}{k_{t+1}}, \tag{37}$$

is positive. Our theory has the following key implications for growing countries: *i.* the effort in human capital accumulation, (36), and the growth rate of real per capita GDP are positively correlated; *ii.* the growth rate of real per capita GDP are higher when there is a liquidity premium on capital, i.e. (37) is strictly positive; *iii.* the liquidity premium eventually disappears. We set $\frac{\lambda_t}{\lambda_{t+1}} = \frac{c_{t+1}}{c_t}$, since utility is logarithmic, and

Andrews test. In the whole sample, the median date for the break is 1978.

[9] In Figure 2 we plot the average real GDP growth rates and the average credit to private sector over GDP ratios, before and after the break date in its real GDP series. We restrict our attention on those economies that have grown faster before the break date. If a country lays above the 45 degree line, then it has experienced a higher credit during the phase of lower growth.

estimated the time discount factor, β, for each country using the real interest rate series for OECD countries from the World Bank data set. We computed (36) and (37) using the series for human capital, consumption, labor and capital shares for OECD countries for the period 1950-2014 from the PTW data set. We found that: *i.* the effort in human capital accumulation is positively (0.704) and significantly correlated with the growth rate of real GDP; *ii.* economies in which (37) is positive grow on average at a rate of 0.054 vis-à-vis 0.033 for the others; *iii.* the probability of having a positive (37) declines as time goes by, being higher before 1978 with a frequency of 0.331 against 0.108 after 1978.

6 Extensions

For simplicity, we have assumed a linear effect of the effort on the redeployability of capital. The central argument is unaffected if any increasing, reasonably well behaved function is assumed instead. Since the model has equilibria whose allocations are inefficient, due to an endogenous market imperfection, other potential sources of inefficiency, such as increasing returns to scale, were left out of the picture. They can be introduced following Lucas (1988) without altering the gist of the paper. Leisure activities have been left out of the model, since it already contains alternative uses of time and effort, but can be added to it, without altering its picture. The population has been kept constant throughout. Population growth can be introduced without altering the picture, rescaling upwards the growth rate of GDP. A phenomenon known as demographic transition can be captured assuming, as in Hansen and Prescott (2002), that the growth rate of the population depends non-linearly on the living standards, represented by consumption per capita. The model can generate a demographic boom followed by the a demographic slow-down for growth miracles and happy de-growth economies, but not stagnant economies. Below, we pursue in greater detail the three following extensions. First, we suggest a way to capture differences in financial and technical development. Second, we consider deviations from perfect competition in the labor market. Finally, we introduce land and agriculture.

6.1 Financial vs Technical Development

So far, we have assumed that redeployability and pledgeability of assets are inestricably linked. However, this assumption can be relaxed. The impact of the effort u on the redeployability of capital can be disentangled from its impact on pledgeability, rewriting the budget constraint as $c_t + k_{t+1} + w_t l_t = y_t + \rho u_{t-1} k_t$, where the parameter ρ captures the technical framework of a country, which contributes to redeploying capital; and the collateral constraint as $w_t l_t \leq \theta u_{t-1} k_t$, where θ captures the legal and financial framework, which contributes to making capital pledgeable and seizable in the event of default. Economies with $\theta > \rho$ have legal and financial institutions that are better than their technical infrastructure, allowing to pledge to third parties even assets that are difficult to redeploy in production, while the reverse is true for economies with $\theta < \rho$, arguably the more common situation. The parameter θ does not affect the growth rate of mature economies, since their collateral constraint is not binding, while ρ affects positively their growth rate, since a more effective way of redeploying capital requires smaller investments in such activity, freeing resources for the accumulation of human capital. Economies that are in the constrained regime are, instead, affected by both parameters, provided $\theta \neq \rho$. Developing countries with $\theta < \rho$ would need to devote more resources to the pledgeability of capital, trying to enlarge their asset base to compensate for their worse institutional framework, at the cost of lower growth in the transitional phase, while developing countries with $\theta > \rho$ would need to devote less resources to the pledgeability of capital, enjoying higher growth in the transitional phase. The characterization of balanced equilibria is similar to the one provided above. Consistently with the view that more innovative projects and technologies are conceived as more difficult to redeploy and pledge as collateral, θ and ρ could also be thought of as being stochastic and negatively correlated with

TFP shocks. This would allow to relate real business cycles à la Prescott to financial business cycles. A technological innovation would make it more difficult to redeploy and pledge the assets used in the new production process, thus, triggering an increase in u, which lowers human capital accumulation and growth, to compensate for the decrease in θ and ρ. For a sufficiently large shock, it may happen that an economy that has already reached its unconstrained balanced path is sent back to a situation with constrained financial markets, with non-negligible consequences for both the growth rate of the economy and the allocation of resources.

6.2 Labor Market Imperfections

The wage has so far been determined competitively. Different ways of setting the wage can be captured as follows. Suppose the overall wage bill is represented by a continuously differentiable function $w(l)$, which is increasing in l with $w(0) = 0$, and is the outcome of a negotiation process between the entrepreneur and the worker. The determination of the effort u at an unconstrained equilibrium remains unaltered with respect to the competitive case. Hence, the growth rate of mature economies is not affected by the nature of the wage determination process. Developing economies are, instead, affected. From the modified equilibrium conditions, when the collateral constraint is binding, we obtain

$$u = \frac{w(l)/l}{w'(l) + w(l)/l} \frac{1+\delta}{\delta}.$$

There are three possible scenarios. If $w(l)/l = w'(l)$, the average and marginal wage are equal, the overall wage bill is linear in labor, and we are back in the competitive case, in which condition (25) holds. If $w(l)/l > w'(l)$, the average exceeds the marginal wage, i.e. the overall wage bill is concave. This is a situation in which the workers have some power to influence the wage determination and appropriate part of the surplus, pushing the wage above the competitive one. In this case, the entrepreneur will have to guarantee a larger amount $w(l)$ with uk relative to the competitive case, and will react increasing effort u, thus, also reducing human capital accumulation and growth. The opposite is true when $w(l)/l < w'(l)$, i.e. the wage bill is convex, and the entrepreneurs have the power to push the wage below the competitive level. In both cases, the presence of an extra imperfection would distort the allocation away from the second best frontier. To close the model, the wage bill may, for instance, be pinned down by generalized Nash bargaining, allowing to determine the effect of the worker's bargaining power on the growth rate of GDP. The allocation of labor can be made frictional assuming that there is a large number of entrepreneurs and workers which are randomly matched according to a matching function as in the labor search literature à la Diamond, Mortensen and Pissarides. The model would then feature equilibrium unemployment which would vary with the equilibrium regime and the growth rate of the economy.

6.3 Land and Agriculture

The reader may wonder what would happen to our basic trade-off if the agents have the opportunity to try to relax or circumvent credit constraints with another asset, in fixed supply, say, land, L, that can be used for both productive and collateral purposes. Suppose the entrepreneur can split optimally one extra unit of time to be devoted to productive effort, e_t, between two technologies, an agricultural one that uses land, and a manufacturing one that uses physical and human capital, both represented by Cobb-Douglas functions. Let y_t^a denote output in agriculture and y_t^m in manufacturing at any point in time. Assume for simplicity that the share of the entrepreneur's effort in the two technologies is the same. The price of land in units of the good at time t is p_t. The overall value of assets is now larger, being equal to the sum of undepreciated capital and the value of land. Hence, the budget constraint is now written as $c_t + k_{t+1} + p_t L_{t+1} + w_t l_t = y_t^m + y_t^a + u_{t-1} k_t + p_t L_t$, and the collateral constraint as $w_t l_t \leq u_{t-1} k_t + p_t L_t$. The choice of effort by the entrepreneur is determined by equating its marginal productivity in the two technologies, giving rise to a condition that represents the importance of agriculture relative to manufacturing, whereby the role of agriculture relative to manufacturing remains the same over time in stagnant economies, while its importance relative to manufacturing declines over time in growing economies, since both physical and human capital increase over time vis-à-vis a constant available amount of land. The price of land satisfies the no-arbitrage condition $p_{t+1} = u_t p_t$, which determines the evolution of the value of land as a function of effort, and implies that the the price of land remains constant in stagnant societies, while it declines over time in growing economies. The equation $p_{t+1} L_{t+1} y_t^m = y_t^a u_t k_{t+1}$, pins down the value of the stock of land as a function of the importance of output in the agricultural sector relative to manufacturing and undepreciated capital, and gives rise to the dynamic feedback between the price of land and the real allocation, discussed by Kiyotaki and Moore (1997). The determination of effort in mature economies is unaffected by the presence of land, since their collateral constraint is not binding. Instead, developing economies, whose credit markets are constrained, are affected by it. Using the modified equilibrium conditions, we obtain

$$u = \frac{y^m}{2y^m + y^a} \frac{1+\delta}{\delta}.$$

In developing economies with a negligible agricultural sector, the growth rate would be determined by (25) as before. On the other hand, developing economies with a larger agricultural sector, in which the value of land is higher, will need to devote less effort to redeployability, freeing resources for human capital accumulation and enjoying higher growth as a consequence. For this to be the case, property rights over land should be sufficiently well enforced to allow the agents to use it as collateral. The characterization is analogous to the case without land. There are still economies that remain stagnant and economies that grow, depending on the productivity of human capital and the rate of patience. In both cases, there are collateral constrained and unconstrained economies, depending on the initial availability of capital and land. In growing economies, as the stock of both physical and human capital grows, agriculture is progressively replaced by manufacturing with a concomitant decline in land values. In economies that grow sufficiently fast, the on-going process of financial deepening allows to compensate for the decline in land values with a larger capital stock to be used as collateral.

7 Policy Intervention

The equilibrium is always at least second best efficient, hence, no policy intervention by a Government with the same commitment abilities as the agents can improve the situation. Assuming that the Government has superior commitment abilities relative to the agents, there is, instead, scope for Government intervention when credit markets are not working smoothly, since the economy operates inefficiently relative to the first best frontier in the constrained regime. We consider two types of intervention, whereby the Government collects taxes to finance the emission of public debt or to establish a legal system.

7.1 Public Debt

First, suppose that the Government has the power to impose lump-sum taxes, T_t, and uses them to finance the emission of public debt in the form of one period bonds that can be acquired at the end of any period at a price π_t in units of the good and pay-off one unit of the good at the end of the following period. The bonds cannot be traded directly for any object before maturity, but can be used as collateral to obtain loans alongside capital. The stock of such bonds at time t is B_t. Only the entrepreneur participates in the bonds market, and his holdings at time t are b_t. The budget constraint of the Government is $\pi_t B_{t+1} + T_t = B_t$, since the new emission and taxation are used to finance the reimbursement of the previous bond issue. The budget constraint of the entrepreneur is now written as $c_t + k_{t+1} + \pi_t b_{t+1} + w_t l_t = y_t + u_{t-1} k_t + b_t - T_t$, and the collateral constraint as $w_t l_t \leq u_{t-1} k_t + b_t$, where the value of the bond holdings extend the amount of pledgeable resources. Relative to the case without bonds, there is an extra Euler equation for bond holdings, that, together with (18), allow to express the price of the bond as an inverse function of effort, u_t. Hence, an economy that grows at a faster rate will enjoy higher prices on its stock of Government bonds relative to economies that grow at slower rates. The determination of effort in mature economies is unaffected by the bond issue, since their collateral constraint is not binding. Instead, developing economies, whose credit markets are constrained, may benefit from a bond issue. With a positive (but not too large) stock of debt, the agents have extra collateral to be pledged and, thus, need to devote less effort to redeployability, freeing resources for human capital accumulation and enjoying higher growth as a consequence. By issuing public bonds, the Government provides the economy with extra commitment power, which is beneficial when the economy is collateral constrained.

7.2 Legal System

Suppose now that the Government does not have access to lump-sum taxation but has the power to tax income imposing a proportional tax, τ. The fiscal revenue, r_t, is used to finance a legal system with judges, public notaries and enforcement officials that may help the entrepreneur pledge (part of) his income. The Government balances its budget, hence, $r_t = \tau_t y_t$. The budget constraint of the entrepreneur becomes $c_t + k_{t+1} + w_t l_t = (1 - \tau_t)\, y_t + u_{t-1} k_t$, and the collateral constraint $w_t l_t \leq \varphi\,(1 - \tau_t)\, y_t + u_{t-1} k_t$, where φ is the fraction of after tax income that can be pledged thanks to the legal system of the country, and is a continuous function $\varphi = \varphi\,(r_t)$ of the fiscal revenue with $\varphi\,(0) = 0$ that is increasing at least for small revenues, so that small interventions may have the hope of being beneficial, while larger interventions may or may not help. Financing the legal system through taxation, the Government makes it possible to partially offset the enforcement and commitment problems that underlie the presence of the collateral constraint, thus, allowing a fraction of the entrepreneur's income to be pledged, which reduces the need to exert effort to redeploy capital, which, in turn, frees resources to be devoted to human capital accumulation, enhancing growth.

8 Conclusion

We have presented a model in which credit market imperfections induce a collateral constraint that can be relaxed investing resources in activities fostering asset redeployability and pledgeability. The resources devoted to such activities are diverted from human capital accumulation, potentially at the expense of economic growth. The model features endogenous structural change in the growth pattern. Economies characterized by a relatively efficient human capital accumulation technology and a small initial capital stock experience first a development stage with sustained growth and tight credit markets, and, eventually, enter a second stage, with lower growth but unhindered credit markets, as in the Uzawa-Lucas growth model. The liquidity premium for physical capital that emerges in the first stage leads to over-investment in physical capital that helps to relax the credit constraint and paves the way for the second stage. This growth pattern seems consistent with the available post WWII evidence, especially for some OECD and Asian countries. The model can be extended in several directions and delivers policy implications.

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A Appendix

A.1 Proofs

Proof of Lemma 1. By equations (9) delayed one period and (12), rearranging we obtain

$$(1-\alpha)\,y_{t+1} - w_{t+1}l_{t+1} = \left(\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta} - \alpha\frac{y_{t+1}}{k_{t+1}} - u_t\right)\frac{w_{t+1}l_{t+1}}{u_t}.$$

Hence, $\frac{1}{\beta} - \alpha\frac{y_{t+1}}{k_{t+1}} - u_t > 0 \Leftrightarrow (1-\alpha)\,y_{t+1} > w_{t+1}l_{t+1}$, which is equivalent to $\zeta_{t+1} > 0$, by (9).■

Proof of Lemma 2. Suppose these terms are equal. This is consistent with the BCCE system only if $(1-\alpha)\,\delta = \beta$, which violates (5).■

Proof of Proposition 1. Substitute (21) and (22) into (20), obtaining one equation in u. The solution is either $\overline{u} = \min\left\{\frac{1+\delta}{2\delta}, 1\right\}$ in the constrained case, or $\widetilde{u} = \min\left\{\frac{(1-\alpha)\delta-\beta\alpha}{(1-\alpha)(\beta+\delta)-\beta\alpha}\frac{1+\delta}{\delta}, 1\right\}$, in the unconstrained case, with $\widetilde{u} > 0$ under (5). By Lemma 2 only these two situations are possible. The rest of the system determines uniquely k, c and $\widehat{c}$.■

Proof of Proposition 2. *a.* $\overline{u} < 1 \Leftrightarrow \delta > 1$; *b.* $\widetilde{u} < 1 \Leftrightarrow \beta > \frac{(1-\alpha)\delta}{(1-\alpha)\delta+\alpha}$.■

Proof of Proposition 3. By (24), the collateral constraint binds at $t = 0$ iff $(1-\alpha)\,y_0 \geq \widehat{c}_0 = k_0 \Leftrightarrow k_0 \leq h_0\,(1-\alpha)^{\frac{1}{1-\alpha}}$.■

Proof of Proposition 4. To check whether a constrained BCCE with growth becomes unconstrained at some finite date, we need to see whether there exists a finite $t > 0$, such that

$$(1-\alpha)\,\widetilde{y}_t = \overline{u}\overline{k}_t,$$

where, on the LHS there is the unconstrained equilibrium value $\widetilde{y}_t$, while on the RHS the constrained equilibrium value $\overline{u}\overline{k}_t$. Therefore, if the equation

$$(1-\alpha)\left(\frac{\beta(1-\alpha)(1+\delta)}{(1-\alpha)(\beta+\delta)-\beta\alpha}\right)^t h_0^{1-\alpha}k_0^{\alpha}=\frac{1+\delta}{2\delta}\left(\frac{1+\delta}{2}\right)^t k_0, \tag{38}$$

has a solution $t^* \in (0,\infty)$, the initially constrained CCE with growth becomes unconstrained at $\lceil t^* \rceil$, the closest integer not smaller than t^*, otherwise the CCE remains indefinitely constrained. Solve (38) for t, obtaining

$$t=\frac{\ln\left((1-\alpha)h_0^{1-\alpha}k_0^{\alpha-1}\right)-\ln\frac{1+\delta}{2\delta}}{\ln\left(\frac{1+\delta}{2}\right)-\ln\left(\frac{\beta(1-\alpha)(1+\delta)}{(1-\alpha)(\beta+\delta)-\beta\alpha}\right)}, \tag{39}$$

where $(1-\alpha)h_0^{1-\alpha}k_0^{\alpha-1} > 1$ and $\frac{1+\delta}{2\delta} < 1$, at an initially constrained CCE with growth. Hence, (39), which is finite, is strictly positive iff $\frac{1+\delta}{2} > \frac{\beta(1-\alpha)(1+\delta)}{(1-\alpha)(\beta+\delta)-\beta\alpha}$, which holds under (5).■

Proof of Proposition 5. In a stagnant CCE, $u=1$ and $k=k_0$ always. The CCE is always constrained if $k_0 < h_0(1-\alpha)^{\frac{1}{1-\alpha}}$, always unconstrained otherwise.■

Proof of Proposition 6 In the constrained regime, by (16) and (17), effort $u_t = \min\left\{\frac{1+\delta}{2\delta},1\right\}$ at all times. The remaining variables are determined as before.■

Proof of Proposition 7. Linearize (28) and (29) around the BCCE, taking first differences, $z_{t+1}-z_t$ and u_t-u_{t-1}. Solve the characteristic equation of the system,

$$(F_z-1-\xi)(G_u-1-\xi)-F_uG_z=0,$$

where H_v is the partial derivative of a function H wrt v evaluated at the BCCE, to find the eigenvalues $\xi_1=\frac{(1-\alpha)\delta+\beta(1-2\alpha)}{\alpha\delta}$ and $\xi_2=\frac{1-\beta}{\beta}$, with $\xi_1>1$ and $\xi_2<1 \Leftrightarrow \beta>\frac{1}{2}$.■

Proof of Proposition 8. In the constrained regime, at the CCE $(1-\alpha)y_t > \widehat{c}_t = w_tl_t = \frac{-v_l(\widehat{c}_t,l_t)l_t}{v_c(\widehat{c}_t,l_t)}$, hence, the CCE allocation violates (31); in the unconstrained regime, the CCE satisfies all the equations (31)-(35), provided μ is chosen appropriately. ■

Table 1: Summary statistics

Description	Source	Mean	Std. Dev.	Min	Max	Obs.
Net growth rate of RGDP	PWT: `rgdpna`	0.035	0.038	-0.349	0.301	1,780
Net growth rate of per capita RGDP	PWT: `rgdpna`, `pop`	0.026	0.036	-0.34	0.163	1,780
Gross growth rate of HC index	PWT: `hc`	0.007	0.006	-0.042	0.042	1,780
Effort in human capital	PWT: `labsh`, `rkna`, `rdgpna`, `hc`	0.003	0.018	-0.028	0.279	1,780
Gross growth rate of per capita consumption	PWT: `rconna`, `pop`	1.024	0.034	0.652	1.229	1,780
Real interest rate	World Bank	0.211	0.243	0.054	2.813	1,475
Liquidity premium	PWT: `labsh`, `rkna`, `rdgpna`, `hc`	-0.074	0.092	-0.758	0.166	1,780
Credit to private sector	World Bank	0.662	0.442	0.005	2.278	1,560

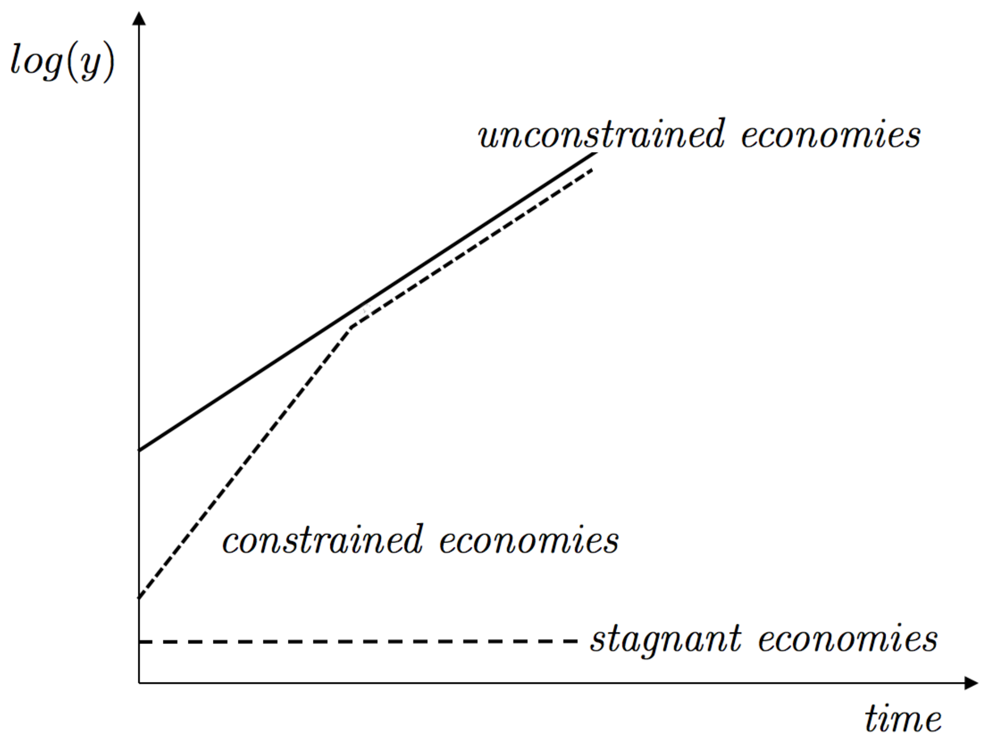

Figure 1: Balanced Growth Paths

Table 2: 10-years average real GDP growth and the capital output ratio

Country	**OLS estimate**
Australia	0.191 ***
Austria	0.092 ***
Belgium	0.073 ***
Canada	0.126 ***
Chile	0.225 ***
Czech Republic	0.050 ***
Denmark	0.079 ***
Estonia	0.114 ***
Finland	0.084 ***
France	0.092 ***
Germany	0.135 ***
Greece	0.089 ***
Hungary	0.045 ***
Ireland	0.136 ***
Israel	0.234 ***
Italy	0.080 ***
Japan	0.111 **
Latvia	0.066 ***
Luxembourg	0.066 ***
Mexico	0.203 ***
Netherlands	0.092 ***
New Zealand	0.126 ***
Norway	0.154 ***
Poland	0.143 ***
Portugal	0.085 ***
Republic of Korea	0.353 ***
Slovakia	0.133 ***
Slovenia	0.072 ***
Spain	0.112 ***
Sweden	0.072 ***
Switzerland	0.073 ***
United Kingdom	0.060 ***
Unites States	0.118 ***

Significance levels: : 10% **: 5% ***: 1%*

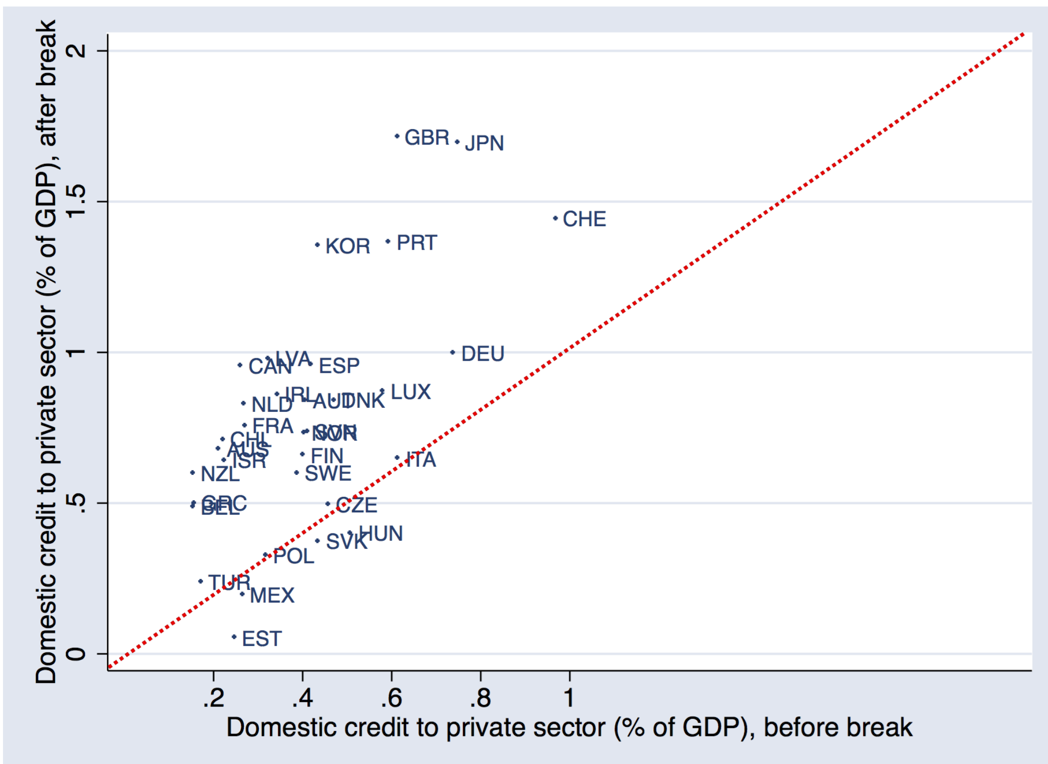

Figure 2: Credit to private sector and real GDP growth rates